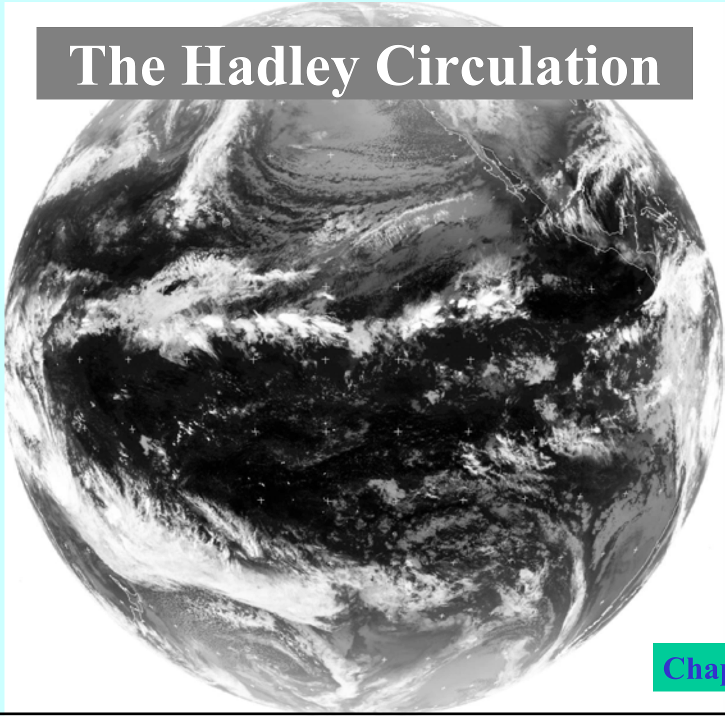
The Hadley Circulation

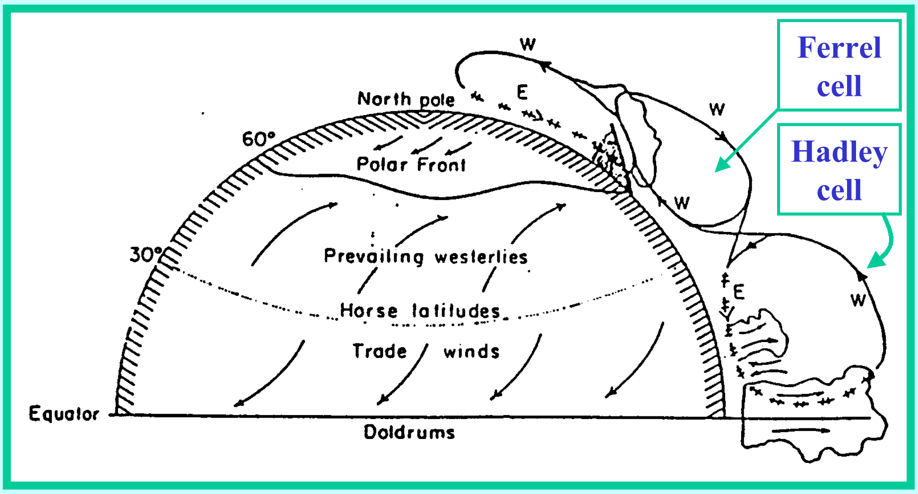


Chapter 4

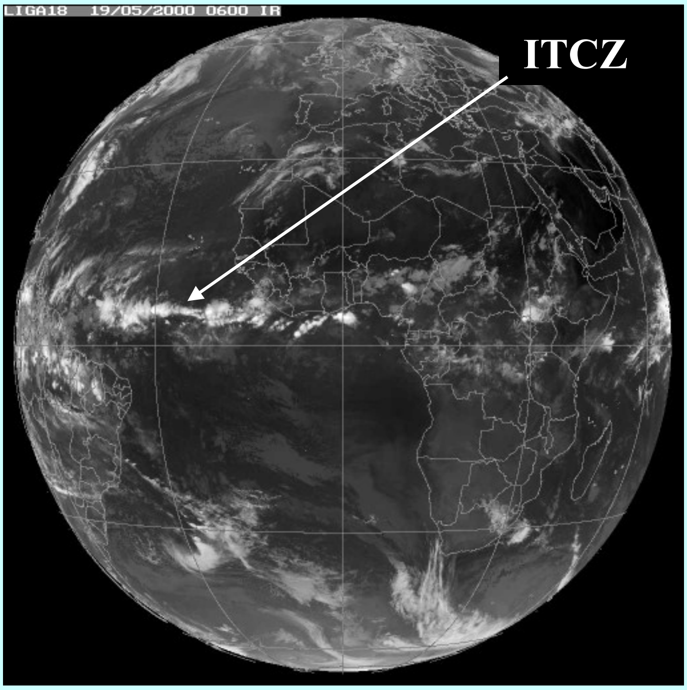
History

- The early work on the mean meridional circulation of the tropics was motivated by observations of the trade winds.
- Halley (1686) and Hadley (1735) concluded that the trade winds are part of a large-scale circulation which occurs due to the latitudinal distribution of solar heating.
- This circulation, now known as the Hadley circulation, consists of upward motion at lower latitudes, poleward motion aloft, sinking motion at higher latitudes and low-level equatorial flow.
- Despite the absence of upper-level observations Hadley deduced that the upper-level flow has a westerly component due to the effect of the earth's rotation.

