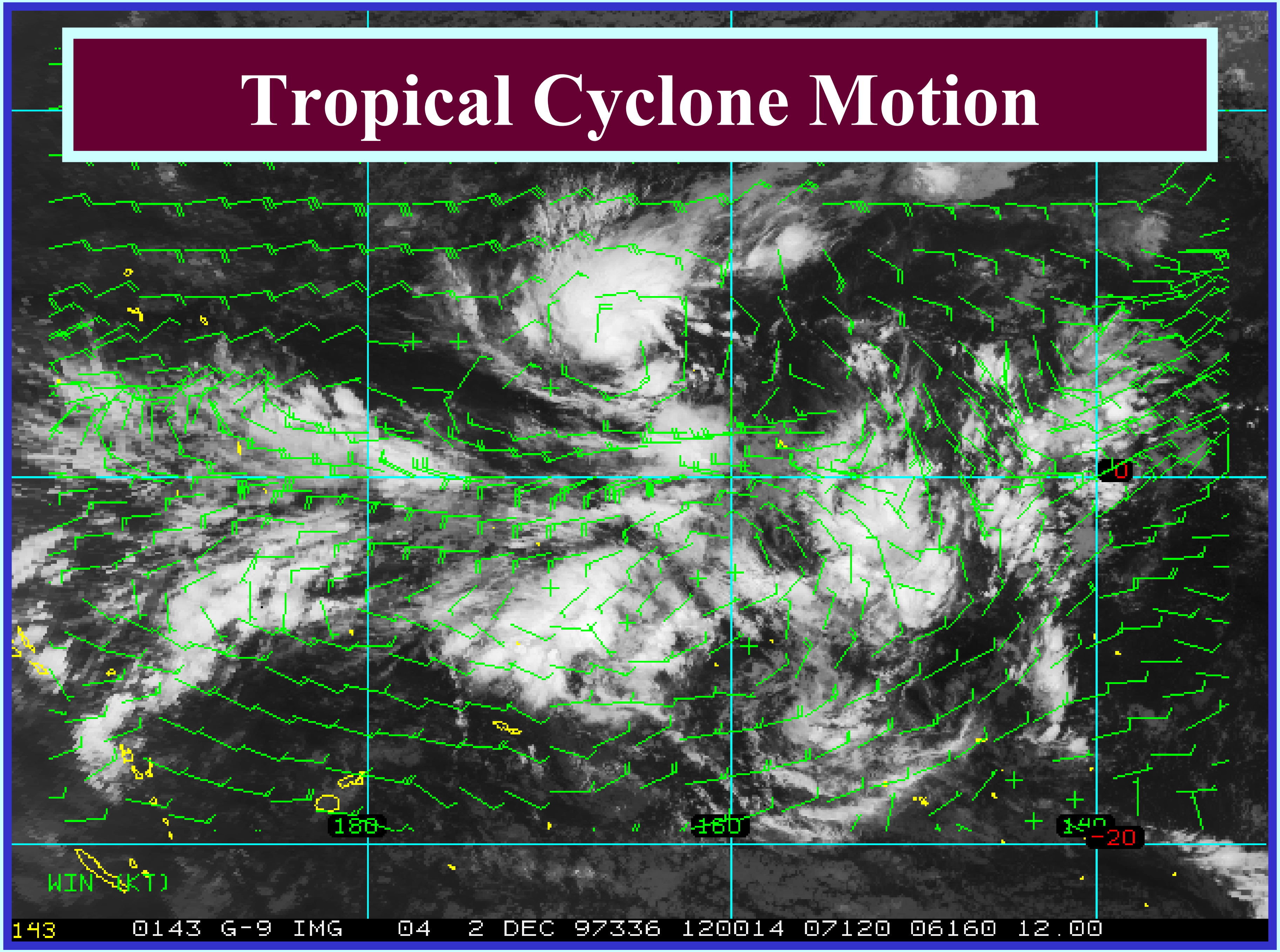
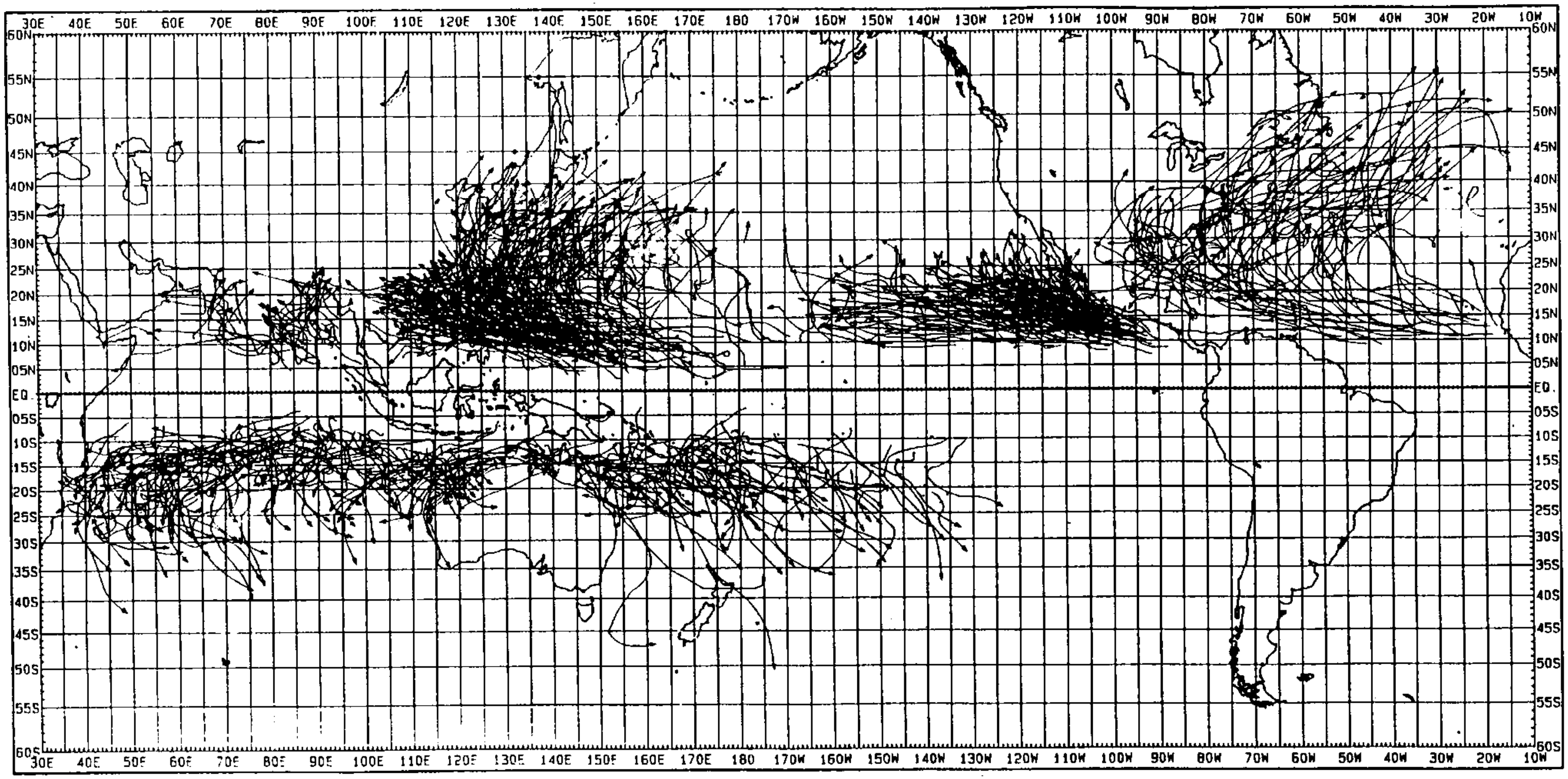
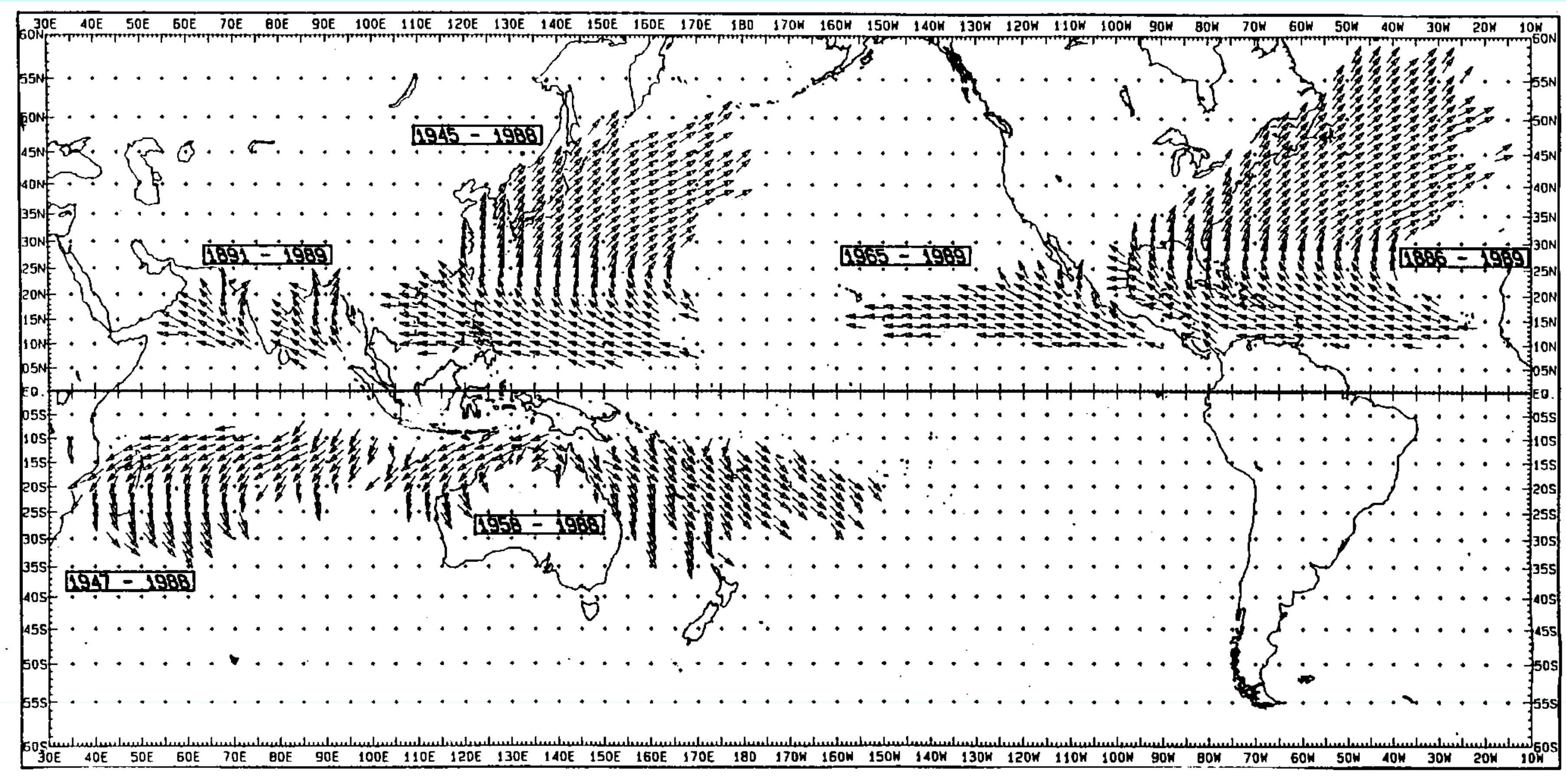
Tropical Cyclone Motion



