Tropical cyclone life cycle in a three-dimensional numerical simulation: Part II: Unbalanced and non-axisymmetric dynamical processes

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Abstract:

The high-resolution, three-dimensional, life-cycle simulation described in Part I is used to explore further aspects of tropical cyclone spin up and early maturity in the framework of the rotating-convection paradigm. Unbalanced and non-axisymmetric processes are a prominent feature of the spin up process. The analyses reveal a reservoir of air with high equivalent potential temperature in the low-level eye region and indicate the importance of this reservoir in supporting convective instability necessary for vortex spin up. The analyses show also that the ability of inflowing air to access this reservoir is linked to the symmetric and asymmetric boundary layer dynamics of the vortex. Analyses of the radial force field provide an improved understanding of the process of spin up within the boundary layer and eyewall. They provide also an explanation for inflow jets sandwiching the upper tropospheric outflow layer that are frequently found in numerical model simulations. Overall, the results point to the significant limitations of a purely axisymmetric and/or balance description of tropical cyclone dynamics.

KEY WORDS Hurricane; tropical cyclone; typhoon; boundary layer; vortex intensification

1 Introduction

The problem of understanding tropical cyclone intensity change has been at the cutting edge of research in recent years, especially in the context of the rapid intensification or rapid decay of storms. This is because of the continued challenges in forecasting intensity change, a phenomenon which involves processes with scales spanning many orders of magnitude. The findings of Persing \textit{et al.} (2013) indicate that strictly axisymmetric models which represent convection in the form of axisymmetric rings, have inherent limitations for understanding the intensification process. For this reason, a paradigm for tropical cyclone intensification that incorporates the three-dimensional nature of deep convection is required. Such a paradigm has been developed over the last decade or more, the so-called rotating-convection paradigm. In this companion study (Part II), we explore further aspects of the vortex evolution in terms of the rotating-convection paradigm, based on the simulation of Part I.

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After reviewing briefly the vortex evolution (section 2) we go on to examine the low-level wind structure in the inner-core region highlighting salient features of the boundary layer dynamics including the contribution of the wind asymmetries to the maximum wind speed (section 3). In section 4 we examine the thermodynamics of vortex spin up with a special focus on the mean and asymmetric structure of equivalent potential temperature, $\theta_e$, and the role of the boundary layer in supporting this structure. In section 5 we quantify the impact of the eddy and wave processes discussed above on the azimuthal-mean tangential and radial wind tendencies during vortex spin up and during the mature phase. The conclusions are presented in section 6.

2 Some aspects of vortex evolution

To provide a context for the analysis of mean and eddy effects we summarize briefly the time evolution of the vortex during its life cycle and present characteristics of the low-level flow field during the spin up phase. We choose a cylindrical polar coordinate system $(r, \lambda, z)$, where $r$ is the radius, $\lambda$ is the azimuth and $z$ is the height, and corresponding velocity components $(u, v, w)$. The axis of coordinates is located at the surface pressure minimum\(^1\).

2.1 Time series of $v_{\text{max}}$

Figure 1 shows a time series of the maximum azimuthally-averaged tangential wind speed ($v_{\text{max}}$) and maximum total wind speed ($vT_{\text{max}}$) during the 16-day integration period with various phases of evolution identified. Following a brief gestation period lasting about 24 h, the vortex begins a period of rapid intensification (RI), lasting from about 24 to 72 h. About 12 hours later, the vortex reaches its mature stage, with a maximum intensity of about 85 m s\(^{-1}\). The mature stage lasts approximately 4 days and from day 7 onwards the vortex progressively decays.

As would be anticipated, $vT_{\text{max}}$ is generally larger than $v_{\text{max}}$ and $vT_{\text{max}}$ begins to increase as soon as deep convection begins to form, well within the gestation period. Typically $vT_{\text{max}}$ is up to 10 m s\(^{-1}\) larger than $v_{\text{max}}$, and the difference between these two quantities is a metric that approximately quantifies the convective asymmetries. The heights of both $v_{\text{max}}$ and $vT_{\text{max}}$ are typically below 1 km, within the frictional boundary layer. The intensity after 16 days as measured by $v_{\text{max}}$ is essentially the same as the starting intensity (15 m s\(^{-1}\)). Part I shows further details of the evolution, both from a three-dimensional and azimuthally-averaged perspective.

\(^1\)The vortex centre location is obtained by finding the minimum in a smoothed pressure field, subject to the requirement that the vortex does not move more than 20 km in a single output time. This requirement prevents the centre-finding algorithm from locking on to a localized region of strong convection. Since there is no ambient vertical shear in the problem, it is reasonable to take the centre location to be independent of height.

3 Low-level radial wind profiles

Whereas Figure 1 shows the time series of maximum wind metrics during the simulated vortex lifecycle, it is of interest to examine the low-level wind structure at particular times. Figure 2 shows the radial structure of the low-level wind field, including the azimuthally-averaged tangential velocity component together with the corresponding gradient wind\(^2\) at the height of the tangent wind maximum (700 m) together with the similarly averaged radial velocity component at the low level model level (25 m) at 48 h during the RI phase and at 96 h, at the end of this phase. Shown also are the radial profiles of maximum total wind speed $vT = \max(\sqrt{u^2 + v^2})$ and maximum total eddy wind speed $vtp$ in the lowest 1 km at each radius. The total eddy wind speed is defined as $\max(\sqrt{u'^2 + v'^2})$, where a prime denotes the difference between the total and azimuthally-averaged variable.