Mean Zonal Circulation

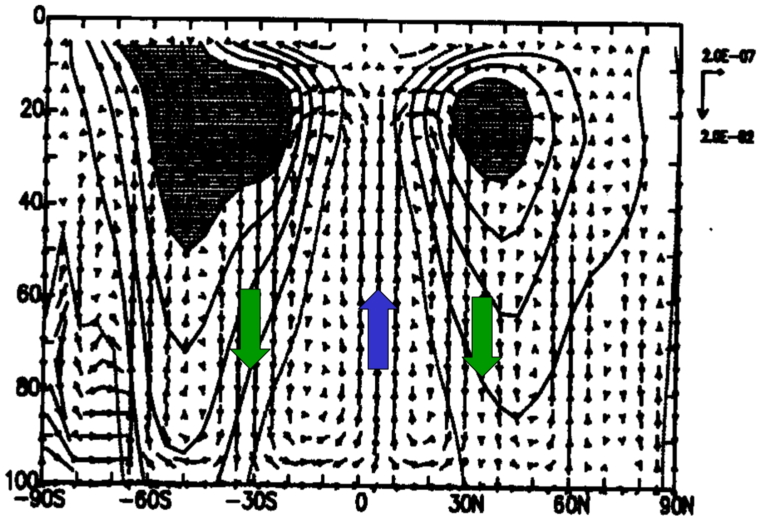


The three-cell meridional circulation pattern after Rossby (1941)