Tropical cyclone tracks (1979-1988)



Mean direction of TC motion



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A numerical study of tropical cyclone motion using a barotropic model. I: The role of vortex asymmetries

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An Analytical Theory of Tropical Cyclone Motion Using a Barotropic Model

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Vortex motion in relation to the absolute vorticity gradient of the vortex environment

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Two-dimensional barotropic flow

$$\frac{\partial}{\partial t}(\zeta + f) + u\frac{\partial}{\partial x}(\zeta + f) + v\frac{\partial}{\partial y}(\zeta + f) = 0, \tag{1.1}$$

where u and v are the velocity components in the x and y directions, respectively. For an incompressible fluid, the continuity equation is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} = 0, \tag{1.2}$$

and accordingly there exists a streamfunction ψ such that

$$u = -\frac{\partial \psi}{\partial y}, \quad v = \frac{\partial \psi}{\partial x}, \tag{1.3}$$

and

$$\zeta = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}, \tag{1.4}$$

The partitioning problem

The partitioning method can be illustrated mathematically as follows. Let the total wind be expressed as $\mathbf{u} = \mathbf{u_s} + \mathbf{U}$, where $\mathbf{u_s}$ denotes the symmetric velocity field and $\mathbf{U}$ is the vortex environment vorticity, and define $\zeta_s = \mathbf{k} \cdot \nabla \wedge \mathbf{u_s}$ and $\Gamma = \mathbf{k} \cdot \nabla \wedge \mathbf{U}$, where $\mathbf{k}$ is the unit vector in the vertical. Then Eq. (1.1) can be partitioned into two equations:

$$\frac{\partial \zeta_s}{\partial t} + \mathbf{c}(t) \cdot \nabla \zeta_s = 0, \tag{1.5}$$

and

$$\frac{\partial \Gamma}{\partial t} = -\mathbf{u_s} \cdot \nabla(\Gamma + f) - (\mathbf{U} - \mathbf{c}) \cdot \nabla \zeta_s - \mathbf{U} \cdot \nabla(\Gamma + f), \tag{1.6}$$

noting that $\mathbf{u_s} \cdot \nabla \zeta_s = 0$, because for a symmetric vortex $\mathbf{u_s}$ is normal to $\nabla \zeta_s$. Equation (1.5) states that the symmetric vortex translates with speed $\mathbf{c}$ and Eq. (1.6) is an equation for the evolution of the asymmetric vorticity. Having solved the latter equation for $\Gamma(\mathbf{x}, t)$, we can obtain the corresponding asymmetric streamfunction by solving Eq. (1.4) in the form $\nabla^2 \psi_a = \Gamma$. The vortex translation velocity $\mathbf{c}$ may be obtained by calculating the speed $\mathbf{U_c} = \mathbf{k} \wedge \nabla \psi_a$ at the vortex centre. In some

The partitioning problem

be obtained by calculating the speed $\mathbf{U_c} = \mathbf{k} \wedge \nabla \psi_a$ at the vortex centre. In some situations it is advantageous to transform the equations of motion into a frame of reference moving with the vortex[2]. Then Eq. (1.5) becomes $\partial \zeta_s / \partial t \equiv 0$ and the vorticity equation (1.6) becomes

$$\frac{\partial \Gamma}{\partial t} = -\mathbf{u_s} \cdot \nabla(\Gamma + f) - (\mathbf{U} - \mathbf{c}) \cdot \nabla \zeta_s - (\mathbf{U} - \mathbf{c}) \cdot \nabla(\Gamma + f). \tag{1.7}$$

Symmetric vortex in a uniform flow

Consider a barotropic vortex with an axisymmetric vorticity distribution embedded in a uniform zonal air stream on an f-plane. The streamfunction for the flow has the form:

$$\psi(x, y) = -Uy + \psi'(r), \tag{1.8}$$

where $r^2 = (x - Ut)^2 + y^2$. The corresponding velocity field is

$$\mathbf{u} = (U, 0) + \left(-\frac{\partial \psi'}{\partial y}, \frac{\partial \psi'}{\partial x}\right), \tag{1.9}$$

The relative vorticity distribution, $\zeta = \nabla^2\psi$, is symmetric about the point $(x - Ut, 0)$, which translates with speed U in the x-direction. However, neither the streamfunction distribution $\psi(x, y, t)$, nor the pressure distribution $p(x, y, t)$, are symmetric and, in general, the locations of the minimum central pressure, maximum relative vorticity, and minimum streamfunction (where $\mathbf{u} = \mathbf{0}$) do not coincide. In particular, there are three important deductions from (1.9):

Symmetric vortex in a uniform flow

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- The total velocity field of the translating vortex is not symmetric, and
- The maximum wind speed is simply the arithmetic sum of U and the maximum tangential wind speed of the symmetric vortex, $V_m = (\partial \psi' / \partial r)_{max}$.
- The maximum wind speed occurs on the right-hand-side of the vortex in the direction of motion in the northern hemisphere and on the left-hand-side in the southern hemisphere.

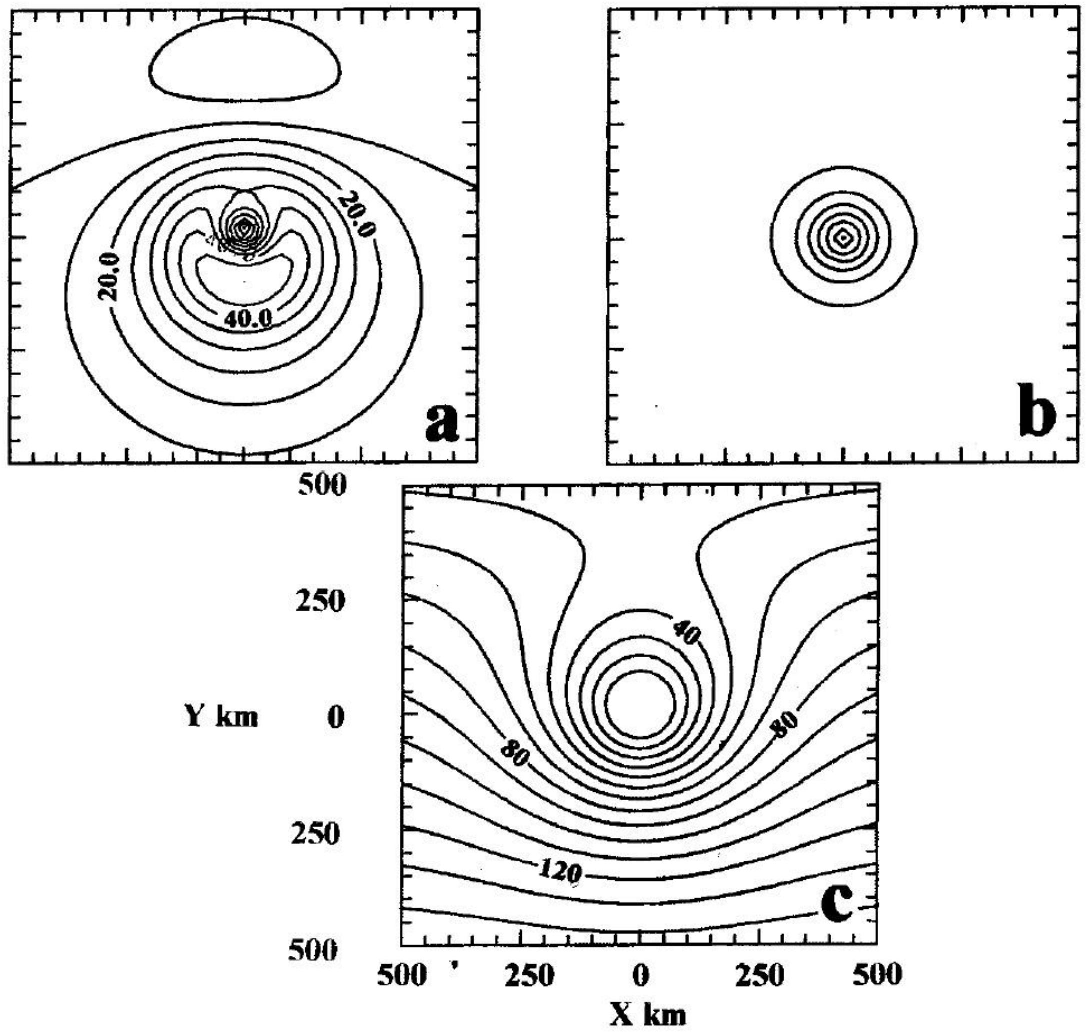


Figure 1.1: Contour plots of (a) total wind speed, (b) relative vorticity, and (c) streamlines, for a vortex with a symmetric relative vorticity distribution and maximum tangential wind speed of 40 m s^{-1} in a uniform zonal flow with speed 10 m s^{-1} on an f-plane. The maximum tangential wind speed occurs at a radius of 100 km (for the purpose of illustration). The contour intervals are: 5 m s^{-1} for wind speed, 2×10^{-4} s^{-1} for relative vorticity and 1×10^{4} m^2 s^{-1} for streamfunction.

Because the vorticity field is Galilean invariant while the pressure field and streamfunction fields are not, it is advantageous to define the vortex centre as the location of maximum relative vorticity and to transform the equations of motion to a coordinate system $(X, Y) = (x - Ut, y)$, whose origin is at this centre[3]. In this frame of reference, the streamfunction centre is at the point $(0, Y_s)$, where

$$U - \Phi(Y_s)Y_s = 0, \tag{1.10}$$

and $\Phi = \psi'(r)/r$. This point is to the left of the vorticity centre in the direction of motion in the northern hemisphere. In the moving coordinate system, the momentum equations may be written in the form

$$\nabla p = \rho\Phi(\Phi + f)(X, Y) + \rho f(0, U). \tag{1.11}$$

The minimum surface pressure occurs where $\nabla p = 0$, which from (1.11) is at the point $(0, Y_p)$ where

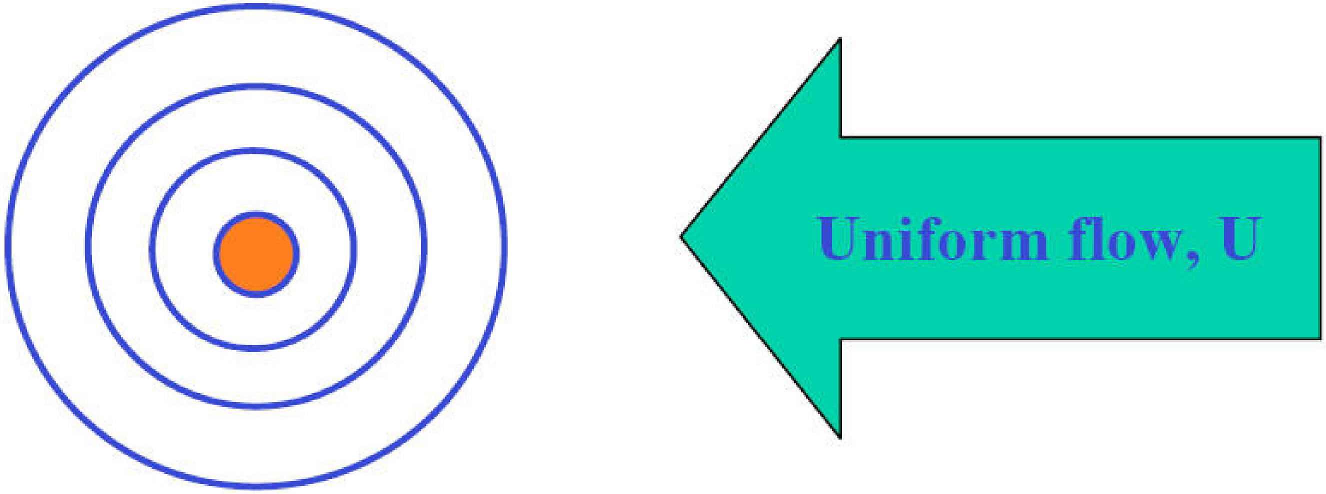
$$Y_p\Phi(Y_p)(\Phi(Y_p) + f) = fU. \tag{1.12}$$

We show that, although Y_p and Y_s are not zero and not equal, they are for practical purposes relatively small.