At 48 h, the tangential wind $v$ is approximately equal to the gradient wind, $v_{\text{gr}}$, beyond a radius of 45 km, but there are small amplitude fluctuations of the difference. The maximum tangential wind is 55.2 m s\(^{-1}\) and occurs at a radius, $r_{\text{max}}$, of 30 km, while the maximum gradient wind is 46.5 m s\(^{-1}\) and occurs at a radius of 34 km. Between radii of 20 km and 45 km, $v$ exceeds $v_{\text{gr}}$, i.e. the tangential wind is supergradient. At $r_{\text{max}}$, $v$ exceeds $v_{\text{gr}}$ by 23.4%. The near surface radial wind has a maximum of 22.7 m s\(^{-1}\) at a radius of 33 km, the maximum being 41% of $v_{\text{max}}$. As expected, $vT$ exceeds $v$ at all radii and has its maximum at a radius inside $r_{\text{max}}$, while $vT - v$ has a maximum at a similar radius. The difference between $vT$ and $v$ reflects the presence of wind asymmetries as highlighted by the profile of $vtp$. Like $vT - v$, the latter has a maximum inside $r_{\text{max}}$. Notably, $vtp$ retains a high degree of variability out to 100 km radius. In other words, the total eddy wind

\(^2\)The gradient wind, $v_{\text{gr}}$, is obtained by solving the quadratic equation $\frac{v^2}{\rho} + f v_y - \frac{2 \rho}{\rho_{\text{gr}}} = 0$. 
speed remains a significant component of the total flow throughout the inner core region.

At 96 h, the situation is similar, but the low-level flow velocity maxima have contracted inwards and amplified considerably. Now $v_{max}$ is 81.8 m s$^{-1}$ while the maximum gradient wind is only 62 m s$^{-1}$ and the near surface maximum inflow is 36.7 m s$^{-1}$. At this time, $r_{vmax}$ is 25 km, while the radius of the maximum gradient wind is 32 km. Further, at $r_{vmax}$, the tangential wind is supergradient by 31.7%. Once again, $vt$ exceeds $v$ at all radii and has its maximum at a radius inside $r_{vmax}$, while $vt - v$ has a maximum at a similar radius. As at 48 h, the difference between $vt$ and $v$ and the eddy wind speed $vtp$ profile reflects the presence of wind speed asymmetries. These asymmetries have increased in magnitude inside $r_{vmax}$, but have decreased notably in relation to $v$ outside $r_{vmax}$ as highlighted by the profile of $vtp$. The total eddy wind speed remains a significant component of the total flow inside $r_{vmax}$. Presumably, the confinement of the maximum wind asymmetries to the region inside $r_{vmax}$ is a reflection of the focussing of the deep convective activity to the eyewall region of the mature vortex. It can be shown that these wind asymmetries contribute to the transport of high $\theta_e$ air from the low-level eye to the developing and mature eyewall (J. Persing 2018, personal communication), see next section.

These findings indicate that the radial component of near-surface flow is a significant fraction of the tangential wind speed and that gradient-wind imbalance is an intrinsic feature of the boundary layer dynamics.

4 Equivalent potential temperature structure

Figures 3 and 4 show horizontal and vertical cross sections of equivalent potential temperature, $\theta_e$, at 48 h during the period of rapid intensification and 96 h just as the mature stage is attained. Although the vortex centre is not exactly at the model Cartesian coordinate centre $(0, 0)$ at these times, the centre remains reasonably close to the coordinate centre throughout the period of rapid intensification and mature phase.

At 48 h the horizontal cross section in Figure 3 panel (a) shows that the updraughts at a height of 1 km are mostly confined to surface values of $\theta_e$ larger than 355 K and they are generally scattered, except for an almost complete ring at a mean radius of about 30 km from the vortex centre. $\theta_e$ has a sharp negative radial gradient just inside this ring. The largest values of $\theta_e$ exceeding 365 K are found in a narrow annular ring of mean radius about 10 km, well inside the ring of strong updraught shown at 1 km altitude. The profile of radial velocity shown in Figure 2(a) indicates that the near surface mean radial flow extends inwards to about 15 km radius, just outside this narrow annular ring. At a height of 6 km (panel (b)) the $\theta_e$ values are generally less than near the surface and there are no values of $\theta_e$ larger than 365 K, and only a few extremely localized regions with values exceeding 360 K.

To gain some perspective on the source of the $\theta_e$ air that comprises the developing eyewall, panels (c) and (d) of Figure 3 show vertical cross sections through the vortex centre at this time (azimuthally-averaged cross sections of $\theta_e$ are shown in Part I, Fig. 5). In a broad sense these cross sections depict the classic $\theta_e$ structure reported long ago by Hawkins and Imbembo (1976), with a $\theta_e$ structure that generally decreases outwards, possesses maxima near the surface and upper troposphere and a minimum near 3 to 4 km altitude. These cross sections reveal also a fine structure that highlight several important features that will be discussed in turn.

At low levels and from about 55 km from the centre, the $\theta_e$ shows very little, if any, augmentation as one moves inwards until the innermost 15 km, where the inflowing air encounters the high $\theta_e$ air of the low-level eye. The lack of systematic augmentation of surface $\theta_e$ outside of the developing eyewall is an indication of a near balance between the surface moisture fluxes that tend to moisten the inflowing layer and mesoscale downdraughts that tend to dry the
Figure 3. Horizontal and vertical cross sections of $\theta_e$ at 48 h, during rapid intensification. Panel (a) shows horizontal cross section of $\theta_e$ at the lowest model level (25 m), along with horizontal wind vectors and updraughts at 1 km altitude. Panel (b) shows horizontal cross section of $\theta_e$ at a height of 6 km, with corresponding horizontal wind vectors and updraughts. Contour interval for $\theta_e$: red contours every 2.5 K up to 365 K, black contours every 2.5 K from 365 K. Updraughts in panels (a) and (b) are indicated by the 1 m s$^{-1}$ contour of vertical velocity (dashed yellow-black contour). Values for the shading of $\theta_e$ are given in the colour bar, in units K. The wind vectors are in relation to the maximum reference vector (30 m s$^{-1}$) at the bottom right. Panels (c) and (d) show vertical cross sections through the vortex centre of $\theta_e$, in the x and y directions. Updraughts in panels (c) and (d) are again indicated by the 1 m s$^{-1}$ contour of vertical velocity (yellow contour). Vortex centre indicated by the hurricane symbol.

In the absence of $\theta_e$ augmentation, deep convection cannot be maintained to drive an inward vorticity flux necessary for intensification above the boundary layer. The emerging updraughts that develop as the air is decelerated and turn upwards are approximately co-located with filaments of enhanced $\theta_e$ air that emanate from the region of $\theta_e$ air larger than 365 K. We can see examples of this on both sides of the cross sections through the developing eyewall. Presumably, the enhanced $\theta_e$ air that is being drawn outwards from the region of maximum surface $\theta_e$ air is enriching the moist air entering the updraught from larger radii. This “fuel” enrichment would contribute to enhancing the local buoyancy of convective updraughts in the eyewall region, itself (J. Persing, personal communication). This process is similar to that envisioned by Braun (2002) and Wang (2016). This fuel enrichment would serve also to mitigate the deleterious effects of mixing with lower $\theta_e$ air just outside of the developing eyewall in association with convective band structures and related eddies.

