Tropical-cyclone intensification and predictability in a minimal three dimensional model

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Abstract: We investigate amplification and predictability of tropical cyclones in the context of a minimal, three-dimensional numerical model. In the prototype problem for intensification starting with a tropical storm strength vortex in a quiescent environment on an $\beta$-plane, the emergent flow in the inner region of the vortex becomes highly asymmetric and dominated by deep convective vortex structures, even though the problem as posed is essentially axisymmetric. The details of the intensification process including the asymmetric structures that develop are highly sensitive to small perturbations in the low-level moisture field at the initial time. This sensitivity is manifest in a significant spread in the intensity of vortices from an ensemble of calculations in which random moisture perturbations are added in the lowest model level. Similar experiments are carried out on a $f$-plane and in the case where there is an anticyclonic shear flow at upper levels. The former set show no significant difference from the $f$-plane calculations in the evolution of intensity, but the latter set show a significantly weaker vortex, contrary to a broadly-held hypothesis that upper-level outflow channels are favourable to intensification.

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1 Introduction

This paper revisits the prototype problem for tropical-cyclone intensification, which considers the evolution of a prescribed, initially cloud-free, axisymmetric, baroclinic vortex over a warm ocean on an $f$-plane. This problem has been the subject of many numerical studies over the years and these can be classified into the following types:

- Hydrostatic axisymmetric models with cumulus parameterization (e.g. Ooyama 1969, Emanuel, 1989, 1995, Nguyen et al. 2002),
- Hydrostatic three-dimensional models with explicit microphysics (e.g., Wang 2001, 2002a, 2002b),
- Non-hydrostatic axisymmetric cloud models (e.g., Willoughby et al. 1984, Rotunno and Emanuel 1987, Persing and Montgomery 2003),
- Non-hydrostatic three-dimensional cloud models (Montgomery et al. 2006),
- Non-hydrostatic three-dimensional mesoscale models (Nguyen et al. 2007, henceforth NSM07).

A significant feature of all the three-dimensional calculations is the emergence of flow asymmetries, despite the symmetry in the formulation except, of course, in the representation of a circular flow on a square grid.

The related problem on a $\beta$-plane is the prototype problem for tropical cyclone motion and has been a topic of much study also (e.g. Flatau et al. 1994, Dengler and Reeder 1997, Wang and Holland 1996, Ritchie and Frank 2007, NSM07). However, in this case the development of flow asymmetries is expected from the lack of symmetry implied by $\beta$. Indeed, it is the wavenumber-one component of these asymmetries that leads to the well-known northwestward motion of the vortex.

The occurrence of asymmetries in $f$-plane calculations was the focus of a study by Zhu and Smith (2003, henceforth ZSU), Zhu and Smith (2002), and many others. The results suggested that the early development of asymmetries was exacerbated by a computational mode in temperature that is generated with the onset of convection. This computational mode does not occur with the CP-grid. However, even with this grid, ZSU found a weak wavenumber-4 asymmetry in the temperature and velocity fields in the lower to middle troposphere. It turns out that these asymmetries are associated with a significant asymmetry in the relative vorticity in this layer.

In recent calculations using a multi-layer model, NSM07 have shown that the flow asymmetries that

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\textsuperscript{*}There have been many more studies of this problem in a barotropic context, but our interest here is focused on baroclinic models with at least three vertical levels to represent the effects of deep convection.
develop are highly sensitive to the surface moisture distribution. When a random moisture perturbation is added in the boundary layer at the initial time, even with a magnitude that is below the accuracy with which moisture is normally measured, the pattern of evolution of the flow asymmetries is dramatically changed and no two such calculations are alike in detail. The same is true also of calculations on a $\beta$-plane, at least in the inner-core region of the vortex, within 100-200 km from the centre. Nevertheless the large-scale $\beta$-gyre asymmetries remain coherent and are similar in each realization so that they survive when one calculates the ensemble mean. The implication is that the inner-core asymmetries on the $f$- and $\beta$-plane result from the onset of model convection and that, like convection in the atmosphere, they have a degree of randomness, being highly sensitive to small-scale inhomogeneities in the low-level moisture distribution which is a well-known characteristic of the real atmosphere (e.g. Weckwerth, 2000).