Consider the case where the inner core is in solid body rotation out to the radius r_m, of maximum tangential wind speed v_m, with uniform angular velocity $\Omega = v_m/r_m$. Then $\psi'(r) = \Omega r$ and $\Phi = \Omega$. It follows readily that $Y_s/r_m = U/v_m$ and $Y_p/r_m = U/(v_m Ro_m)$, where $Ro_m = v_m/(r_m f)$ is the Rossby number of the vortex core which is large compared with unity in a tropical cyclone. Taking typical values: $f = 5 \times 10^{-5}\ s^{-1}$, $U = 10$ m s^{-1}, $v_m = 50$ m s^{-1}, $r_m = 50$ km, $Ro_m = 20$ and $Y_s = 10$ km, $Y_p = 0.5$ km, the latter being much smaller than r_m. Clearly, for weaker vortices (smaller v_m) and/or stronger basic flows (larger U), the values of Y_s/r_m and Y_p/r_m are comparatively larger and the difference between the various centres may be significant.

Tropical cyclone motion

f-plane



Symmetric vortex $\boldsymbol{\zeta} = \boldsymbol{\zeta}(r,t)$, $\mathbf{v} = \mathbf{v}(r,t)$

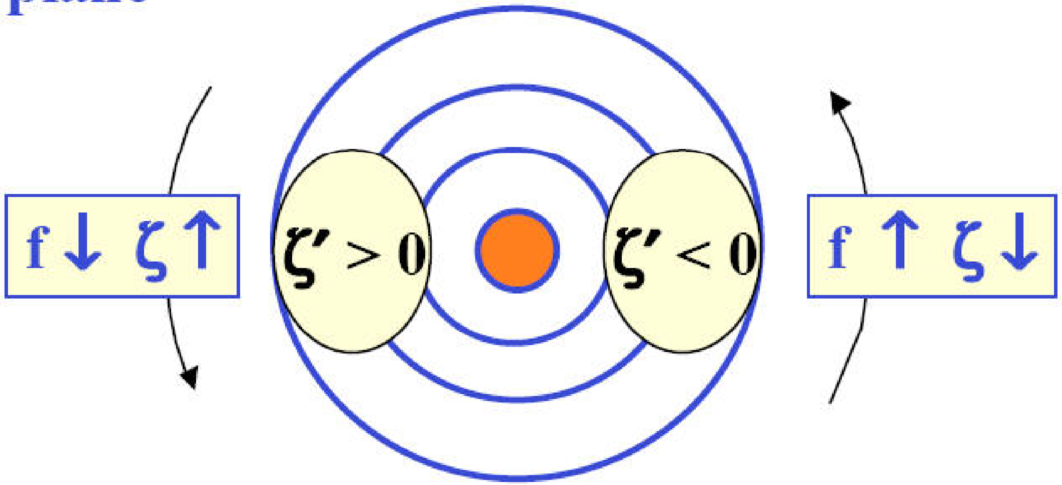
$$\frac{\partial \zeta}{\partial t} + \mathbf{u} \cdot \nabla \zeta = 0 \qquad \mathbf{v} \cdot \nabla \zeta = 0 \quad \Rightarrow \quad \frac{\partial \zeta}{\partial t} + U \frac{\partial \zeta}{\partial x} = 0$$

$$\mathbf{u} = U\mathbf{i} + \mathbf{v}$$

Tropical cyclone motion

β-plane

$f = f_o + \beta y$



Symmetric vortex $\zeta = \zeta(r,t)$, $v = v(r,t)$

$$\frac{\partial \zeta}{\partial t} + \mathbf{u} \cdot \nabla \zeta + \beta y = 0 \qquad \mathbf{v} \cdot \nabla \zeta \approx 0$$

Vortex motion on a beta-plane

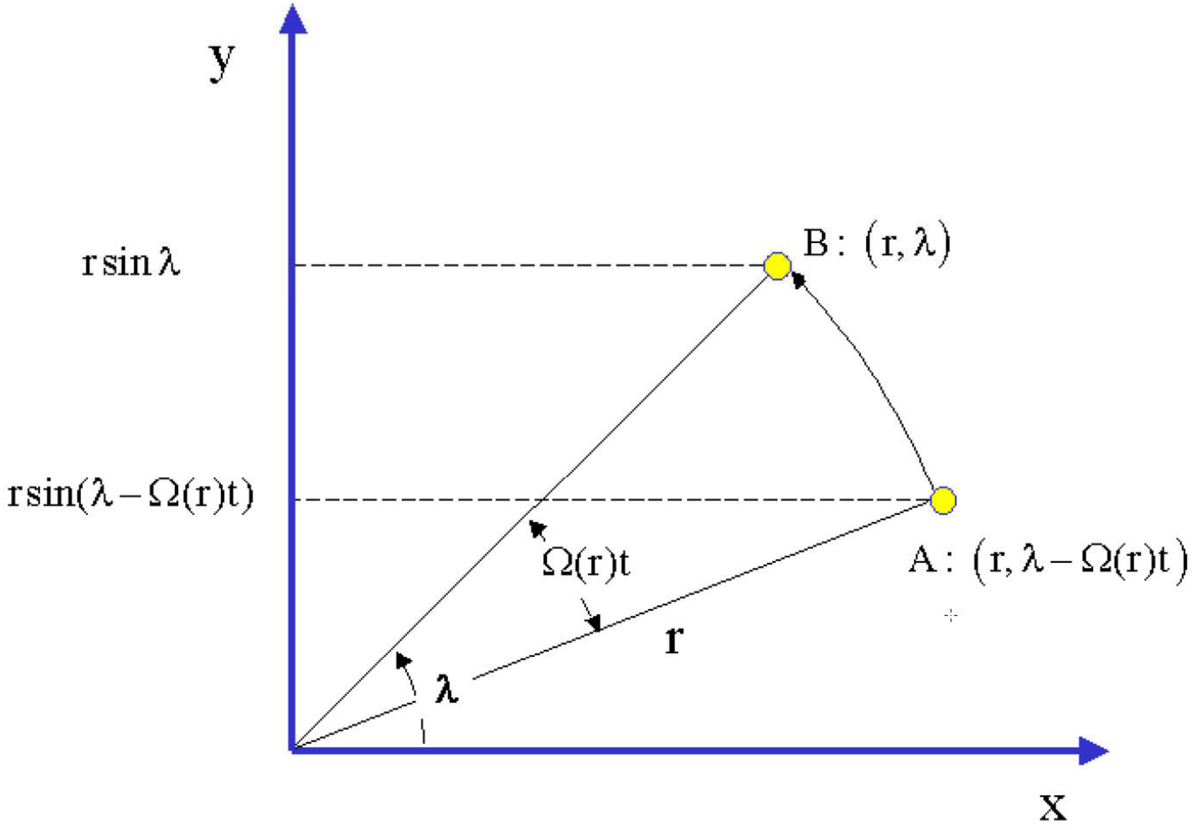


Figure 1.2: An air parcel moving in a circular orbit of radius r with angular velocity $\Omega(r)$ is located at the point B with polar coordinates (r, λ) at time t. At time $t = 0$ the parcel was located at point A with coordinates $(r, \lambda - \Omega(r)t)$. During this time it undergoes a meridional displacement $r[\sin\lambda - \sin(\lambda - \Omega(r)t)]$.

Vortex motion on a beta-plane

the moving vortex (we discuss the reason for the vortex movement below). Consider an air parcel that at time t is at the point with polar coordinate (r, λ) located at the (moving) vortex centre (Fig. 1.2). This parcel would have been at the position $(r, \lambda - \Omega(r)t)$ at the initial instant, where $\Omega(r) = V(r)/r$ is the angular velocity at radius r and $V(r)$ is the tangential wind speed at that radius. The initial vorticity of the parcel is $\zeta_s(r) + f_0 + \beta r \sin(\lambda - \Omega(r)t)$ while the vorticity of a parcel at its current location is $\zeta(r) + f_0 + \beta r \sin\lambda$. Therefore the vorticity perturbation $\zeta_a(r, \lambda)$ at the point (r, λ) at time t is $\zeta(r) - \zeta_s(r)$, or

$$\zeta_a(r, \lambda) = \beta r[\sin\lambda - \sin(\lambda - \Omega(r)t)]$$

or

$$\zeta_a(r, \lambda) = \zeta_1(r, t)\cos\lambda + \zeta_2(r, t)\sin\lambda, \tag{1.13}$$

where

$$\zeta_1(r, t) = -\beta r \sin(\Omega(r)t), \quad \zeta_2(r, t) = -\beta r[1 - \cos(\Omega(r)t)]. \tag{1.14}$$

Vortex motion on a beta-plane

$$\zeta_1(r,t) = -\beta r \sin(\Omega(r)t), \quad \zeta_2(r,t) = -\beta r[1 - \cos(\Omega(r)t)]. \tag{1.14}$$

We can now calculate the asymmetric streamfunction $\psi_a(r,\lambda,t)$ corresponding to this asymmetry using Eq. (1.4). The solution should satisfy the boundary condition that $\psi \to 0$ as $r \to \infty$. It is reasonable to expect that ψ_a will have the form:

$$\psi_a(r,\lambda) = \Psi_1(r,t)\cos\lambda + \Psi_2(r,t)\sin\lambda, \tag{1.15}$$

and it is shown in Appendix 3.4.1 that

$$\Psi_n(r,t) = -\frac{r}{2}\int_r^\infty \zeta_n(p,t)\,dp - \frac{1}{2r}\int_0^r p^2\zeta_n(p,t)\,dp \quad (n = 1,2), \tag{1.16}$$

The Cartesian velocity components $(U_a, V_a) = (-\partial\Psi_a/\partial y, \partial\Psi_a/\partial x)$ are given by

$$U_a = \cos\lambda\sin\lambda\left[\frac{\Psi_1}{r} - \frac{\partial\Psi_1}{\partial r}\right] - \sin^2\lambda\frac{\partial\Psi_2}{\partial r} - \cos^2\lambda\frac{\Psi_2}{r}, \tag{1.17}$$

$$V_a = \cos^2\lambda\frac{\partial\Psi_1}{\partial r} + \sin^2\lambda\frac{\Psi_1}{r} - \cos\lambda\sin\lambda\left[\frac{\Psi_2}{r} - \frac{\partial\Psi_2}{\partial r}\right]. \tag{1.18}$$

Vortex motion on a beta-plane

In order that these expressions give a unique velocity at the origin, they must be independent of λ as $r \to 0$, in which case

$$\left.\frac{\partial \Psi_n}{\partial r}\right|_{r=0} = \lim_{r\to 0} \frac{\Psi_n}{r}, \quad (n = 1, 2).$$

Then

$$(U_a, V_a)_{r=0} = \left[-\left.\frac{\partial \Psi_2}{\partial r}\right|_{r=0}, \left.\frac{\partial \Psi_1}{\partial r}\right|_{r=0} \right], \tag{1.19}$$

and using (1.16) it follows that

$$\left.\frac{\partial \Psi_n}{\partial r}\right|_{r=0} = -\frac{1}{2} \int_0^\infty \zeta_n(p, t)\, dp. \tag{1.20}$$

If we make the reasonable assumption that the symmetric vortex moves with the velocity of the asymmetric flow across its centre, the vortex speed is simply

$$\mathbf{c}(t) = \left[-\left.\frac{\partial \Psi_2}{\partial r}\right|_{r=0}, \left.\frac{\partial \Psi_1}{\partial r}\right|_{r=0} \right], \tag{1.21}$$

which can be evaluated using (1.14) and (1.20).