Zonal mean winds – Annual mean

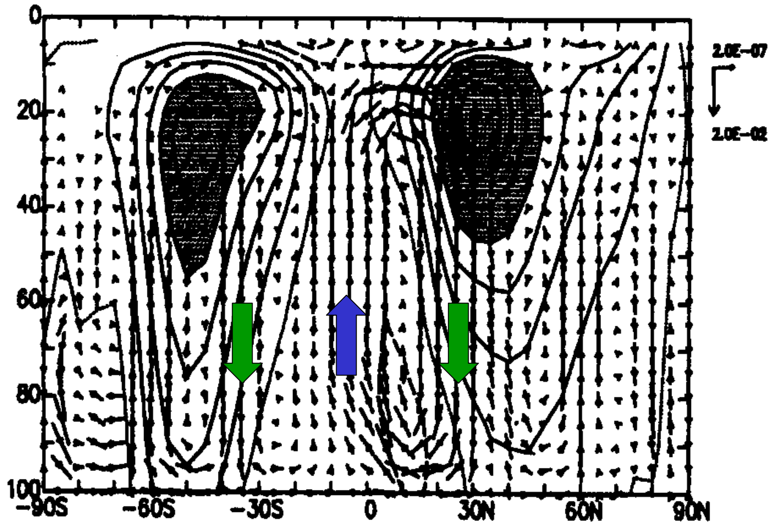
pressure



SH latitude NH

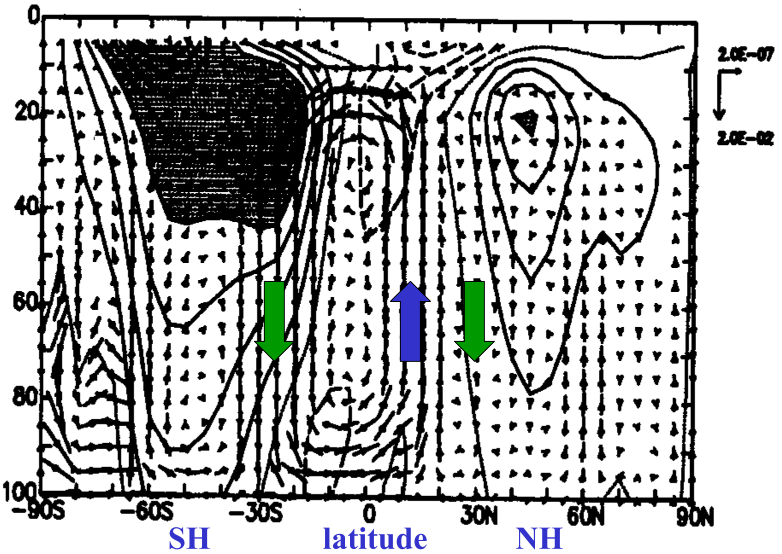
Zonal mean winds - DJF

pressure

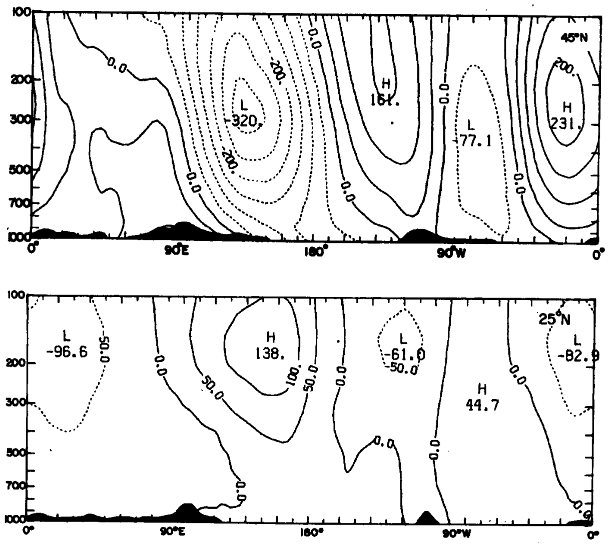


SH latitude NH

Zonal mean winds - JJA



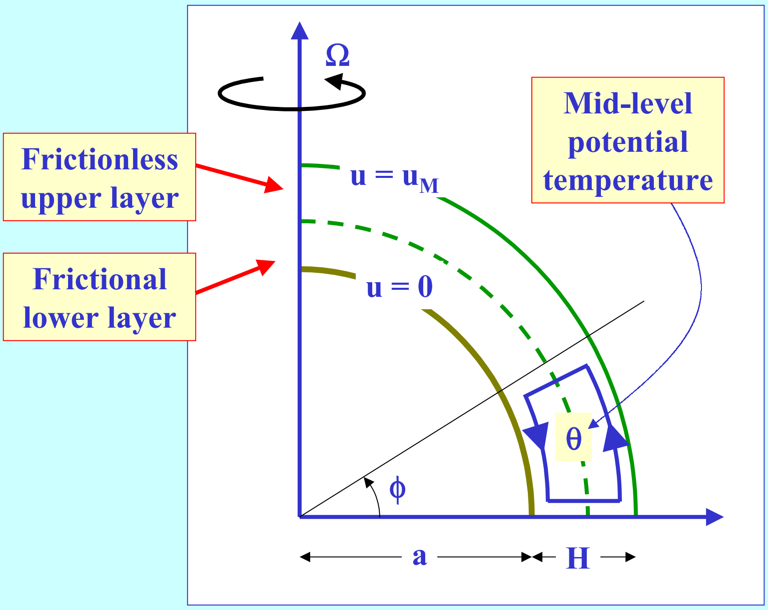
Deviations of geopotential height from the zonal time mean, Φ′



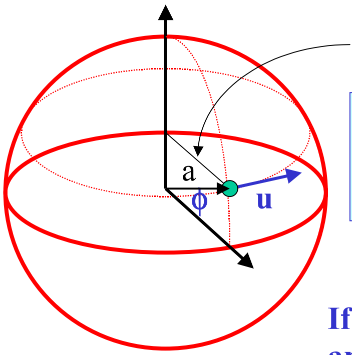
From Gill, 1982

The Held-Hou model of the Hadley circulation

- The Held-Hou model is symmetric about the equator and assumes steady, linear, axisymmetric flow in hydrostatic balance.
- The main features are
  - a simplified representation of solar heating,
  - the use of angular momentum conservation and thermal wind balance.
- Aim: to predict the strength and the width of the Hadley circulation.
- The model has two-levels on the sphere with equatorward flow at the surface and poleward flow at height H.



Absolute angular momentum on a sphere



$$M_{abs} = ua\cos\phi + \Omega a^2\cos^2\phi = (\Omega a\cos\phi + u)a\cos\phi$$

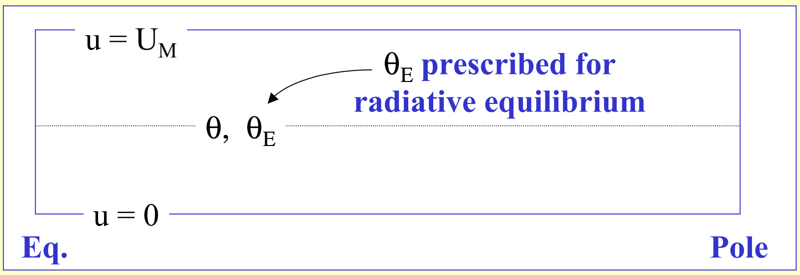
If $u = 0$ **at the equator,** $M_{abs} = \Omega a^2$**, and if** M_{abs} **is conserved,**

$$(\Omega a\cos\phi + u)a\cos\phi = \Omega a^2$$

$$u = \frac{\Omega a^2(1-\cos^2\phi)}{a\cos\phi} = \frac{\Omega a\sin^2\phi}{\cos\phi}$$