The recognition that the inner-core asymmetries are associated with convective processes motivated us to revisit the prototype intensification problem using the minimal three-dimensional hurricane model described by ZS03. The re-examination is important because the model is potentially useful for developing a basic understanding of tropical-cyclone dynamics and has been used to study the inner-core asymmetries when the model is coupled to a simple ocean model (Zhu et al. 2004). The present version of the model has a higher horizontal resolution than that of ZS03 (10 km instead of 20 km) and has a simple explicit representation of moist processes, but no convective parameterization scheme. In particular we investigate the structure and evolution of the flow asymmetries and the range of variability of the vortex intensity and structure when the boundary-layer moisture is slightly perturbed. This study complements that of NSM07.

The random nature of the inner-core asymmetries calls for a new methodology to assess differences between two particular flow configurations, because the results of a single deterministic calculation in each configuration may be unrepresentative of a model ensemble in each configuration. That means one needs to compare the ensemble means of suitably perturbed ensembles of the two configurations. We illustrate this methodology in two examples of idealized flows.

The paper is structured as follows. In section 2 we give a brief description of the model. Then, in sections 3 and 4 we compare ensemble calculations of vortex evolution on an $f$-plane and on $\beta$-plane, where the ensembles are generated by adding small moisture perturbations at low levels. In section 5 we apply the foregoing methodology to explore the effects of adding an anticyclonic shear flow to the upper model level. This problem is of interest because it has often been supposed that the presence of outflow channels in the upper troposphere is conducive to tropical-cyclone intensification (see e.g. Sadler 1976, 1978; Holland and Merrill 1984). In the calculations shown here, the presence of the shear flow is found to be detrimental to intensification. The conclusions are given in section 6.

2 The model

The minimal hurricane model is that described in ZS03. It is fully three-dimensional and based on the hydrostatic primitive equations formulated in $\sigma$-coordinates $(x, y, \sigma)$, where $\sigma = (p - p_{top})/(p_{s} - p_{top})$, $p$ being the pressure, $p_{s}$ the surface pressure, and $p_{top}$ the pressure at the top of the domain. The vertical differencing is carried out on a Charney-Phillips grid (CP-grid) shown in Fig. 1. The model equations and the advantages of CP-grid are discussed in ZS03. The model is divided vertically into four layers of unequal depth in $\sigma$: the lowest layer has depth 0.1 and the three layers above have depths 0.3. Some of the calculations are carried out on an $f$ plane and some on a $\beta$-plane. Newtonian cooling is used to represent the effect of radiative cooling. Simply the moist static energy is relaxed to its environmental value on a prescribed timescale, $\tau$. The turbulent flux of momentum to the sea surface and the fluxes of sensible heat and water vapour from the surface are represented by bulk aerodynamic formulae. The surface drag coefficient, $C_{D}$, is calculated from the formula used by Shapiro (1992):

$$C_{D} = (1.024 + 0.05366 R_{F} |u_{0}|) \times 10^{-3},$$

where $R_{F} = 0.8$ is used to reduce the boundary layer wind, $u_{0}$, to the 10-m level. The surface exchange coefficients for moisture and heat are assumed to be equal to each other and to $C_{D}$.

Moist processes are represented in this version of the model by the simplest explicit scheme in which condensation occurs when the air becomes supersaturated at a grid point. At such points the excess water is assumed to precipitate out and the latent heat of the condensed water goes to increase the air temperature. The scheme, which is described in detail by ZSU, involves an iterative procedure. There is no parameterization of deep convection as in the original model.