Vortex motion on a beta-plane

If we make the reasonable assumption that the symmetric vortex moves with the velocity of the asymmetric flow across its centre, the vortex speed is simply

$$\mathbf{c}(t) = \left[-\left.\frac{\partial \Psi_2}{\partial r}\right|_{r=0}, \left.\frac{\partial \Psi_1}{\partial r}\right|_{r=0} \right], \qquad (1.21)$$

which can be evaluated using (1.14) and (1.20).

The assumption is reasonable because *at the vortex centre* $\zeta \gg f$ and the governing equation (1.1) expresses the fact that $\zeta + f$ is conserved following the motion. Since the symmetric circulation does not contribute to advection across the vortex centre (recall that the vortex centre is defined as the location of the maximum relative vorticity), advection must be by the asymmetric component. With the method of partitioning discussed in section 3.2, this component is simply the environmental flow by definition. The slight error committed in supposing that ζ is conserved rather than $\zeta + f$ is equivalent to neglecting the propagation of the vortex centre. The track error amounts to no more than a few kilometers per day which is negligible compared with the actual vortex displacements (e.g., see Fig. 1.6).

Some calculations

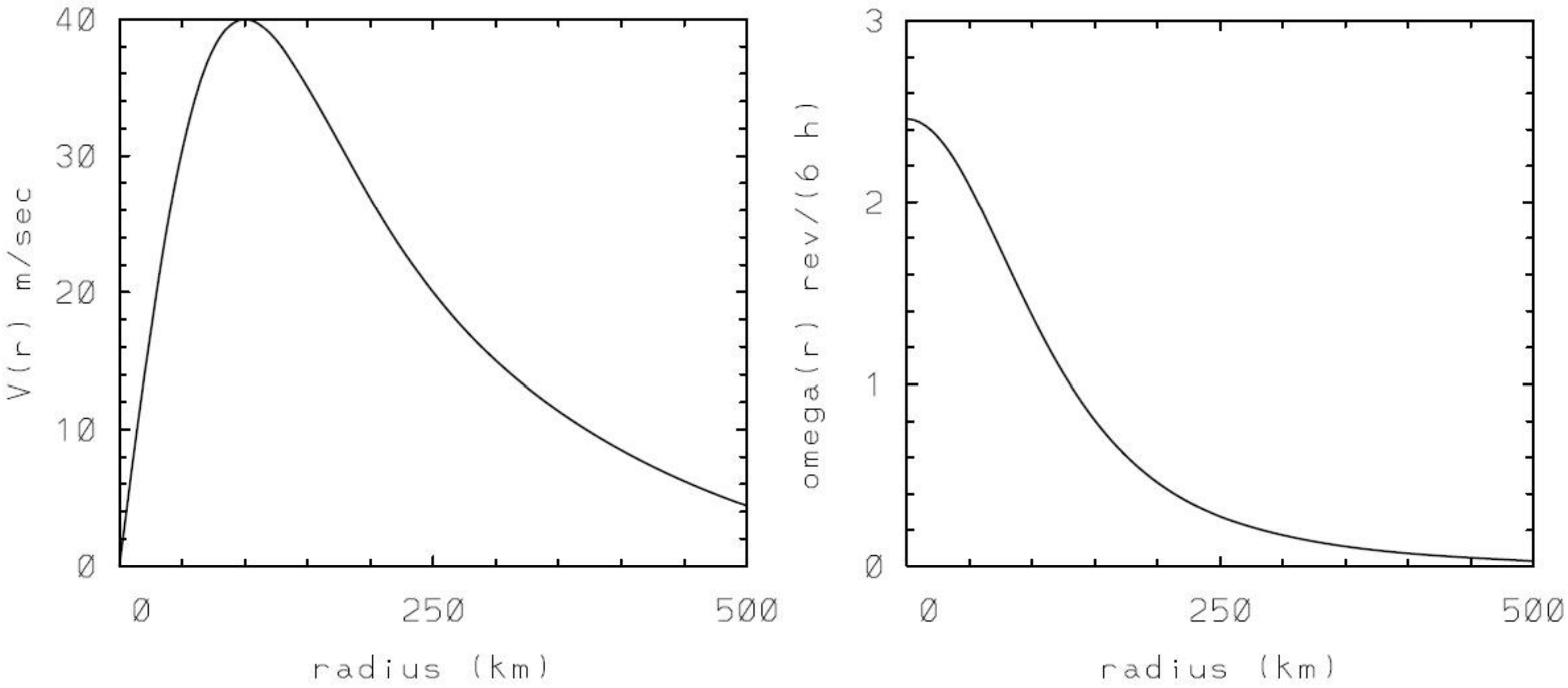
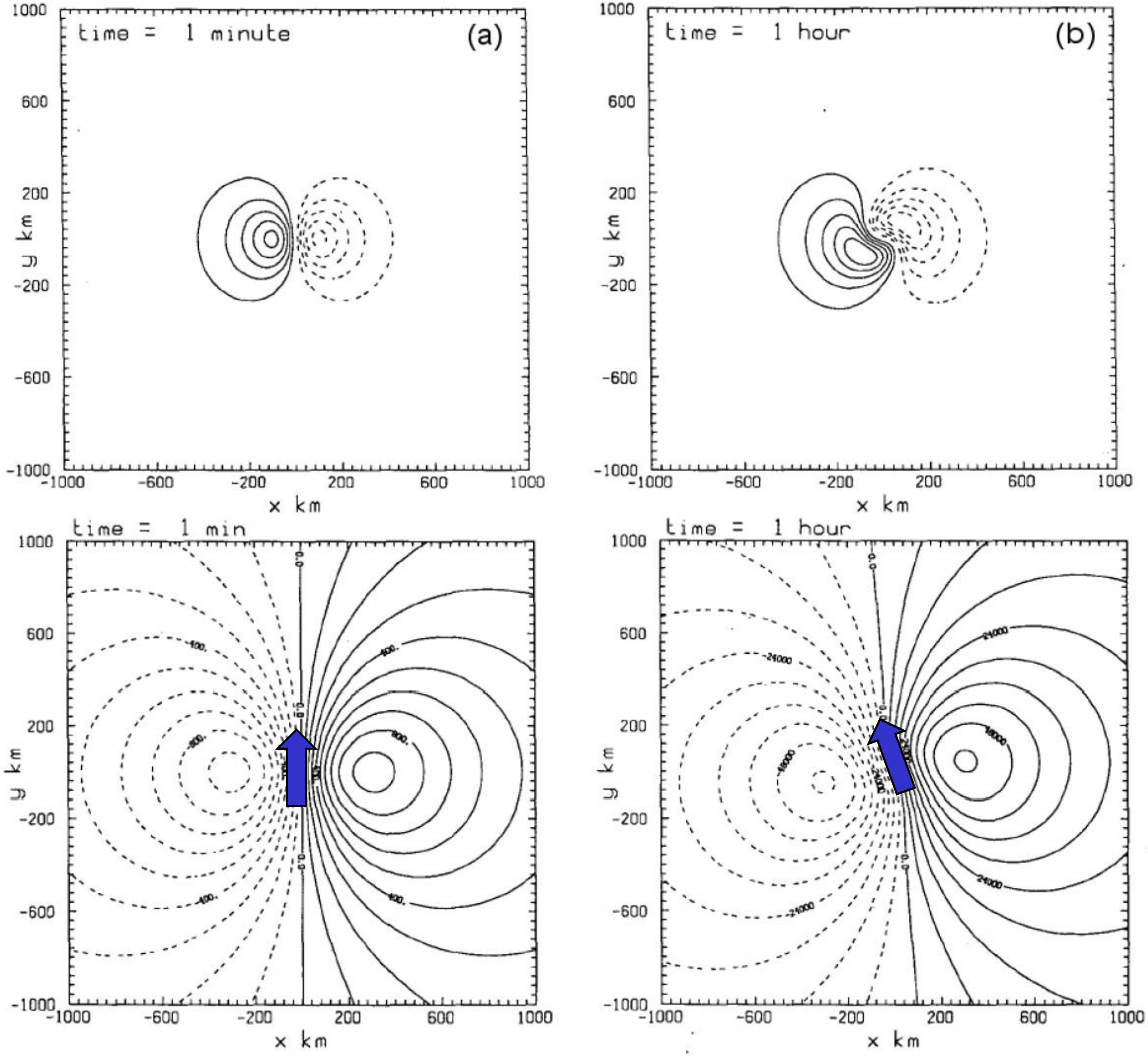


Figure 1.3: (left) Tangential velocity profile $V(r)$ and (right) angular velocity profile $\Omega(r)$ for the symmetric vortex.