At 96 h (Figure 4), the horizontal cross sections depict a well-defined ring-like updraught at 1 km altitude (panel (a)). There are no strong updraughts (> 1 m s$^{-1}$) outside of this ring. The surface $\theta_e$ values have generally increased over the domain shown, and inner-core values of $\theta_e$ exhibit a monopolar structure with a maximum just above 370 K. At 6 km altitude (panel (b)), the inner $\theta_e$ structure is approximately circular with the highest values occurring within an almost circular ring with a mean radius of approximately 30 km. Outside of this ring, $\theta_e$ values decrease approximately monotonically in the core region with a comparatively slower decrease at larger radii. Outside the inner core region there are a number of convective bands to the north and west of the vortex centre at this time. Asymmetric updraught structures are evident in the inner core region also.

The vertical cross sections at 96 h exhibit similar structural features as seen at 48 h. However, the elevated values of $\theta_e$ in the core region have generally increased in magnitude. Now, values of $\theta_e$ at the surface exceed 370

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K and values exceeding 360 K are found in deep, narrow filaments on the inner edge of the eyewall updraughts. Again, the vertical cross sections indicate that the air in the eyewall is being enriched from the pool of high $\theta_e$ values in the eye at low levels.

The foregoing analyses serve to highlight subtle, but important thermodynamic features of the inner-core vortex. In particular, the analyses point to the importance of the pool of high $\theta_e$ air in the low-level eye region in supporting convective instability essential for vortex spin up. The ability of inflowing air to access this “fuel reservoir” depends importantly on the underlying boundary layer dynamics of the vortex. In turn, the boundary layer dynamics depends on the vortex evolution above the boundary layer, which, itself, depends on the ability of the convection to amplify the winds above the boundary layer by the classical mechanism discussed in Part I. In section 5 we present an analysis of mean and eddy processes in order to build a more complete understanding of the role of convective eddies and the associated waves they generate on the dynamics of vortex intensification.

5 Analysis of mean and eddy processes

As noted in Part I, only mature storms are approximately axisymmetric and then only in their inner core region. In particular, in the genesis and early intensification stages, deep convection is not organized into ring-like structures and the flow is quite asymmetric. Thus the flow generated by relatively disorganized convection is quite asymmetric.
Nevertheless, it is still possible to derive understanding from an azimuthally averaged perspective, but with suitable modifications to take into account the asymmetries. We remind the reader that localized coherent structures project on both mean and eddy terms in the equations for azimuthally-averaged quantities. For flows that are punctuated by highly localized convective structures, the eddy terms would be expected to make an important contribution to the evolution of azimuthal mean quantities, requiring a modification to the axisymmetric view of tropical cyclone intensification.

Some years ago, Persing et al. (2013) compared the results of idealized numerical experiments to examine the difference between tropical cyclone evolution in three-dimensional and strictly axisymmetric model configurations. They used the CM1 model with a horizontal grid spacing of 3 km. The results indicated that 3-D eddy processes associated with vortical plume structures made a broadly favorable contribution to the intensification process, enhancing the radial contraction of the maximum tangential velocity and promoting a vertical extension of tangential winds through the depth of the troposphere. Further, the resolved 3-D eddy momentum fluxes above the boundary layer exhibit counter-gradient characteristics during a key spin-up period, and more generally, they are not solely diffusive.

Again because of its fundamental importance for tropical cyclone dynamics, we repeat here and extend the Persing et al. (2013) (the term \(V_{ppg}\) in their Equation (12)) for the subgrid scale motions are given by

\[
\frac{\partial}{\partial t} \left( \gamma r d\lambda \right) = -\frac{1}{\rho r} \frac{\partial p}{\partial \lambda} + F_r,
\]

and set any dependent variables \(\gamma\) as the sum of a mean part \(\bar{\gamma}\) and an asymmetry (or eddy) \(\gamma'\), i.e. \(\gamma = \bar{\gamma} + \gamma'\). Then, noting that, by definition, \(\bar{\gamma} = 0\), we obtain, for example, the azimuthally-averaged tangential form of the velocity tendency equation:

\[
\frac{\partial \bar{v}}{\partial t} = -\bar{u} \frac{\partial \bar{v}}{\partial r} \frac{V_{m\zeta}}{V_{m\nu}} - \bar{w} \frac{\partial \zeta}{\partial z} \frac{V_{e\zeta}}{V_{ev}} - \bar{w} \frac{\partial \bar{v}}{\partial z} + \bar{F}_\lambda. \tag{4}
\]

Here, we have neglected the azimuthally-averaged pressure gradient term involving perturbations of density in the azimuthal direction, which, as noted by Persing et al. (2013) (the term \(V_{ppg}\) in their Equation (12)) is tiny compared with all other terms. The five terms on the right hand size of the Eq. (4) are, in order: \(V_{m\zeta}\) is the mean radial influx of absolute vorticity, \(V_{m\nu}\) is the mean vertical advection of mean tangential momentum, \(V_{e\zeta}\) is the eddy radial vorticity flux, \(V_{ev}\) is the vertical advection of eddy tangential momentum, and \(V_{d}\) is the combined mean horizontal and vertical diffusive tendency of tangential momentum, given by:

\[
\bar{F}_\lambda = \frac{1}{r^2 \rho} \left( \frac{\partial \bar{v}^2 \zeta}{\partial r} \frac{V_{dr}}{V_{dz}} + \frac{1}{\rho} \frac{\partial \bar{p} \zeta}{\partial \lambda} \right), \tag{5}
\]

where the stress tensors (e.g. Landau and Lifshitz (1966, p51)) for the subgrid scale motions are given by

\[
\frac{\tau_{r\lambda}}{\rho} = K_{m,h} \left( \frac{1}{r} \frac{\partial u}{\partial \lambda} + \frac{1}{r} \frac{\partial v}{\partial \lambda} \right), \tag{6}
\]

\[
\frac{\tau_{\lambda z}}{\rho} = K_{m,v} \left( \frac{1}{r} \frac{\partial u}{\partial \lambda} + \frac{1}{r} \frac{\partial v}{\partial \lambda} \right), \tag{7}
\]

and \(K_{m,h}\) and \(K_{m,v}\) are the model output horizontal and vertical diffusivities, respectively.