The initial vortex is axisymmetric and baroclinic. The tangential wind profile is that used by Smith et al. (1990), but with different parameters: the maximum tangential wind speed is 15 m s$^{-1}$ at level-4 at a radius of 120 km and

![Figure 1. The Charney-Phillips vertical grid used in the model.](Image)
its magnitude reduces to zero at level-1. The initial mass and geopotential fields are obtained by solving the inverse balance equation in the same way as Kurihara and Bender (1980). The far-field temperature, geopotential height and humidity structure are based on the mean West Indies sounding (Jordan 1957). The horizontal grid spacing of the model is 10 km and the integration time step is 3 s. The ocean surface temperature is 26.3°C. The experiments are performed on an $f$-plane at 20°N. As in ZS03, the value of $\tau$ is taken to be 0.1 K (day)$^{-1}$. The vortex centre is obtained by calculating a centre of weighted relative vorticity evaluated over the area 400 km $\times$ 400 km centred on the location of maximum relative vorticity.

Three main sets of calculations are carried out. The first set, the control set, consists of a standard calculation and 10 additional ones in which the moisture fields throughout the innermost domain are randomly perturbed at the surface (i.e. at level-4$\frac{1}{2}$ and level-3$\frac{1}{2}$). In the latter, the magnitude of the mixing ratio perturbation lies in the range of ($-0.5$ g kg$^{-1}$, $0.5$ g kg$^{-1}$). The integration time is 96 h. All three sets of runs are summarized in Table 1.

### Table I. The numerical experiments.

<table>
<thead>
<tr>
<th>No.</th>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A0</td>
<td>Control</td>
<td>Control experiment on an $f$-plane, quiescent environment.</td>
</tr>
<tr>
<td>A1-A10</td>
<td>$f$-plane ensemble</td>
<td>Ten ensemble members. See text for details.</td>
</tr>
<tr>
<td>B0-B10</td>
<td>$\beta$-plane ensemble</td>
<td>Same as A0-A10, except on $\beta$ plane</td>
</tr>
<tr>
<td>C0-C10</td>
<td>$f$-plane ensemble</td>
<td>Same as A0-A10, except with an anticyclonic upper-level shear flow</td>
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3 Experiments on an $f$-plane

3.1 Vortex evolution in the control experiment

We describe first the development of the initial axisymmetric vortex in the $f$-plane experiments (A0-A10 in Table 1). Figure 3.1 shows time-series of the maximum, azimuthally-averaged, total wind speed near the top of the boundary layer ($\sigma = 0.95$), during a 96 h integration in these experiments. The average is performed about the centroid of the vertical component of relative vorticity. We call this average wind speed $V_{\text{max}}$ and use it to characterize the intensity of the vortex at any given time. As in many previous calculations relating to the present thought experiment, the vortex evolution begins with a gestation period during which it slowly decays due to surface friction. However, the boundary layer progressively moistens due to evaporation from the underlying sea surface. The imposition of friction from the initial instant leads to a breakdown of gradient-wind balance in a shallow boundary layer and thereby to net inward force (e.g. Smith 1968). This force drives inflow in the boundary layer, which through continuity is accompanied by outflow above the layer. The early decay of the vortex is a result of this outflow, combined with the conservation of absolute angular momentum as air parcels move outwards.

The moist inflowing air cools as it rises out of the boundary layer and expands. Eventually condensation occurs in some grid columns near the radius of maximum tangential wind speed, $r_m$. Then, as shown below, “model” convective clouds develop rapidly and soon afterwards the vortex rapidly intensifies.