time = 1 minute
(a)
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y km
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200
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-600
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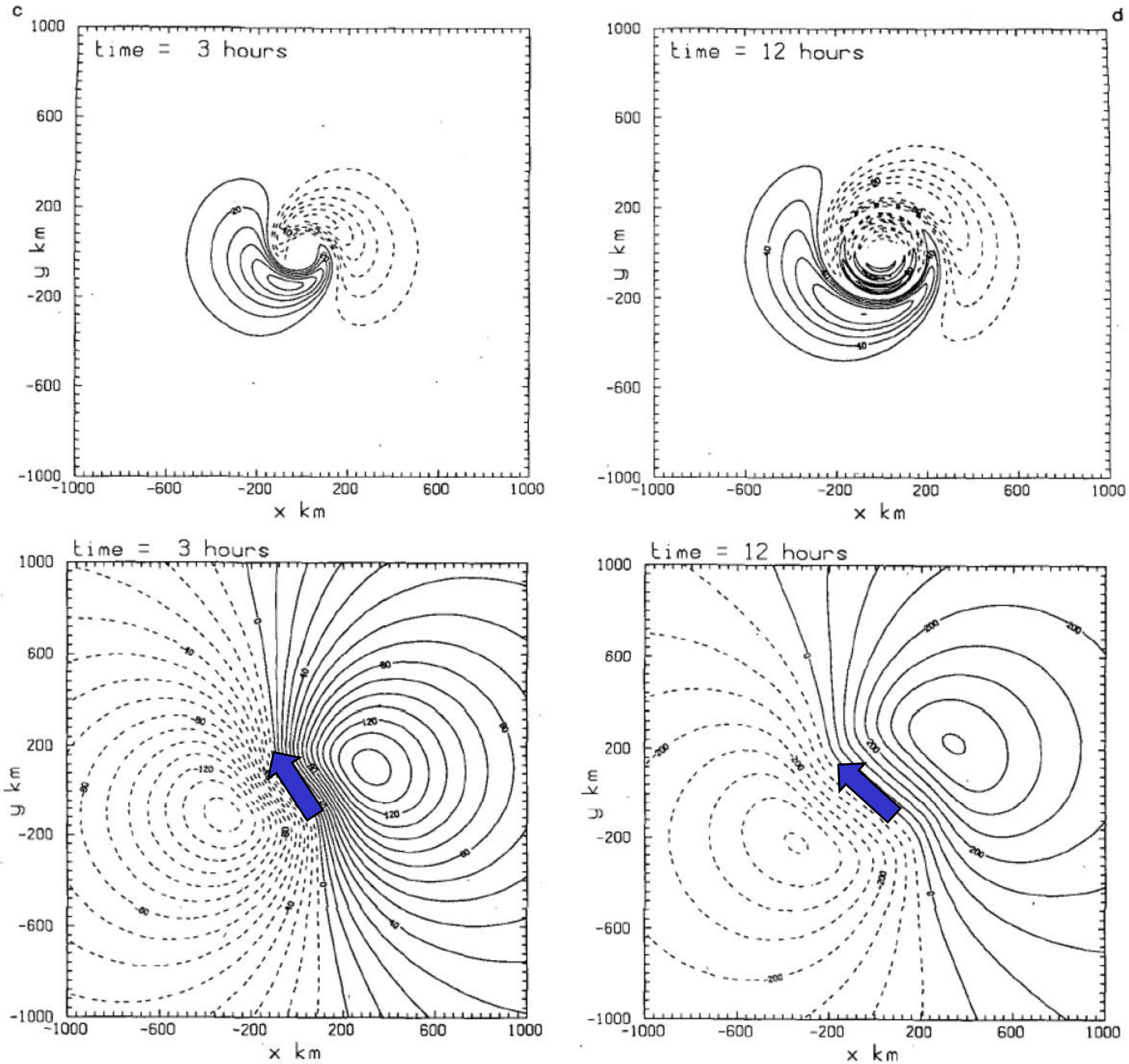
c
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time = 3 hours
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time = 12 hours
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-600
-1000

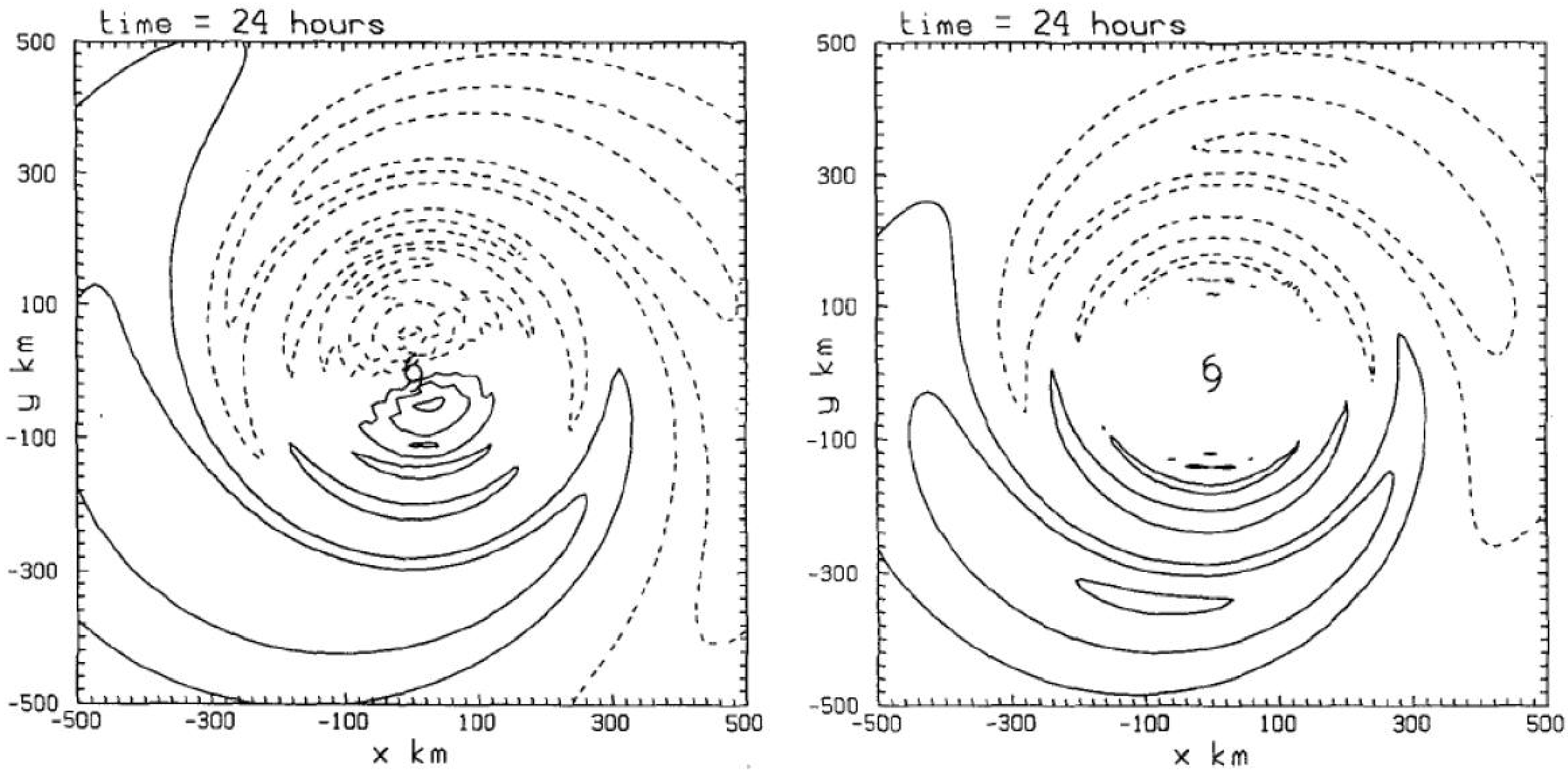
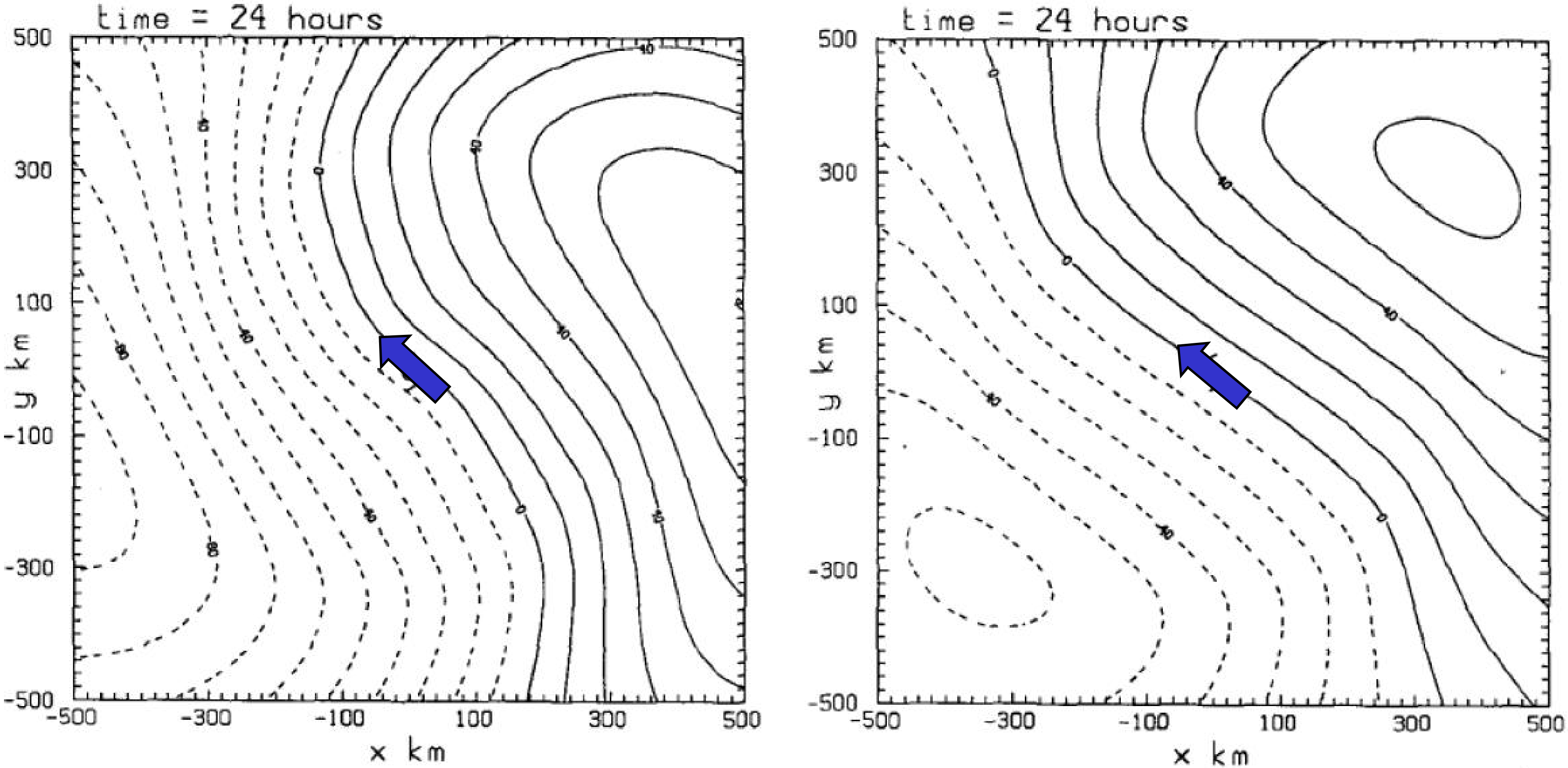


Figure 1.5: Comparison of the analytically-computed asymmetric vorticity and streamfunction fields (upper right and lower right) with those for the corresponding numerical solutions at 24 h. Only the inner part of the numerical domain, centred on the vortex centre, is shown (the calculations were carried out on a 2000 km $\times$ 2000 km domain). Contour intervals are 5×10^{-6} s^{-1} for ζ_a and 10^5 m^2 s^{-1} for ψ_a. The tropical cyclone symbol represents the vortex centre.