Radiative equilibrium

The thermal structure of the atmosphere is characterized by the midlevel potential temperature, θ**.**

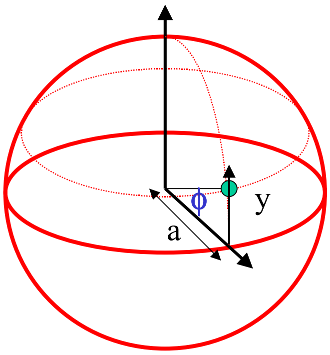


$$\theta_E(\varphi) = \theta_0 - \frac{1}{3}\Delta\theta\left(3\sin^2\varphi - 1\right)$$

Radiative processes are represented using a Newtonian cooling with timescale τ_E **given by**

$$\frac{D\theta}{Dt} = \frac{\theta_E - \theta}{\tau_E}$$

Near equatorial approximation



$$\sin\phi \approx \phi \approx \frac{y}{a}, \quad \cos\phi \approx 1$$

$$\theta_E(y) = \theta_{E0} - \Delta\theta \frac{y^2}{a^2}$$

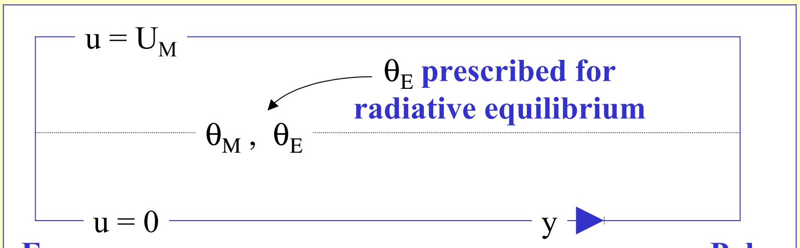
$$\theta_{E0} = \theta_0 + \tfrac{1}{3}\Delta\theta$$

$$U_M = \frac{\Omega}{a} y^2$$

Thermal wind balance

We assume that θ (= θ_M) and u (= U_M) are in thermal wind balance.

$$f\frac{\partial u}{\partial z} = -\frac{g}{\theta}\frac{\partial \theta}{\partial y}$$

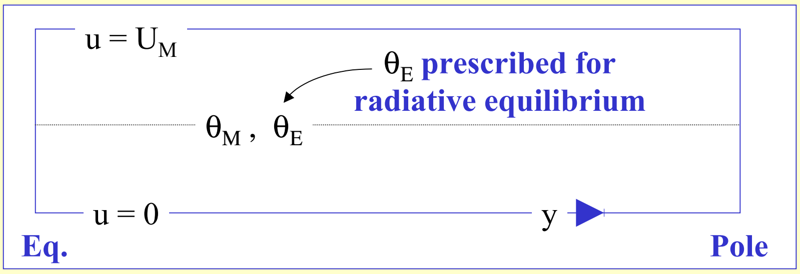


$$f = 2\Omega \sin\phi \approx \frac{2\Omega y}{a}$$

$$\frac{\partial u}{\partial z} = \frac{U_M}{H} = \frac{\Omega}{aH} y^2$$

$$\frac{\partial \theta}{\partial y} = -\frac{2\Omega^2 \theta_0}{a^2 g H} y^3$$

Solution for θ_M



$$\frac{\partial\theta}{\partial y} = -\frac{2\Omega^2\theta_0}{a^2 gH} y^3 \quad \Rightarrow \quad \theta_M = \theta_{M0} - \frac{\Omega^2\theta_0}{2a^2 gH} y^4$$

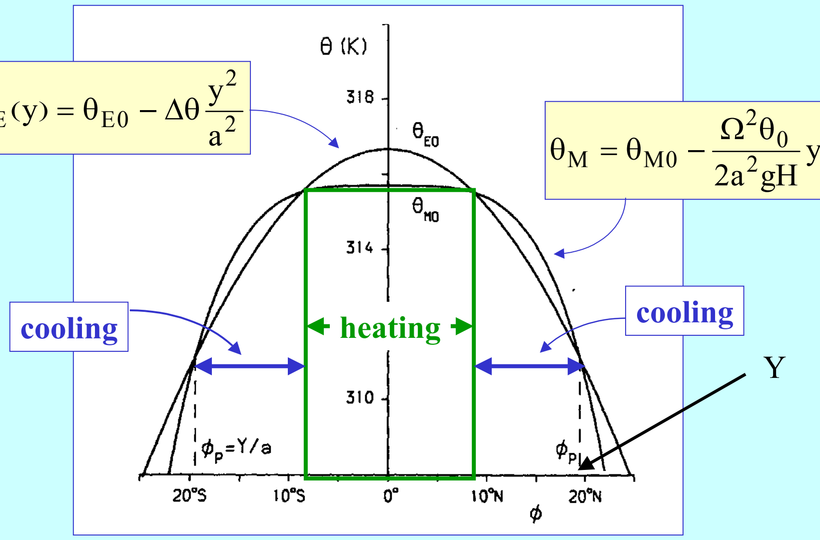
"M" used to remind us that θ has been derived using conservation of angular momentum

