Tropical cyclones. Problem Sheet 2, SS2011

Exercise 1

Show that for a two-dimensional incompressible flow in the (x,y)-plane, the streamfunction ψ and vorticity ζ are related by the equation

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = \zeta.$$

- Sketch the streamfunction field associated with two line vortices of equal strength separated by a distance a in a large body of fluid.
- Show that the vortices will tend to rotate around each other.

[Hint: you may think of the vortices being represented by delta functions $\zeta_o\delta(x - \frac{1}{2}a)\delta(y)$ and $\zeta_o\delta(x + \frac{1}{2}a)\delta(y)$, where ζ_o is the strength of the vortices, and you may assume that $\psi \to 0$ as the distance from the vortices increases. Think in terms of the membrane analogy discussed in class.]

- Sketch the streamfunction field that would arise if the vortices had the same strength but opposite signs.
- Show that, in the latter case, the vortices will tend to move together at right angles to the line joining them.

Exercise 2

Show that with the coordinate transformation

$$X = x + v/f, \qquad Z = z,$$

$$\frac{\partial^2\psi}{\partial z^2} = \frac{\partial^2\psi}{\partial Z^2} + \frac{2S}{f}\frac{\partial^2\psi}{\partial X\partial Z} + \frac{S^2}{f^2}\frac{\partial^2\psi}{\partial X^2} + \frac{1}{f}\frac{\partial S}{\partial z}\frac{\partial\psi}{\partial X}$$

Exercise 3

In the slab-symmetric model for the Hadley circulation discussed in class, the Sawyer-Eliassen equation is:

$$\left(N^2 + \frac{\partial b}{\partial z}\right)\frac{\partial^2\psi}{\partial x^2} - 2fS\frac{\partial^2\psi}{\partial x\partial z} + f\zeta_a\frac{\partial^2\psi}{\partial z^2} = \frac{\partial\dot{V}}{\partial z} - \frac{\partial\dot{B}}{\partial x}.$$

- Write down the form of the equation for a barotropic vortex with zonal wind $v(x)$ and no diabatic heating when $\partial b/\partial z = 0$.

- Sketch the circulation induced by a frictional force $\dot{V} = -Fz$ for $z < h$ and $\dot{V} = 0$ for $z > h$, where h is typical of the depth of the atmospheric boundary layer (about 1 km) when $v = v_o x \exp(-x^2/x_o^2)$. Here F, h v_o and x_o are positive constants.

Exercise 4

Show that the inertial stability parameter I given by

$$I^2 = \frac{1}{r^3}\frac{\partial M^2}{\partial r}$$

is equal to $\xi\zeta_a$, where ξ is twice the absolute angular velocity and ζ_a is the absolute vorticity.

Show that the potential vorticity of the axi-symmetric flow with tangential wind speed $v(r, z)$ and potential temperature $\theta(r, z)$ is positive if the slope of the M-surfaces is larger than that of the isentropic surfaces, where M is the absolute angular momentum.