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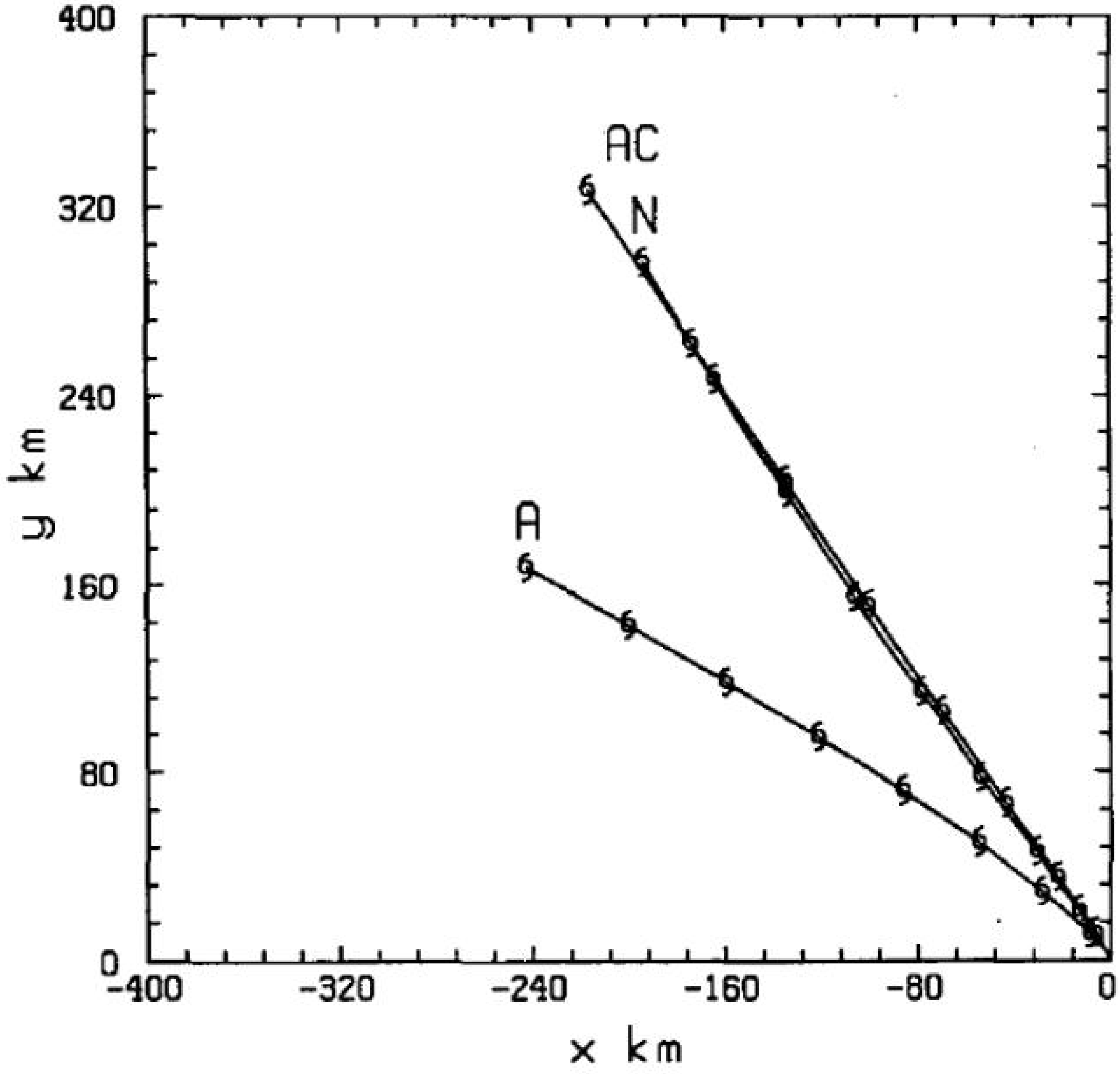
Vortex track

The vortex track, $\mathbf{X}(t) = [X(t), Y(t)]$ may be obtained by integrating the equation $d\mathbf{X}/dt = c(t)$, and using (1.20) and (1.21), we obtain

$$\left[\begin{array}{c} X(t) \\ Y(t) \end{array}\right] = \left[\begin{array}{l} \frac{1}{2}\int_0^\infty \left\{\int_0^1 \zeta_2(p,t)dt\right\} dp \\ -\frac{1}{2}\int_0^\infty \left\{\int_0^1 \zeta_1(p,t)dt\right\} dp \end{array}\right]. \tag{1.22}$$

With the expressions for ζ_n in (1.14), this expression reduces to

$$\left[\begin{array}{c} X(t) \\ Y(t) \end{array}\right] = \left[\begin{array}{l} -\frac{1}{2}\beta\int_0^\infty r\left[t - \frac{\sin(\Omega(r)t)}{\Omega(r)}\right] dr \\ \frac{1}{2}\beta\int_0^\infty r\left[\frac{1-\cos(\Omega(r)t)}{\Omega(r)}\right] dr \end{array}\right]. \tag{1.23}$$



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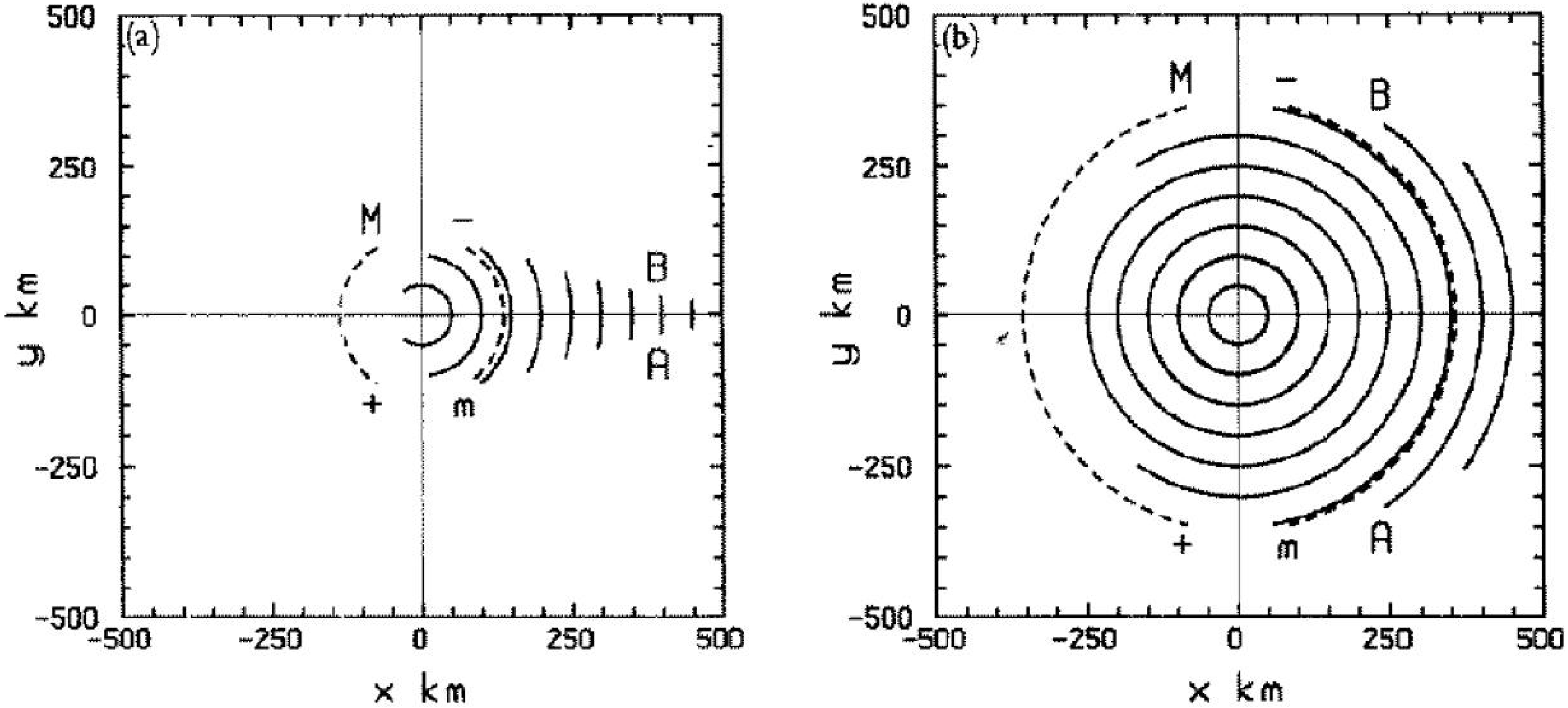
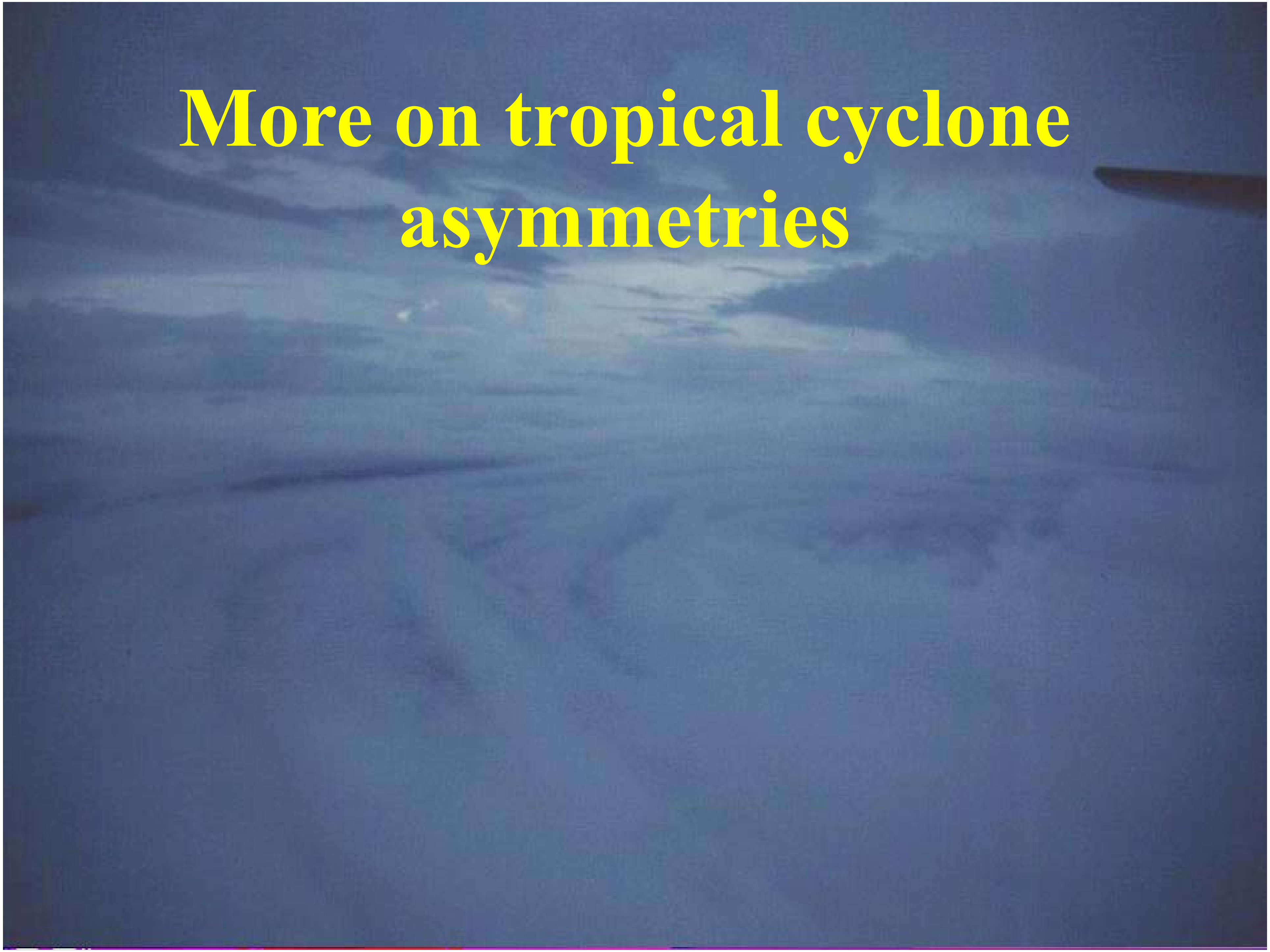


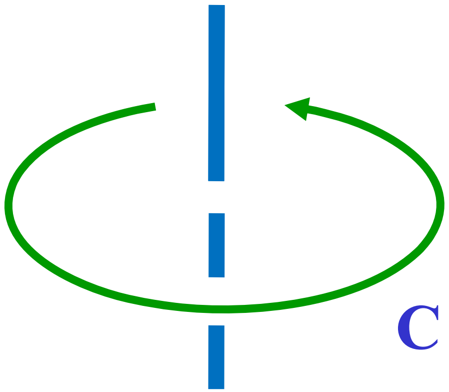
Figure 1.8: Approximate trajectories of fluid parcels which, for a given radius, give the maximum asymmetric vorticity contribution at that radius. The figures refer ro the case of motion of an initially-symmetric vortex on a β-plane with zero basic flow at (a) 1 h, (b) 24 h. The particles arc assumed to follow circular paths about the vortex centre (e.g. AB) with angular velocity $\Omega(r)$, where Ω decreases monotonically with radius r. Solid lines denote trajectories at 50 km radial intervals. Dashed lines marked 'M' and 'm' represent the trajectories giving the overall axisymmetric vorticity maxima and minima, respectively. These maxima and minima occur at the positive and negative ends of the relevant lines.