The azimuthally-averaged radial momentum equation can be written as

\[
\frac{\partial u}{\partial t} + \frac{\bar{u} u'}{\rho} + \left( \frac{u'}{r} \frac{\partial u'}{\partial r} + \frac{v'}{r} \frac{\partial u'}{\partial \lambda} \right) = -\frac{w'}{\rho} \frac{\partial p}{\partial r} + \frac{F_r}{U_{magf}} + \frac{F_d}{U_{efgf}}.
\]

Here, \(\bar{F}_r\) is the combined mean horizontal and vertical diffusive tendency of radial momentum, given by:

\[
\bar{F}_r = \frac{1}{r^2 \rho} \left( \frac{\partial \bar{p} \bar{r}}{\partial r} \frac{U_{dh}}{V_{dz}} + \frac{1}{\rho} \frac{\partial \bar{p} \bar{r}}{\partial \lambda} \right), \tag{9}
\]
where the stress tensors (e.g. Landau and Lifshitz, 1966, p51) for the subgrid scale motions\(^4\) are given by

\[
\tau_{rr} = 2K_{mh} \left( \frac{\partial u}{\partial r} \right),
\]
\[
\tau_{\theta\theta} = 2K_{mh} \left( \frac{1}{r} \frac{\partial v}{\partial \theta} + \frac{u}{r} \right),
\]
\[
\tau_{r\theta} = K_{mv} \left( \frac{\partial w}{\partial r} + \frac{\partial u}{\partial \theta} \right).
\]

To facilitate interpretation, we have chosen this pseudo-Lagrangian form in which the left-hand-side represents the material acceleration in the radial direction following the horizontal wind\(^5\). The individual terms on this side represent: the local tendency of the mean radial velocity, the mean radial advection of radial momentum per unit mass, \(U_{mr}\), and the mean horizontal advection of eddy radial momentum, \(U_{ch}\) per unit mass. The terms on the right-hand-side of the equation are in order: \(U_{mv}\) is minus the mean vertical advection of mean radial momentum per unit mass and \(U_{cv}\) is minus the eddy vertical advection of eddy radial momentum per unit mass; \(U_{maf}\) and \(U_{eaqf}\) are the mean and eddy gradient force per unit mass, respectively; and \(U_d\) is the combined mean radial and vertical diffusive tendency of radial momentum.

Note that we have chosen to write \(U_{maf}\) with \(\bar{\rho}\) in the denominator. This choice requires that the azimuthal variation of \(\rho\) be retained in the definition of \(U_{eaqf}\), and assumes that \(|\rho - \bar{\rho}| < < \rho\).

5.1.1 Attributes of the mean-eddy flow partitioning

The foregoing partitioning of the flow into an azimuthal mean and eddy contributions is a natural one for an isolated vortex, especially when applied to vortex waves (e.g. Reasor and Montgomery 2015 and refs.). Nevertheless, care is required when interpreting the individual contributions when strong and highly azimuthally-localized features punctuate the vortex flow in a particular annulus. This is because such localized features project on both components of the partition. For example, an individual updraught within this annulus will project both into the mean and eddy components. However, the mean updraught will have a small positive value while the eddy will have a large positive value in its particular location, but a small negative value elsewhere, a consequence of the partitioning result that \(\overline{w'} = 0\). The small negative value for the eddy will identically cancel the small positive value from the mean outside the region of the eddy.

\(^4\)The expression for \(\overline{v'}\) corrects the expression given in Persing et al. (2013, their Eq. (20)). The difference is found to be negligible.

\(^5\)This choice is different from that for the tangential momentum equation and means that the sum of terms on the right-hand-side can be interpreted as forces that produce pseudo-material acceleration in the radial direction. Such a choice is perhaps less appropriate in the tangential momentum equation as one would then lose the near form of the radial vorticity flux term.

Another issue with this partitioning is that, because the vortex centre is not exactly stationary, there is a weak flow across the vortex axis, even in the problem studied here where the vortex environment is quiescent. Since there is no source or sink of mass at the axis, both \(\bar{u}\) and \(\bar{v}\) must vanish at the axis, the latter since the vorticity at the axis is finite. As a result, both mean tendency terms must be zero implying that the sums of terms on the right-hand-sides of Equations (4) and (8) must sum to zero at the axis. Because one of the terms in the expressions for \(U_{ch}\) and \(U_{eaqf}\) involve \(v'/r\), and \(v'\) may be finite at the axis, these terms must cancel. However, on a finite mesh, this cancellation may be susceptible to appreciable numerical discretization error.

Despite the foregoing cancellation issue, the formulation in terms of mean and eddy components is generally useful for providing insight, providing this attribute of the partitioning is borne in mind. For example, the effects of eddy momentum fluxes associated with velocity perturbations due to a single updraught on the tangential-mean velocity tendency were investigated by Kilroy and Smith (2016) and a conceptual framework for the interpretation of these eddy fluxes was given.

5.2 Mean tangential velocity tendency equation

5.2.1 Spin-up phase

Figure 5 shows radius-height plots of the three hour time averaged terms in the azimuthally-averaged tangential velocity tendency equation at 48 h, which is during the period of rapid intensification (Figure 1). The time average is based on model output saved every 15 minutes. Panels (a) and (b) show the contributions to the tendency from the mean radial vorticity flux and vertical advection, while panels (c) and (d) show the corresponding eddy contributions. Panels (e) and (f) show the sum of the mean and eddy terms, respectively. In each panel shown the mean eyewall updraught is highlighted by the thick yellow contour, which shows the location of the 0.5 m s\(^{-1}\) vertical velocity isopleth, and the while the pink contours show the ± 1 m s\(^{-1}\) radial velocity.