the equatorial temperature

Equilibrium temperature, actual temperature

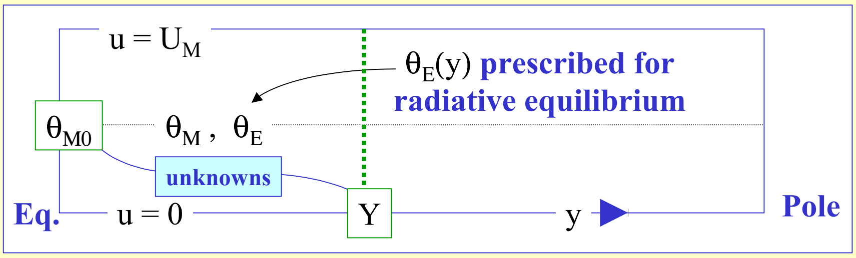
$$\theta_E(y) = \theta_{E0} - \Delta\theta \frac{y^2}{a^2}$$

$$\theta_M = \theta_{M0} - \frac{\Omega^2\theta_0}{2a^2 gH} y^4$$



From James (1994)

Constraint on θ_M



$$\theta_M = \theta_{M0} - \frac{\Omega^2\theta_0}{2a^2gH}y^4$$

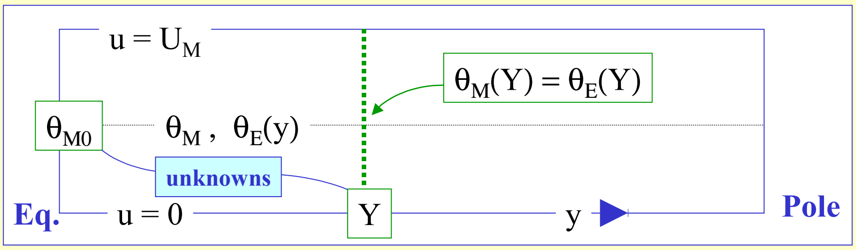
$$\theta_E(y) = \theta_{E0} - \Delta\theta\frac{y^2}{a^2}$$

Steady state $\Rightarrow$ there can be no net heating of an air parcel when it completes a circuit of the Hadley cell:

$$\int_0^Y \frac{D\theta}{Dt}dy = 0 \qquad \frac{D\theta}{Dt} = \frac{\theta_E - \theta_M}{\tau_E}$$

$$\Rightarrow \quad \int_0^Y \theta_M\, dy = \int_0^Y \theta_E\, dy$$

Solution for θ_{M0} and Y



$$\theta_M = \theta_{M0} - \frac{\Omega^2\theta_0}{2a^2gH}y^4$$

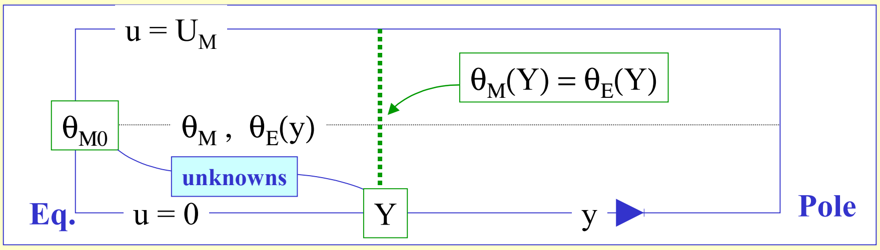
$$\theta_E(y) = \theta_{E0} - \Delta\theta\frac{y^2}{a^2}$$

$$\int_0^Y \theta_M\, dy = \int_0^Y \theta_E\, dy \quad \Rightarrow \quad \theta_{M0} - \frac{\Omega^2\theta_0}{10a^2gH}Y^4 = \theta_{E0} - \frac{\Delta\theta}{3a^2}Y^2$$

Assume that $\theta_M(Y) = \theta_E(Y)$

$$\Rightarrow \quad \theta_{M0} - \frac{\Omega^2\theta_0}{2a^2gH}Y^4 = \theta_{E0} - \frac{\Delta\theta}{a^2}Y^2$$

Solution for θ_{M0} and Y



$$\theta_{M0} - \frac{\Omega^2\theta_0}{10a^2gH}Y^4 = \theta_{E0} - \frac{\Delta\theta}{3a^2}Y^2$$

$$\theta_{M0} - \frac{\Omega^2\theta_0}{2a^2gH}Y^4 = \theta_{E0} - \frac{\Delta\theta}{a^2}Y^2$$