The changes in vortex structure during the later period of rapid intensification, between 42 and 48 h, are exemplified by those in the control calculation A0. In this experiment there is no moisture perturbation in the boundary layer. The evolution is highlighted by the fields of vertical velocity at $\sigma = 0.9$ (Fig. 3), the vertical component of relative vorticity, (Fig. 4), and total wind speed at $\sigma = 0.95$ (Fig. 5). At 42 h, there are four "convective cells" located at the corners of a square, located at a radius of about 50 km from the vortex centre (Fig. 3a). The pattern of evolution of the relative vorticity (Fig. 4) and total wind field (Fig. 5) are similar to that of the vertical velocity. Moreover, comparing Figs. 3 and 4 it is seen that the updrafts possess significant local rotation. The rotation is locally enhanced as relative vorticity is stretched and amplified. As time proceeds the rotating updrafts circle around the vortex axis cyclonically and spiral inwards so that they progressively interact and merge (Fig. 4). Thus although the calculation begins with an axisymmetric vortex in a quiescent environment on an $f$-plane, the vortex intensification process is intrinsically non-axisymmetric.
The foregoing evolution is similar to that in the high-resolution cloud-resolving vortex simulations described by Hendricks, et al. (2004), Montgomery et al. (2006), and NSM07. These simulations, which used horizontal grid spacings of between 5 km and 1.67 km, showed that intense vorticity anomalies are produced by buoyant cores growing in the rotation-rich environment of an incipient vortex and that these convective cores subsequently undergo merger and axisymmetrization. Following these authors we refer to the rotating updrafts as "vortical hot towers", or VHTs. Of course, with a horizontal grid spacing of 10 km, the VHTs in our model are not adequately resolved and they are hydrostatic.

After a slight decline in intensity between 53 h and 56 h, the vortex begins again to intensify to reach what might be described as a quasi-steady state, the period after about 60 h (Fig. 3.1). The mean intensity between 60 h and 96 h is 40.3 m s$^{-1}$, but there are significant fluctuations during this period, the standard deviation being 3.8 m s$^{-1}$. As in the calculations of NSM07, it is found that these fluctuations are associated with major convective outbreaks and axisymmetrization of the vortex is never complete. The question is: how significant are the individual fluctuations in intensity? This question is addressed in the next section.

### 3.2 Vortex evolution in the ensemble experiments

Figure 3.1 shows also the evolution of intensity of the ensemble calculations A01-A10, in which the low level moisture field is randomly perturbed. It is notable that the onset of the rapid intensification occurs earlier than in the control calculation A0 in all these calculations. While this result may be surprising at first sight, the reason is that although the moisture perturbation in a particular ensemble member is spatially random, there are always some positive perturbation that initiate deep convection earlier. In fact, grid-scale saturation at $\sigma = 0.9$ occurs at about 18 h in all the ensembles, which is about 2 h earlier than in the control run and this leads to a correspondingly earlier development of the VHTs. We see this in the pattern of relative vorticity in the ensemble members A01-A03 at 38 h (Fig. 6), the fields similar to that in the control experiment at 40 h. While the differences between ensemble members at this time are slight, the solutions rapidly diverge as shown in the corresponding vorticity fields at 45 h in A0 and 43 h in A1-A3 shown in Fig. 7. These differences provide an explanation for the differences in intensity between the control experiment and the ensembles evident in Fig. 3.1 and provides an answer to the question raised in subsection 3.1. It is manifestly clear that the differences in intensity are not significant: rather they are random features associated with random perturbations of low-level moisture.

### 4 Experiments on a $\beta$-plane

At this stage it is of interest to examine how the evolution of the flow asymmetries is affected by the presence of a $\beta$-effect, the relevant question being: to what extent does the azimuthal wavenumber-one asymmetry imposed by $\beta$ impose a stamp on the asymmetries that develop? For this reason we repeated the ensemble $f$-plane experiments described in section 3.1 for the $\beta$-plane. These experiments are designated B0-B10 in Table 1. A time-series of the ensemble average of the azimuthally-averaged total wind speed maxima in the boundary layer for experiments A0-A10 is compared with that for the experiments B0-B10 in Fig. 4. There is virtually no difference between the two curves until about 40 h and there is no significant difference after about 63 h when the two curves lie well within one standard deviation of the variability in each ensemble (indicated by the vertical bars in Fig. 4). In the intervening period, which marks the second part of the period of rapid intensification, the intensity in the ensemble mean intensity on the $f$-plane is a little larger, by up to 5 m s$^{-1}$, compared with the mean intensity on the $\beta$-plane. Even then the deviation hardly exceeds one standard deviation of the variability in each ensemble. We conclude that there is no significant difference between the intensity in the $f$- and $\beta$-plane calculations. Note, however, that it is necessary to carry out ensemble calculations to demonstrate this lack of a significant difference: had we chosen to compare two ensemble members, one from each set of calculations, we may have arrived at a different conclusion. This possibility is indicated by the substantial variation in intensity between individual members in the case of the $f$-plane calculations shown in Fig. 3.1, a result that is true also of those on the $\beta$-plane.