The mean vorticity influx (panel (a)) is large and positive in a shallow layer near the surface, marking the boundary layer inflow region, and it is large and negative in a narrow sloping sheath just above this layer. It is large and negative also in the eyewall updraught, extending into the upper tropospheric outflow layer. There are two small regions of positive vorticity flux, one above the sheath which is a manifestation of the centrifugal recoil effect (see e.g. Montgomery and Smith 2017) and one in the upper troposphere on the inside of the eyewall updraught. The principal features in the mean tendency from vertical advection (panel (b)) are similar to those in panel (a), but are opposite in sign so that there is considerable cancellation as seen in panel (e). This cancellation is a reflection of the fact that above the boundary layer,
Figure 5. Radius-height plots of the three hour time averaged terms in the azimuthally-averaged tangential wind tendency equation at 48 h. The time averaging is centred on the time shown. Contour intervals are: thin positive contour 1 m s$^{-1}$ h$^{-1}$, thick positive contour 5 m s$^{-1}$ h$^{-1}$, thin negative dashed contour is the zero contour, thick negative dashed contours 0.5 m s$^{-1}$ h$^{-1}$ and 5 m s$^{-1}$ h$^{-1}$. Thick black contour ± 20 m s$^{-1}$ h$^{-1}$. Shading as indicated on the side bar. The yellow contour shows the 0.5 m s$^{-1}$ vertical velocity, while the pink contours show ± 1 m s$^{-1}$ radial velocity. Solid contours positive, dashed contours negative.
absolute angular momentum is approximately conserved (see Appendix of Smith et al. 2009).

From the perspective of the mean dynamics, the main contribution to the spin-up of the tangential wind is associated with the import of mean cyclonic absolute vorticity in the boundary layer and its vertical advection into the eyewall updraught. There is however a region spanning between 1 and 3 km where the mean tendencies give a spin down effect. This spin down is more than negated by the vertical eddy momentum transport discussed below.

The contribution to the mean tendency from the eddies are confined mainly to the eyewall updraught region. The eddy vorticity flux term is mostly negative, but there is a region of positive tendency above about 11 km in the outflow region. The vertical advection of eddies is relatively large and positive below a height of 4 km, but has more of a dipole structure above this level with positive tendencies on the inner part of the eyewall updraught and negative tendencies on the outside. The combined eddy tendency in panel (f) shows a pattern that is quite similar to the sum of the mean tendencies (panel (e)), but of opposite sign.\(^6\) Also, the combined eddy tendency in the eyewall region spanning approximately 1 to 3 km more than compensates the spin down effect from the combined mean terms. In essence, the vertical momentum transport in this layer is taking place principally by the convective eddy structures.

The horizontal diffusive tendency shows a weak dipole structure below a height of 4 km (panel (g)), while the vertical diffusive tendency is negative in a shallow layer near the surface with a small region of positive tendency below the base of the eyewall updraught. Comparing the radial and vertical eddy terms with the corresponding sub grid-scale diffusion terms shows that the pattern of the eddy terms is generally quite different from that of the diffusion terms. The discrepancy in the pattern of tendencies implies that the resolved eddy contributions cannot be regarded simply as a down-gradient diffusive process. Persing et al. (2013) arrived at a similar conclusion in their study.

Comparing panels (e) and (h) one sees that the horizontal vorticity influx is generally larger than the mean vertical diffusion tendency. Even after adding the slight negative tendency from the combined eddy term near the surface, the net tendency of tangential wind is positive in the inner-core boundary layer. In essence, this is confirmation that the nonlinear boundary layer spin up mechanism is operating to spin up the maximum tangential wind in the boundary layer. Another manifestation of this spin up mechanism is the outward sloping region of positive values below about 4 km in height in panel (d). The positive values of \(\nu'\), which presumably contribute to the supergradient excess, are being lofted to give the values of \(V_{ev}\) and must be generated near the surface. These values must be a result of the nonlinear boundary layer spin-up mechanism, which acts on the asymmetric component of flow also.

The time-averaged tangential velocity tendency calculated directly from the model output is shown in panel (j). This tendency is simply the difference in tangential velocity over the 3 hour period divided by the 3 hour time span. The panel shows clearly that there is spin up throughout much of the eyewall updraught, including the boundary layer beneath the eyewall and extending to approximately 60 km radius.

This direct tendency can be compared with the estimate from the sum of tendency terms (panel (i)) on the right-hand-side of Equation (4). While the direct calculation is a little smoother, it does show a sloping band of strong positive tendency, mostly inside the 1 m s\(^{-1}\) eyewall updraught contour. The most notable discrepancy between these two panels is found in the corner flow region near where the boundary layer inflow terminates (see Figure 5(c) from Part I). Possible sources for the error in the sum of tendency terms are discussed by Persing et al. (2013, p12318). In brief, there are principally three sources of error: the sampling of output data, the evaluation of parameterized internal diffusion and surface momentum fluxes, and the use here of centred spatial differences to calculate advection. (The CMI model uses a 5th order upstream advection scheme.) The nature of these errors become most apparent in the boundary layer where both the second and third sources of error become especially prevalent.

5.2.2 Mature phase

Figure 6 shows similar radius-height plots to those in Figure 5, but at 96 h, which is after the period of rapid intensification (Figure 1). The individual tendency terms have structures that are similar to their counterparts in Figure 5, but generally larger in magnitude, except in panel (j), where, consistent with the time being at the start of the mature phase, the local mean tangential wind tendency calculated from the left-hand-side of Equation (4) is weaker than that in Figure 5(j). In particular, there is still a strong mean tendency associated with the radial influx of vorticity in the boundary layer (panel (a)), which is opposed by the negative tendency due to friction (panel (h)). Significantly, the strong negative mean tendency \(V_{mc} + V_{mv}\), where the air ascends as it exits the boundary layer is opposed by a positive tendency by the sum of the eddy terms \(V_{ec} + V_{ev}\). In other words, the eddies are playing an important role to maintain the swirling flow in the eyewall. In the upper part of the eyewall, there is much cancellation between the sums \(V_{mc} + V_{mv}\) and \(V_{ec} + V_{ev}\).