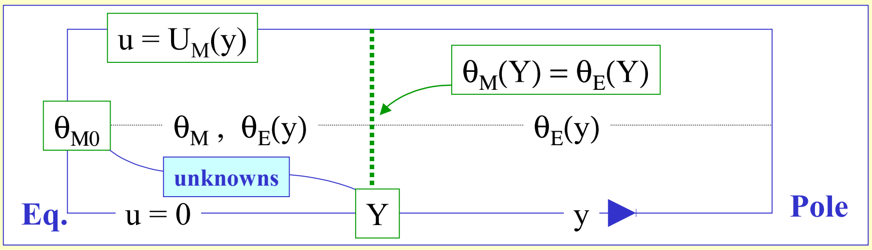
$$Y = \left(\frac{5\Delta\theta gH}{3\Omega^2\theta_0}\right)^{1/2}$$

$$\theta_{M0} = \theta_{E0} - \frac{5\Delta\theta^2 gH}{18a^2\Omega^2\theta_0}$$

Take $\theta_0 = 255$ K, $\Delta\theta = 40$ K **and** H = 12 km $\Rightarrow$ $\underbrace{Y \approx 2400 \text{ km}}$ **and** $\theta_{M0} \approx 0.9$ K **cooler than** $\theta_E(0)$.

≈ in agreement with obs.

Meridional variation of U_M



$U_M = \frac{\Omega}{a}y^2$ **for** $y \le Y$.

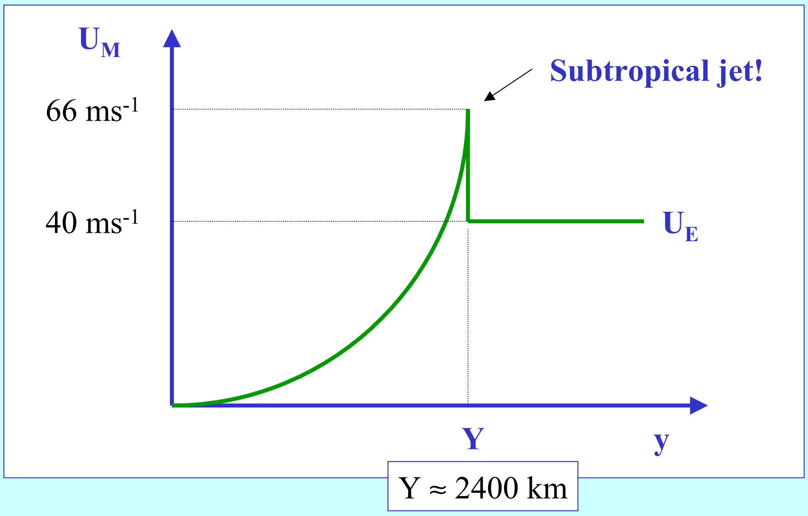
The zonal wind increases quadratically with y **to reach a maximum value of approximately** 66 m s^{-1} **at** y = Y.

Assume that for y > Y, U_M **is in thermal wind balance with** $\theta_E(y)$.

$$U_E = \frac{\Delta\theta gH}{\Omega a\theta_0}$$

U_E **is** 40 ms^{-1}

Zonal wind



Strength of the Hadley circulation in the model

By symmetry v = 0 **at the equator. Then**

$$\frac{D\theta}{Dz} = \frac{\theta_{E0}-\theta_{M0}}{\tau_E} \quad \Rightarrow \quad w\frac{\partial\theta}{\partial z} = \frac{\theta_{E0}-\theta_{M0}}{\tau_E}$$

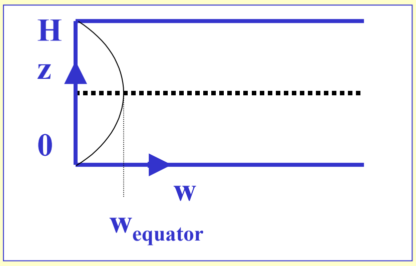
Assume constant Brunt-Väisälä frequency, N.

$$\Rightarrow \quad w_{equator} = \frac{g}{\theta_0 N^2}\frac{\left(\theta_{E0}-\theta_{M0}\right)}{\tau_E}$$

Using $\tau_E \sim 15$ **days and** $N \sim 10^{-2}$ s^{-1} **gives** $w \sim 0.27$ **mm** s^{-1}

Strength of the Hadley circulation in the model

$$w_{equator} = \frac{g}{\theta_0 N^2}\frac{(\theta_{E0} - \theta_{M0})}{\tau_E}$$



$$w = 4w_{equator} z(H - z)$$

$$\left.\frac{\partial w}{\partial z}\right|_{z=H} = -4w_{equator} H$$

$$\Rightarrow \left.\frac{\partial v}{\partial y}\right|_{z=H} \approx 4w_{equator} H \Rightarrow v|_{z=H} \approx 4w_{equator} Hy$$

$$w_{equator} \sim 0.27 \text{ mm s}^{-1} \Rightarrow v(Y)|_{z=H} \approx 4w_{equator} HY \sim 21 \text{ cm s}^{-1}$$