More on tropical cyclone asymmetries



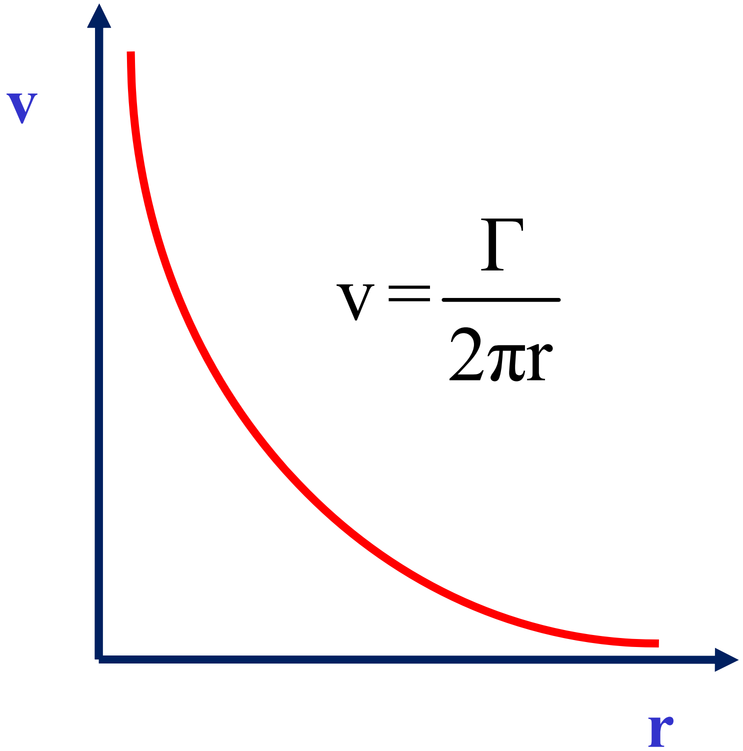
A line vortex

A line vortex has the tangential wind profile:

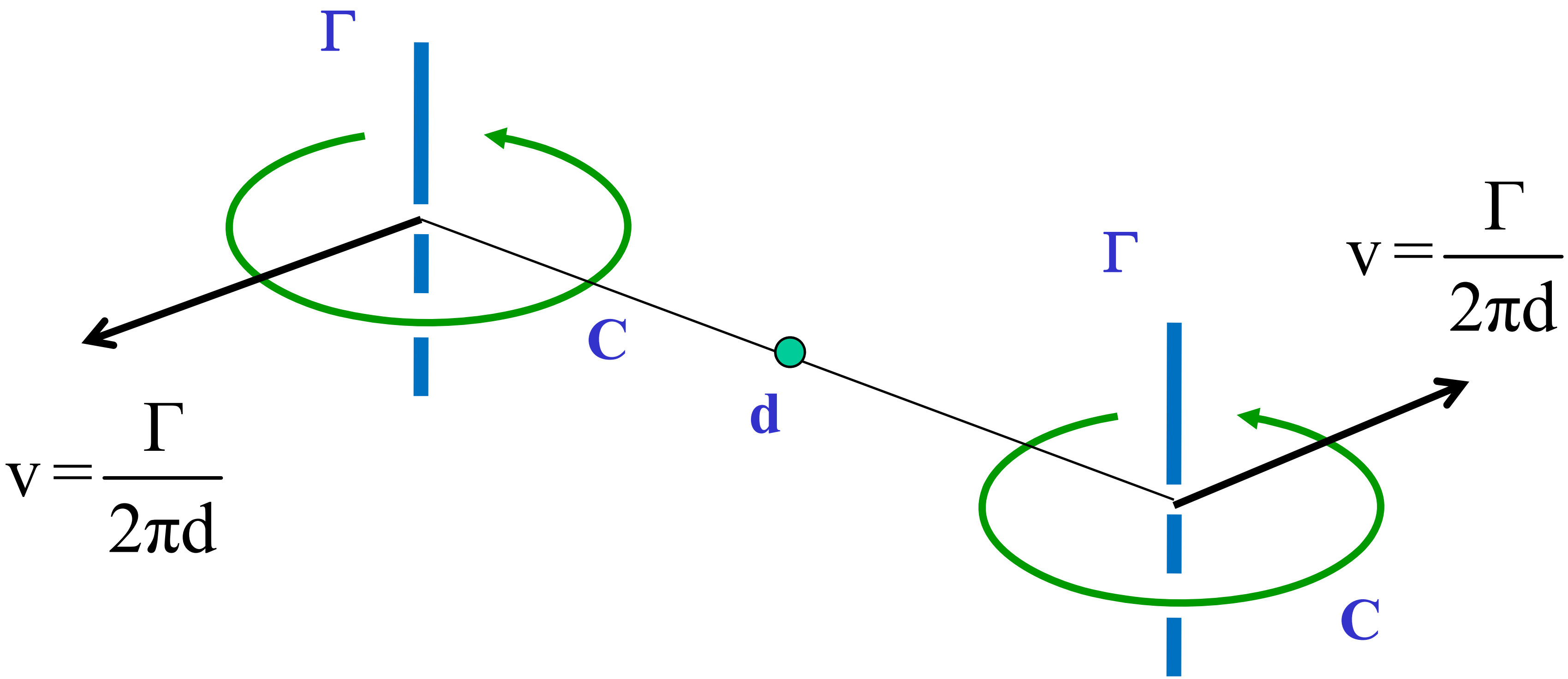


$$\text{Circulation} = \oint_C 2\pi r v dr = \Gamma$$

$$\zeta = \frac{1}{r}\frac{\partial}{\partial r} r v = 0$$

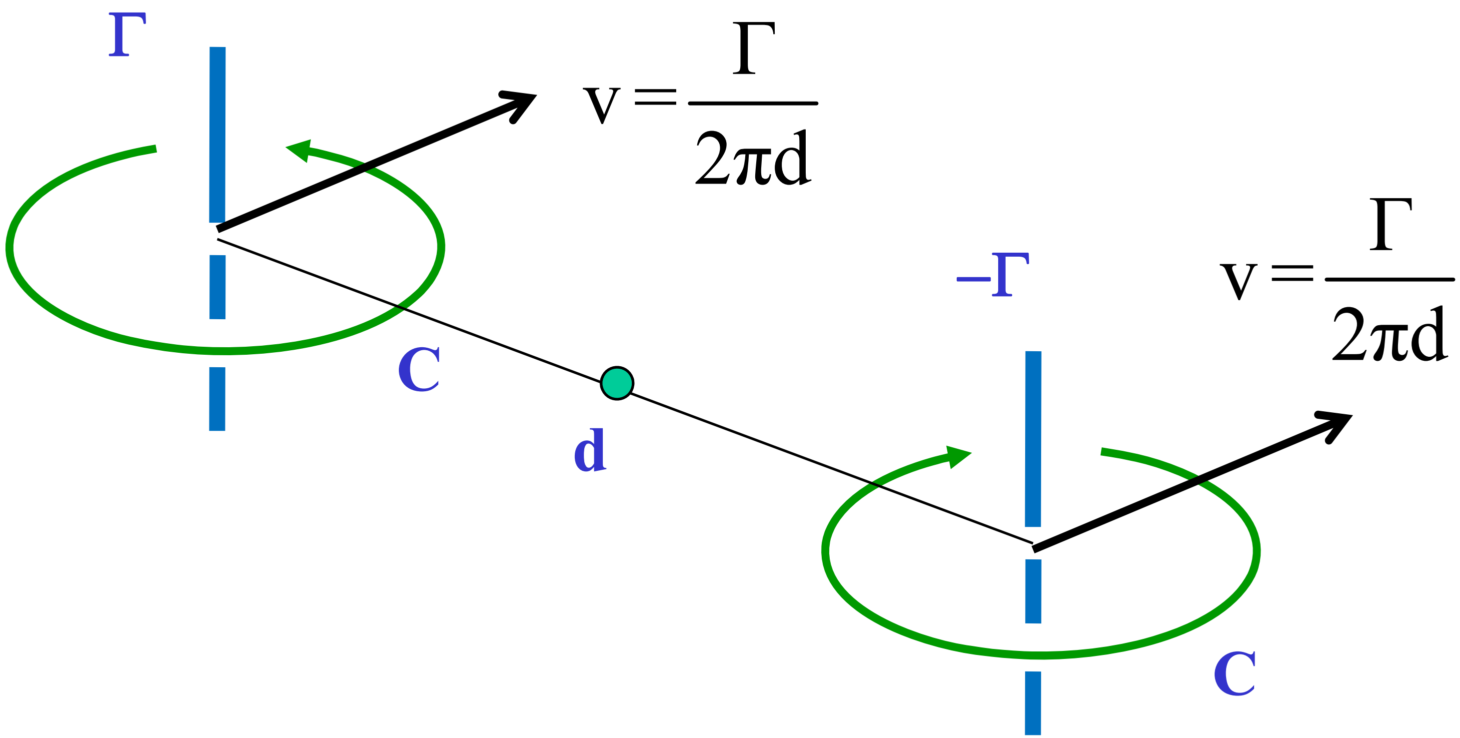


Two-vortex interaction: line vortices: same sign



Vortices rotate around each other about their common centre

Two-vortex interaction: line vortices, opposite signs



Vortices translate in the direction normal to the line between them with speed

$$v = \frac{\Gamma}{2\pi d}$$

Reference

Fundamentals of Geophysical Fluid Dynamics J. McWilliams (2006) CUP
Chapter 3: Barotropic and Vortex Dynamics