The inner-core asymmetries in the $\beta$-plane calculations are again dominated by random "model" convective bursts as in the $f$-plane experiments and do not show up as a coherent asymmetry in the ensemble mean. This feature is exemplified by the azimuthal wavenumber-one component of relative vorticity, fields of which are shown in Fig. 9. Nevertheless, a coherent large-scale asymmetry is apparent in the ensemble average fields and corresponds with the associated $\beta$-gyres familiar in in barotropic calculations (see e.g. Fiorino and Elsberry 1989, Smith et al. 1990, Smith and Ulrich, 1990, 1993). Note that the strength of these asymmetries increases with time during the 12 h period from 48 h to 60 h and their scale increases as would be expected from barotropic theory (Smith and Ulrich 1993). A similar result was obtained in the high-resolution calculations of NSM07.

Our conclusions here differ significantly from those of a very recent paper by Ritchie and Frank (2007). They compared two deterministic experiments of a similar type to ours using a version of the Pennsylvania State University-National Center for Atmospheric Research fifth-generation Mesoscale Model (MM5) with 5 km horizontal resolution. They found that the vortex on the $\beta$-plane "quickly develops a persistent wavenumber-one asymmetry in its inner core". Based on our ensemble experiments and those of NSM07, we question the validity.
Figure 3. Vertical velocity fields of the control experiment A0 at (a) 42 h, (b) 44 h, (c) 46 h, and (d) 48 h. Contour interval is $0.5 \times 10^{-5}$ s$^{-1}$ for positive values (in red) and $1.0 \times 10^{-5}$ s$^{-1}$ for negative values (in blue). The negative values indicate ascending motions. The zero contour is not plotted.

Figure 4. The vertical component of the relative vorticity fields of the control experiment A0 at (a) 42 h, (b) 44 h, (c) 46 h, and (d) 48 h. Contour interval is $0.25 \times 10^{-5}$ s$^{-1}$ for positive values (in red) and $0.05 \times 10^{-5}$ s$^{-1}$ for negative values (in blue). The zero contour is not plotted.
Figure 5. Total wind fields of the control experiment A0 at (a) 42 h, (b) 44 h, (c) 46 h, and (d) 48 h. Wind speeds higher than 30 m s\(^{-1}\) are coloured red and the contour interval is 5 m s\(^{-1}\).

Figure 6. Vertical component of the relative vorticity in the (a) control experiment A0 at 40 h, and in three of the ensemble experiments (b) A01, (c) A02, and (d) A03 at 38 h. Contour interval is 0.25 \(\times 10^{-5}\) s\(^{-1}\) for positive values (in red) and 0.05 \(\times 10^{-5}\) s\(^{-1}\) for negative values (in blue). The zero contour is not plotted.
of this conclusion. Unfortunately, it is not possible to compare the details of vortex evolution in our control experiments with theirs as they show only coarse-grain, horizontal fields of potential vorticity and only at the mature stage of vortex development. Nevertheless, we note that they found little difference in the intensity between their $f$- and $\beta$-plane experiments, consistent with our results.