Summary

- **Observations show that the strength of the meridional flow in the Hadley circulation is approximately 1 m s^{-1}.**
- **Thus although the Held-Hou model provides a reasonable estimate of the geometry of the Hadley circulation it gives a very poor estimate of the strength of the circulation.**
- **The Held-Hou model predicts that the width of the Hadley cell is inversely proportional to the planetary rotation rate.**

$$Y = \left(\frac{5\Delta\theta gH}{3\Omega^2\theta_0}\right)^{1/2}$$

- **This prediction has been confirmed in more realistic models of planetary atmospheres.**

Summary

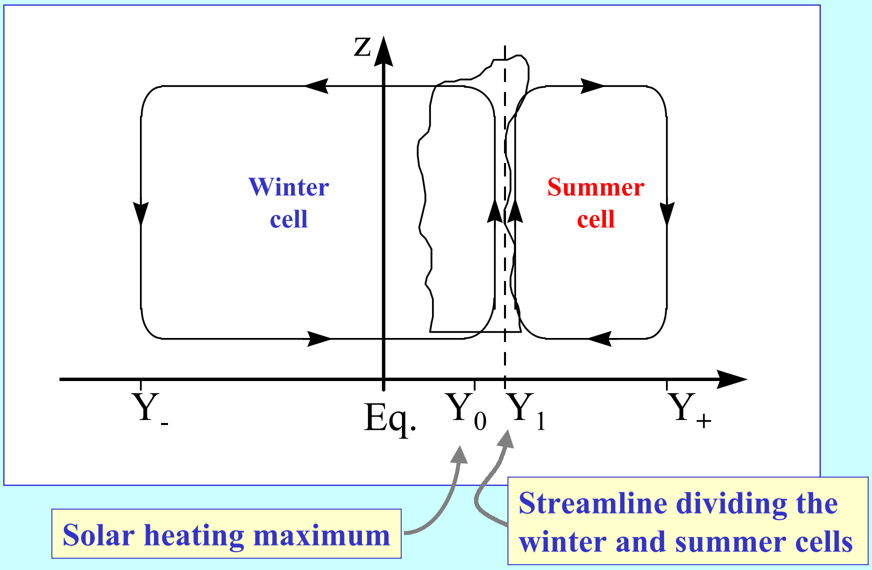
$$Y = \left(\frac{5\Delta\theta gH}{3\Omega^2\theta_0} \right)^{1/2}$$

- At low rotation rates the Hadley cells extend far polewards and account for most of the heat transport from equator to pole.
- At high rotation rates the Hadley cells are confined near the equator and baroclinic waves poleward of the Hadley circulations are responsible for a significant proportion of the heat transport.
- For more details see, for example, James (1994, Ch. 10).
- Although the Held-Hou model gives a reasonable estimate for the size of the Hadley circulation it gives a very poor estimate of its strength.

Summary

- A better model can be formulated by relaxing one of the assumptions of the Held-Hou model, namely that of symmetry about the equator.
- Although the annual mean solar heating is symmetric about the equator, the heating at any given time is generally not. Thus the response to the solar forcing is not necessarily symmetric about the equator.
- We saw earlier that although the annual mean Hadley circulation is symmetric about the equator, the monthly mean Hadley circulation may be very asymmetric.
- Lindzen and Hou (1988) extended the Held-Hou model to allow for such an asymmetry whilst retaining the other assumptions described above.

The extended Held-Hou Model



Extensions

Radiative processes are represented again using a Newtonian cooling with timescale τ_E given by

$$\frac{D\theta}{Dt} = \frac{\theta_E - \theta}{\tau_E}$$

The equilibrium potential potential temperature is

$$\theta_E(y) = \theta_{E0} - \frac{\Delta\theta}{a^2}(y^2 - Y_o^2)$$

θ_E is a maximum at Y_o

Use conservation of absolute angular momentum

Extensions (cont)

Conservation of absolute angular momentum

$$\Rightarrow \quad U_M = \frac{\Omega}{a}\left(y^2 - Y_1^2\right)$$

Thermal wind balance

$$\frac{\partial\theta}{\partial y} = -\frac{2\Omega^2\theta_0}{a^2 gH} y\left(y^2 - Y_1^2\right)$$

$$\Rightarrow \quad \theta_M(y) = \theta_M(Y_1) - \frac{2\theta_0\Omega^2}{4a^2 gH}\left(y^2 - Y_1^2\right)^2$$

Extensions (cont)

$$\theta_M(y) = \theta_M(Y_1) - \frac{2\theta_0\Omega^2}{4a^2 gH}\left(y^2 - Y_1^2\right)^2$$