5.3 Mean radial momentum equation

5.3.1 Spin-up phase

Figure 7 shows radius-height plots of the three hour time-averaged terms in the azimuthally-averaged, pseudo-Lagrangian radial momentum equation at 48 h. The

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\(^6\) This tendency for cancellation is perhaps not surprising bearing in mind that the coherent eddy structures have compensating imprints on both the mean and eddy contributions in this simple azimuthal mean and eddy partitioning.

\(^7\) See e.g. Smith and Montgomery (2016).
Figure 6. Radius-height plots of the three hour time averaged terms in the azimuthally-averaged tangential wind tendency equation at 96 h. The time averaging is centred on the time shown. Contour intervals are: thin positive contour 1 m s\(^{-1}\) h\(^{-1}\), thick positive contour 5 m s\(^{-1}\) h\(^{-1}\), thin negative dashed contour is the zero contour, thick negative dashed contours 0.5 m s\(^{-1}\) h\(^{-1}\) and 5 m s\(^{-1}\) h\(^{-1}\). Thick black contour ± 20 m s\(^{-1}\) h\(^{-1}\). Shading as indicated on the side bar. The yellow contour shows the 0.5 m s\(^{-1}\) vertical velocity, while the pink contours show ± 1 m s\(^{-1}\) radial velocity. Solid contours positive, dashed contours negative.
Figure 7. Radius-height plots of the three hour time-averaged terms in the azimuthally-averaged radial wind tendency equation at 48 h. The time averaging is centred on the time shown. Contour intervals are: thin positive contour 1 m s\(^{-1}\) h\(^{-1}\), thick positive contour 5 m s\(^{-1}\) h\(^{-1}\), thin negative dashed contour is the zero contour, thick negative dashed contours 0.5 m s\(^{-1}\) h\(^{-1}\) and 5 m s\(^{-1}\) h\(^{-1}\). Thick black contour ± 20 m s\(^{-1}\) h\(^{-1}\). Shading as indicated on the side bar. The yellow contour shows the 0.5 m s\(^{-1}\) vertical velocity, while the pink contours show ± 1 m s\(^{-1}\) radial velocity. Solid contours positive, dashed contours negative.
azimuthal and time mean radial velocity is superposed on all panels so as to help facilitate interpretation of the radial momentum dynamics. Panels (a) and (b) show the mean eddy agradient force fields, $U_{magf}$ and the mean radial advection ($U_{mr}$) tendency. As expected, $U_{magf}$ is strongly negative in a shallow surface-based layer at radii beyond about 30 km. This negative region corresponds with the frictional boundary layer and it is primarily this negative force that drives the boundary layer inflow. Recall that in Part I we showed that much of the low-level inflow is a result of boundary layer dynamics and that the “suction effect” of deep convection plays a secondary role.

The magnitude of $U_{magf}$ increases with decreasing radius to about 30 km (near the surface) and then declines rapidly. The corresponding tangential winds in this region of negative values are subgradient. The mean radial winds progressively increase with decreasing radius in this layer (Part I, Figure 5(c)), reaching values of about 20 m s$^{-1}$, which is a significant fraction (about 35%) of the tangential winds in the same region.

Inside this 30 km radius and in a shallow layer that slopes upwards with radius, $U_{magf}$ is strongly positive and serves to deaccelerate the inflow and even accelerate the flow outwards just above the boundary layer into the eyewall updraught. In this positive region, the tangential winds are supergradient. Just above this positive region is a shallow sloping region of negative $U_{magf}$. This and the two regions of alternating sign below it represent the primary driving forces for the standing centrifugal wave near the base of the eyewall. Above the negative region, $U_{magf}$ is generally positive, being particularly strong in the eyewall updraught and in the outflow region to more than 100 km radius. This pattern is not surprising because air with high angular momentum is being transported vertically by deep convective cores in the developing eyewall as is evident in Figure 5(f).

At radii larger than about 100 km, there is a layer straddling the tropopause ($\approx 16$ km) in which $U_{magf}$ is negative and has a magnitude up to 5 m s$^{-1}$ h$^{-1}$. This layer of subgradient mean tangential wind deepens progressively with radius and would serve not only to decelerate the outflow, but to accelerate inflow.

The structure of the mean radial advection $U_{mr}$ shows a shallow layer of negative values beyond a radius of about 35 km at the surface where the flow in the boundary layer is accelerating inwards. It shows also shallow sloping layers of alternating positive and negative values, generally confined within a radius of 50 km and a height of about 3 km. The lowermost layer of positive values coincides with the radii where the inflow is decelerated sharply before ascending into the eyewall. The alternating negative and positive layers above are further signatures of the standing centrifugal wave near the base of the eyewall seen in Figures 5(a).

In the upper troposphere, the radial flow accelerates outwards as the air exits the eyewall, but the region of outward acceleration is sandwiched by layers where $U_{mr} < 0$ outside a radius of about 125 km. Beyond this radius, the mean acceleration in the upper troposphere is inwards so that outward flowing air is being decelerated, consistent with the pattern of $U_{magf}$. Moreover, inspection of the mean inflow at this same time shows that there are regions of inflow sandwiching the outflow layer and these regions overlap with that of negative $U_{magf}$ and $U_{mr}$. Where this occurs, the flow in the inflow layers will be accelerated inwards.

The radius-height structure of the mean vertical advection $U_{mv}$ in panel (c) shows a series of layers in which $U_{mv}$ has alternating sign. In the lowest layer, near the base of the developing eyewall updraught, $U_{mv} < 0$, indicating a contribution to an increase of the radial inflow by the vertical advection of inward radial momentum from near the surface. In the sloping layer above, $U_{mv} > 0$, reflecting the mean vertical transport of positive radial momentum associated with the strong outflow of air just above the boundary layer seen in Figure 5(c) of Part I. Again, these features coincide approximately with the standing centrifugal wave referred to above and are a response to the mean agradient force distribution ($U_{magf}$) seen in Figure 7(a). The layer of negative $U_{mv}$ between about 5 and 8 km height is associated with the upward transport of mean radial inflow in eyewall updraught from the lower troposphere seen in Figure 5(c) of Part I. The layered pattern of $U_{mv}$ above about 8 km is consistent also with the layered pattern of inflow and outflow in the upper troposphere together with that of vertical velocity seen also in Figure 5(c) of Part I. For example, the upward transport of mean inward radial momentum from the inflow layer just below the mean outflow layer together with the downward transport of inward radial momentum from the inflow layer just above the mean outflow layer contribute to the layer of negative tendency centred at an altitude of about 12 km.