5 Experiments with an upper-level anticyclonic shear flow on an $f$-plane

It is commonly believed that a favourable condition for tropical cyclone intensification is the existence of so-called outflow channels in the upper troposphere that provide a means of assisting the removal of air that rises in the eyewall. As far as we are aware, this idea was first put forward by Sadler (1976) and it seems to have gained much credence amongst forecasters, some of whom see it as a mechanism for "sucking" air upwards in the storm and thereby enhancing the secondary circulation as well as the low-level convergence. However, one might argue that the deep convection is a response to conditional instability and that observed outflow channels are generated by the inner-core convection, itself. This idea is supported by the scale analysis of tropical motions by Charney (1963), which shows that in the absence of diabatic forcing, synoptic-scale flows are horizontally non-divergent, a finding that would apply specifically to outflow channels.

To our knowledge these ideas have not been tested using numerical models and such is the motivation of this section. Our plan is to use the minimal model and the ensemble methodology described above to test the hypothesis that the presence of outflow channels is favourable to intensification. To do this we carried out an additional set
of experiments identical to A0-A10, except that an upper-
level anticyclonic shear flow was imposed at the model-
level $\sigma = 0.15$, where there is initially no motion in the
previous experiments. We refer to these experiments as
C0-C10. The meridional wind shear

$$\frac{du}{dy} = 2.5 \times 10^{-6} \text{sech}^2 \left( \frac{y}{400} \right) \text{ s}^{-1},$$

where $y$ is the meridional coordinate in km. Then the
wind speed is 10 m s$^{-1}$ at $y = \pm 1000$ km. We ask now
the question: does the presence of the shear flow enhance
the intensification rate of the model tropical-cyclone?

Figure 10 shows time-series of the ensemble average of
the azimuthally-averaged total wind speed maximum
in the $f$-plane experiments A0-A10 (red line) and shear-flow experiments C0-C10 (blue line). The corresponding vertical bars mark the standard deviation among
ensemble members at given time.

36 h, but the intensification begins slightly earlier in
the case of a quiescent environment and the subsequent
intensification is more rapid. In fact, during the rapid
intensification phase, between 36 and 72 h, the mean
intensity in the cases with "favourable" outflow channels
is less than that in the case of a quiescent environment
by up to 20 m s$^{-1}$. During the mature stage, after about
72 h, the difference is only about 5 m s$^{-1}$. The spread in
intensity (the vertical bars in Fig. 10) is similar in both
sets of experiments, being higher for some periods and less
for others until 60 h. Afterwards the spread is larger in the
shear-experiment set than in the $f$-plane set. The reason
for these differences can be traced to the fact that the
upper-level shear flow is associated through thermal wind
balance with a warm anomaly over the cyclone core. This
warm anomaly reduces the degree of convective instability
created by surface evaporation and leads to a weaker
secondary circulation within the cyclone. In other words,
these simple idealized calculations, at least, do not support
the hypothesis that outflow channels are favourable to
intensification.

6 Conclusions

We have shown that the predictability of hurricane inten-
sity using a minimal hurricane model is limited by the sen-
sitivity of the solutions to small, but random perturbations
to the low-level moisture field. The results are in line with
those of NSM07, who used a more complex, although also
considerably idealized nonhydrostatic model.

The sensitivity to random moisture perturbations is
larger with the 10 km grid size used here compared with
the 5 km resolution grid size used by NSM07, presumably
because the implied horizontal scale of moisture fluctua-
tions is larger. This sensitivity is quantified in terms of
the standard deviation of the intensity of a set of ensemble
calculations.
Together with that of NSM07, our study represents a new approach to understanding hurricane dynamics using models. It adopts the view that single deterministic calculations may have features that are not significant when one takes into account the variability associated with the uncertainty in the low-level moisture distribution. In this view, only features that survive in an ensemble mean can be regarded as robust.

We showed two examples of this approach. One in a comparison of vortex evolution on an $f$-plane with that on a $\beta$-plane and the other examining the role of apparently favourable outflow channels. In the latter case, the presence outflow channels was found to be detrimental to intensification because the warm upper-level thermal anomaly between them has a stabilizing effect on deep convection in the intensifying vortex.

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References


