Tropical cyclones. Problem Sheet 3, SS2014

Exercise 1

The Ertel potential vorticity, P, is defined as

$$P = \frac{(\omega + 2\mathbf{\Omega}) \cdot \nabla\theta}{\rho}.$$

and has the useful property that, for the general adiabatic motion of a rotating stratified fluid, it is materially conserved, i.e.,

$$\frac{DP}{Dt} = 0.$$

Show that for an axisymmetric vortex with velocity vector $(0, v(r, z), 0)$,

$$P = \frac{1}{\rho}\left(-\frac{\partial v}{\partial z}\frac{\partial \theta}{\partial r} + (\zeta + f)\frac{\partial \theta}{\partial z}\right). \tag{1}$$

Exercise 2

Show that P as defined in Eq. (1) is proportional to $\nabla\theta \wedge \nabla M$, where M is the absolute angular momentum.

Show that $P > 0$ when the the slope of the M-surfaces are more steeply inclined to the horizontal than the θ-surfaces and $P < 0$ when the θ-surfaces are more steeply inclined to the horizontal than the M-surfaces.

Note that when the M- and θ-surfaces coincide, $P = 0$.

Exercise 3

Show that the inertial stability parameter, I, given by

$$I^2 = \frac{1}{r^3}\frac{\partial M^2}{\partial r}$$

is equal to $\xi\zeta_a$, where ξ is twice the absolute angular velocity and ζ_a is the absolute vorticity.

Exercise 4

Show that for a two-dimensional incompressible flow in the (x,y)-plane, the streamfunction ψ and vorticity ζ are related by the equation

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = \zeta.$$

- Sketch the streamfunction field associated with two line vortices of equal strength separated by a distance a in a large body of fluid.

- Show that the vortices will tend to rotate around each other.

[Hint: you may think of the vortices being represented by delta functions $\zeta_o \delta(x - \frac{1}{2}a)\delta(y)$ and $\zeta_o \delta(x + \frac{1}{2}a)\delta(y)$, where ζ_o is the strength of the vortices, and you may assume that $\psi \to 0$ as the distance from the vortices increases. Think in terms of the membrane analogy discussed in class.]

- Sketch the streamfunction field that would arise if the vortices had the same strength but opposite signs.

- Show that, in the latter case, the vortices will tend to move together at right angles to the line joining them.