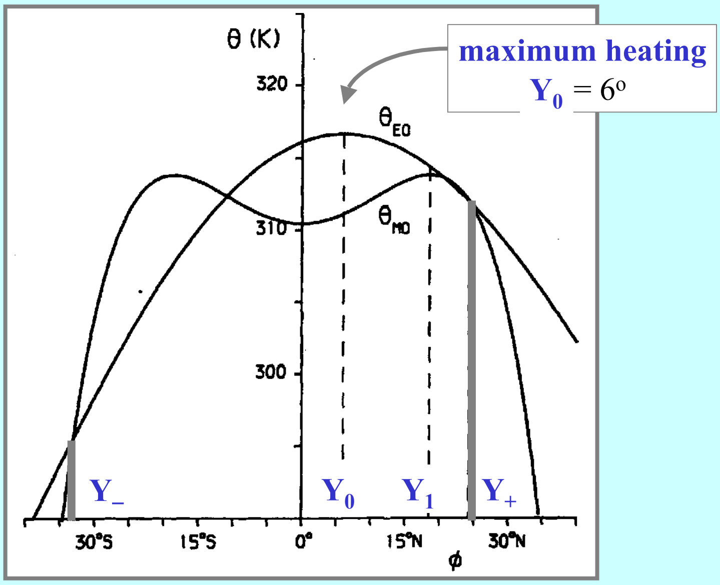
$$\theta_E(y) = \theta_{E0} - \frac{\Delta\theta}{a^2}(y^2 - Y_o^2)$$

$$\frac{D\theta}{Dt} = \frac{\theta_E - \theta}{\tau_E}$$

$$\int_{Y_1}^{Y_+} (\theta_E - \theta_M)\,dy = 0 \quad \text{and} \quad \int_{Y_1}^{Y_-} (\theta_E - \theta_M)\,dy = 0$$

+ **continuity of potential temperature at** $y = Y_+$ **and** $y = Y_-$.

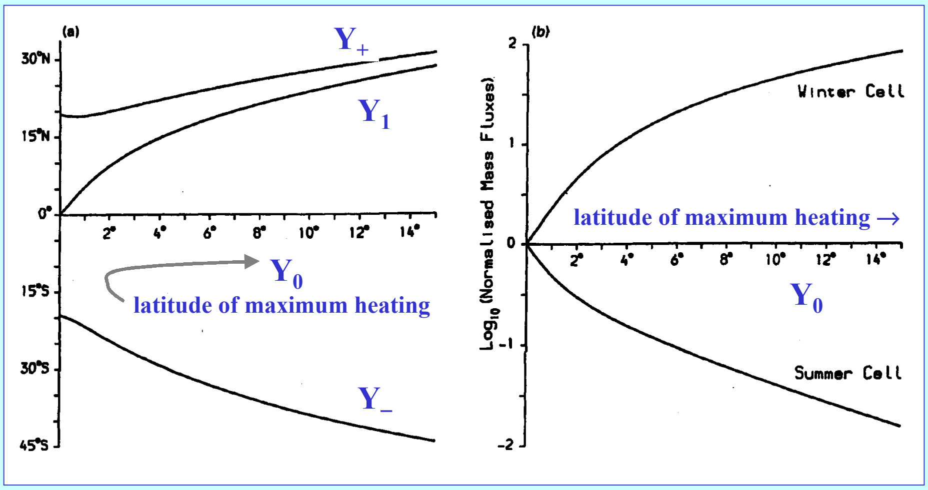
Four unknowns: Y_1, Y_+, Y_-, **and** $\theta_M(Y_1)$.



The Held-Hou model for asymmetric heating.

(From James, 1994)

Results of the Held-Hou model for asymmetric heating with varying latitude of maximum heating.



(a) Poleward extent of the summer and winter circulations and of the latitude of the dividing streamline. (b) Mass flux carried by the winter and summer cells.

A recent reference:

Polvani & Sobel, 2001:
The Hadley circulation and the weak temperature approximation.
***J. Atmos Sci.*, 59, 1744-1752.**

About θ_e

Equivalent potential temperature

First law of thermodynamics

$$\frac{dq}{T} = c_p d\ln\theta \quad \Rightarrow \quad \frac{D}{Dt}\ln\theta = \frac{1}{c_p T}\frac{Dq}{Dt}$$

$$\frac{Dq}{Dt} = -L\frac{Dw_s}{Dt}$$

condensation rate

$$\frac{D}{Dt}\ln\theta = -\frac{L}{c_p T}\frac{Dw_s}{Dt} \approx -\frac{D}{Dt}\left(\frac{Lw_s}{c_p T}\right)$$

$$\ln\theta_e = \ln\theta + (Lw_s / c_p T)$$

$$\Rightarrow \quad \frac{D}{Dt}\ln\theta_e = 0$$

Equivalent potential temperature in the tropics