The eddy agradient force field, $U_{eagf}$, in Figure 7(d) indicates that the asymmetric part of the tangential flow is not in gradient wind balance. This eddy term contributes to a positive radial acceleration throughout the troposphere out to between 40 and 70 km radius, where it represents an additional centrifuge effect to that of the mean term $U_{magf}$. In particular, $U_{eagf}$ shows strong positive values near the surface at radii between approximately 10 and 30 km near where the surface inflow terminates (see again Figure 5(c) of Part I), reinforcing the deceleration of the boundary layer inflow. The relatively large values of $U_{eagf}$ near the axis should not be taken too seriously and are presumably due to numerical inaccuracy as discussed in section 5.1.1.

Figures 7(e) and 7(f) show the time-averaged and azimuthally-averaged sub-grid-scale tendencies, $U_{dr}$ and $U_{dz}$. The radial diffusion of radial momentum is relatively small except in the region where the radial flow terminates and turns up into the eyewall. The vertical diffusion of radial momentum shows a very shallow layer of strong positive tendency beyond a radius of about 30 km, which is a manifestation of surface friction slowing down the inflow. Above this layer lies a somewhat thicker layer of negative tendencies.
tendency, which is associated with the vertical diffusion of inward radial momentum through the inflow layer. This diffusion becomes particularly strong near where the boundary layer inflow terminates.

Figure 7(g) shows minus the eddy vertical advection of eddy vertical momentum per unit mass, $U_{ev}$. The structure of this term, like $U_{mv}$, exhibits a series of layers in which $U_{ev}$ has alternating sign. In the lower troposphere, the $U_{ev}$ field broadly reinforces that of $U_{mv}$. In the upper troposphere there is a degree of cancellation between $U_{mv}$ and $U_{ev}$, but the eddies can be seen to be reinforcing the strong outflow in the layer between about 11 and 14 km in height. This layer lies below the axis of the upper-level outflow layer, which implies that the eddies are acting to transport radial momentum against the mean gradient in this layer, i.e. they are countergradient. In contrast, the layer of negative $U_{ev}$ in the height range 14-16 km lies above the axis of the outflow layer and acts to slow down the outflow in this layer. Further, the layer of negative $U_{ev}$ in the height range 8-11 km overlaps with the pronounced inflow layer just below the outflow layer and acts to reinforce the inflow in this layer. These features are consistent with the upper-level pattern of the vertical eddy momentum flux shown in Persing et al. (2013; Figure 15(e)) and contribute to a strengthening of the primary outflow layer, itself, as well as the inflow layer below the outflow layer.

Figure 7(h) shows the local mean change in radial wind averaged over the three hour period ($\Delta u$), which if divided by the time period would give the local tendency $\partial u / \partial t$. The figure indicates that over this time interval there has been a strengthening of the overturning circulation. In particular the low-level inflow has strengthened, the outflow just above the boundary layer where the boundary layer terminates has strengthened, and the upper-level outflow has strengthened. In addition, the layers of inflow sandwiching the upper-level outflow layer have strengthened, as has a region of inflow being drawn in by the developing eyewall updraught in a layer between 4 and 8 km in height.

5.3.2 Overall radial momentum budget at 48 h

Although the $\Delta u$ field clearly indicates the change in the radial flow during the three hour time period, one can explain this change only in terms of the radial force field and the response thereto. The essence of the radial momentum budget is encapsulated in Figures 7(i) and 7(j), which show the sum of the time-averaged and azimuthally-averaged tendencies on each side of Equation (8). Panel (i) shows the pseudo-Lagrangian radial acceleration, while panel (j) shows the forces leading to this acceleration. Generally, the principal features of these two fields match each other quite well, despite there being a few local discrepancies in detail that are presumably associated with interpolation errors and the like (see section 5.2.1).

There is a net inward force field through much of the lower troposphere including much of the developing eyewall region, itself. This inward force is particularly strong in the inner-core boundary layer. Near the surface and just below the eyewall updraught, this force exceeds 20 m s$^{-1}$ h$^{-1}$. Inside the developing eyewall, near its inner edge, and throughout much of the upper troposphere out to a radius of about 120 km, there is strong positive radial acceleration, again exceeding 20 m s$^{-1}$ h$^{-1}$ in the upper troposphere. The net force is associated primarily with the pattern of mean agradient force and the mean and eddy vertical advection terms (panels (a), (c) and (e)).

The existence of this force provides an explanation for the occurrence of inflow layers that sandwich the upper-tropospheric outflow layer.

In summary, the flow in much of the inner-core region is significantly unbalanced in the radial direction, especially at low levels and in the upper troposphere. Thus, while the assumption of gradient wind balance may be a reasonable first approximation through much of the middle troposphere during spin up, the assumption cannot be justified in the inner-core boundary layer or in the upper troposphere in the region of strong outflow. The radial force fields demonstrate that even Ekman balance (i.e. a balance between the linearized form of $U_{magf}$ and $U_{aef}$) is strongly violated in the inner-core boundary layer, where the terms $U_{mv}$, $U_{ev}$ and $U_{magf}$ are comparable in magnitude to $U_{magf}$ and cannot be neglected in a zero-order approximation.

5.3.3 Mature phase

Figure 8 shows similar radius-height cross sections to those in Figure 7, but at 96 h. Once again, the tendencies in the individual panels have structures that are similar to their counterparts in Figure 7, but the fields are generally larger in magnitude consistent with the quadratic nonlinearity of the dynamics. In particular, there is good agreement between the mean pseudo-Lagrangian tendencies shown in panel (i) and the sum of the radial force contributions in panel (j). Moreover, the degree of radial force imbalance has increased since 48 h and the region of imbalance has increased in area. These increases are particularly striking in both the lowest 2 km and in the upper troposphere between about 10 and 16 km. Note the negative force in the latter region, which would provide an explanation for the increased strength of the upper tropospheric inflow layers. Finally, note also the strengthening of the supergradient winds near the base of the eyewall in panel (a) between 48 h and 96 h and also in the upper part of the eyewall.

5.4 Vertical profiles of frictional forces

In the foregoing analyses of the radial and tangential momentum equations during the spin up and mature phases of the vortex lifecycle, certain important details of the flow in the corner region beneath the developing eyewall are difficult to discern. In this region, it is of interest to examine the magnitude and vertical structure of the vertical divergence of the subgrid-scale stress terms, i.e.,
Figure 8. Radius-height plots of the three hour time-averaged terms in the azimuthally-averaged radial wind tendency equation at 96 h. The time averaging is centred on the time shown. Contour intervals are: thin positive contour 1 m s$^{-1}$ h$^{-1}$, thick positive contour 5 m s$^{-1}$ h$^{-1}$, thin negative dashed contour is the zero contour, thick negative dashed contours 0.5 m s$^{-1}$ h$^{-1}$ and 5 m s$^{-1}$ h$^{-1}$. Thick black contour $\pm$ 20 m s$^{-1}$ h$^{-1}$. Shading as indicated on the side bar. The yellow contour shows the 0.5 m s$^{-1}$ vertical velocity, while the pink contours show $\pm$ 1 m s$^{-1}$ radial velocity. Solid contours positive, dashed contours negative.
Figure 9. Vertical profiles of azimuthally averaged and three-hour-time averaged (a,c) $V_{dz}$ and (b,d) $U_{dz}$ at (a,b) 48 h and (c,d) 96 h. The red curve (labeled ‘30 km’) is a profile taken at a radius of 30 km, while the blue curve (labeled ‘50 km’) is taken at a radius of 50 km. The units on the abscissa are m s$^{-1}$ h$^{-1}$.

Figure 9 shows vertical profiles of the azimuthally-averaged and three-hour-time averaged vertical divergence contributions $V_{dz}$ and $U_{dz}$ at 48 h and 96 h and at 30 km and 50 km radii. These low-level vertical profiles typify the structure and relative magnitude of the $V_{dz}$ and $U_{dz}$ force fields in the corner region of the boundary layer. Panels (a) and (c) show the expected deceleration of the tangential wind in the boundary layer. At 30 km radius and at 48 h, the maximum value of $|V_{dz}|$ is approximately 16 m s$^{-1}$ h$^{-1}$ and occurs at a height of 200 m. At 96 h the corresponding maximum of $|V_{dz}|$ has increased in value to approximately 23 m s$^{-1}$ h$^{-1}$ and it occurs near a height of 350 m. The values of $|V_{dz}|$ at 50 km radius are somewhat smaller in magnitude and the maximum occurs at a slightly lower altitude.

Perhaps surprisingly is the structure of the vertical profiles of the $U_{dz}$ force field. While one certainly expects the radial component of frictional force in the radial direction to be positive in the low-level inflow layer of the vortex as discussed above (reflecting the tendency of the nearly stationary sea surface to decelerate the strong inflow near the surface), the magnitude of the decelerating force is even larger than that of the corresponding tangential component of force $|V_{dz}|$ in the boundary layer. For example, at 30 km radius and at 48 h the maximum $U_{dz}$ exceeds 30 m s$^{-1}$ h$^{-1}$ and occurs at approximately 100 m height. At 96 h the corresponding maximum $U_{dz}$ has increased significantly in value to approximately 63 m s$^{-1}$ h$^{-1}$ and the maximum has moved slightly downwards below 100 m height. In other words, during spin up the radial component of frictional force is comparable to or greater than that of the tangential component in the inner core region. The disparity between the radial component of force and the tangential component of force increases significantly as the vortex reaches maturity, wherein there is a very sharp vertical gradient of radial momentum near the surface.

The foregoing results indicate that the vertical contribution to the radial component of force should be retained at zero order in any theoretical formulation of realistic vortex spin up and quasi-steady maximum intensity.

6 Conclusions
We have used the high-resolution, three-dimensional, life-cycle simulation described in Part I to explore further aspects of the vortex spin up in the framework of the rotating-convection paradigm.

As in previous studies, the maximum tangential and radial winds are found to occur within and near the top of the vortex boundary layer. Vortex spin up in the boundary layer is accompanied by supergradient winds that exceed the gradient wind by approximately 25% - 35%. Reflecting the strong radial inflow in the boundary layer, the maximum tangential winds occur approximately 10 km inside the maximum gradient winds. The wind asymmetries, associated in part with the asymmetric deep convection, make a substantive contribution (∼ 30%) to the maximum wind speed inside the radius of maximum wind speed.
wind asymmetries contribute to the transport of high $\theta_e$ air from the low-level eye to the developing and mature eye-wall.

Diagnostic analyses of the $\theta_e$-structure within 100 km of the vortex centre highlight some subtle, but important features of the inner-core thermodynamics. The analyses show a reservoir of high $\theta_e$ air in the low-level eye region and indicate the importance of this reservoir in supporting convective instability necessary for vortex spin up. The analyses show also that the ability of inflowing air to access this reservoir is linked to the symmetric and asymmetric boundary layer dynamics of the vortex.

As is well known, the boundary layer dynamics cannot be considered strictly in isolation as it depends on the vortex evolution above the boundary layer, which, itself, depends on the ability of the convection to amplify the winds above the boundary layer by the classical mechanism discussed in Part I. A more complete understanding of the flow within and above the boundary layer is afforded by an analysis of both the azimuthal-mean tangential and radial wind tendencies during vortex spin up and the beginning of the mature phase. Here we have extended previous work and quantified the role of convective eddies and the associated waves they generate on the mean tangential and radial momentum dynamics of the intensifying vortex.

In particular, an analysis of the tangential momentum equation shows that the eddies make a substantial contribution to the spin up of the eye-wall region confirming previous findings. An analysis of the force fields in the radial momentum equation highlights a significant degree of gradient wind imbalance in much of the inner-core vortex. This analysis provides also an explanation of forces responsible for the occurrence of inflow layers sandwiching the upper tropospheric outflow layer. Finally, it indicates that the vertical contribution to the radial component of frictional force should be retained at zero order in any theoretical formulation of realistic tropical cyclone evolution.

The results of this study point to the significant limitations of a purely axisymmetric and/or balance description of tropical cyclone dynamics.

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