Vorticity and velocity distribution

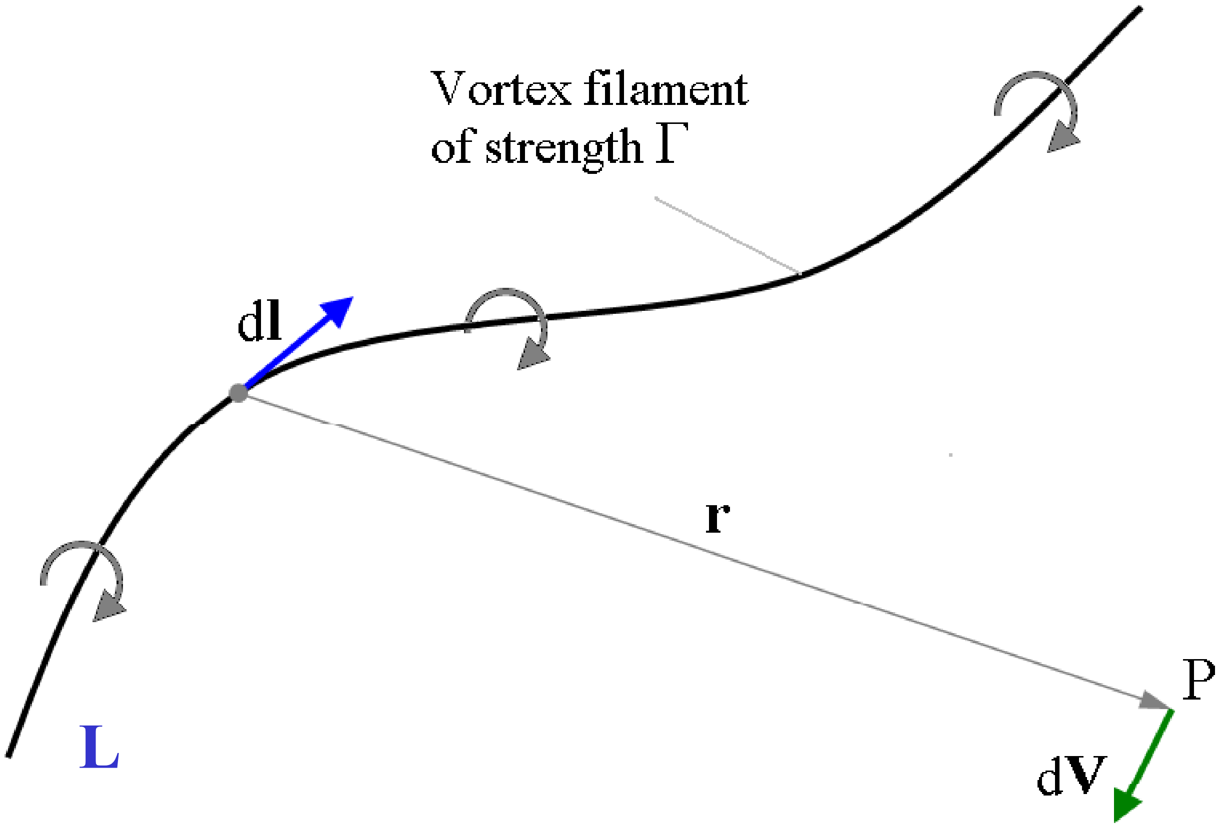
$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = \zeta$$

$$u = -\frac{\partial \psi}{\partial y} \qquad v = \frac{\partial \psi}{\partial x}$$

Can "invert" the vorticity to obtain the streamfunction when suitable boundary conditions on the streamfunction are given.

Biot-Savart law

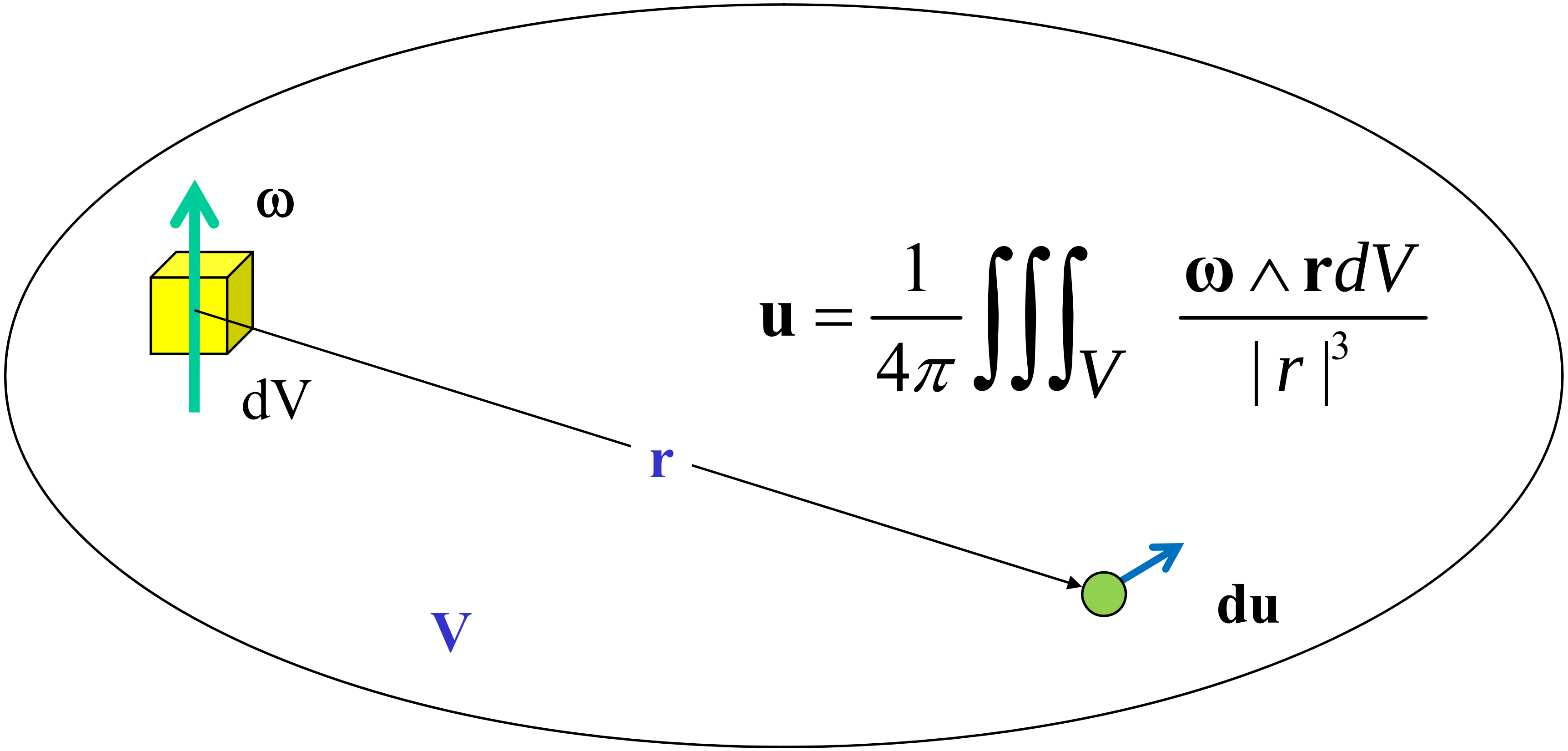
Biot-Savart law



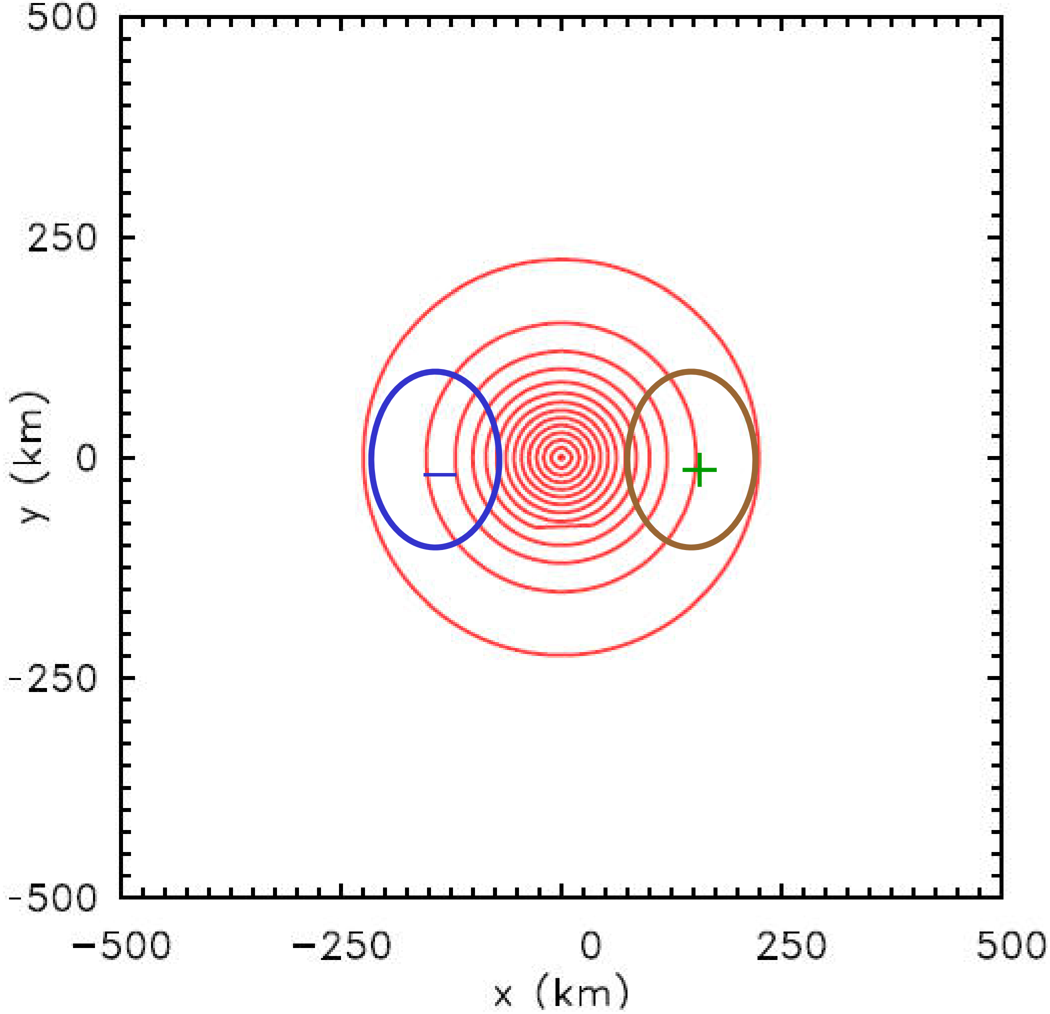
$$\mathbf{V} = -\frac{\Gamma}{4\pi}\int_L \frac{\mathbf{r} \wedge d\mathbf{l}}{r^3}$$

Biot-Savart law

Integral over a volume V



$$\mathbf{u} = \frac{1}{4\pi}\iiint_V \frac{\boldsymbol{\omega} \wedge \mathbf{r}\, dV}{|r|^3}$$



+ wavenumber-one vorticity asymmetry

The partitioning problem

Recall, for a moving reference frame

$$\frac{\partial \Gamma}{\partial t} = -\mathbf{u_s} \cdot \nabla(\Gamma + f) - (\mathbf{U} - \mathbf{c}) \cdot \nabla \zeta_s - (\mathbf{U} - \mathbf{c}) \cdot \nabla(\Gamma + f). \qquad (1.7)$$

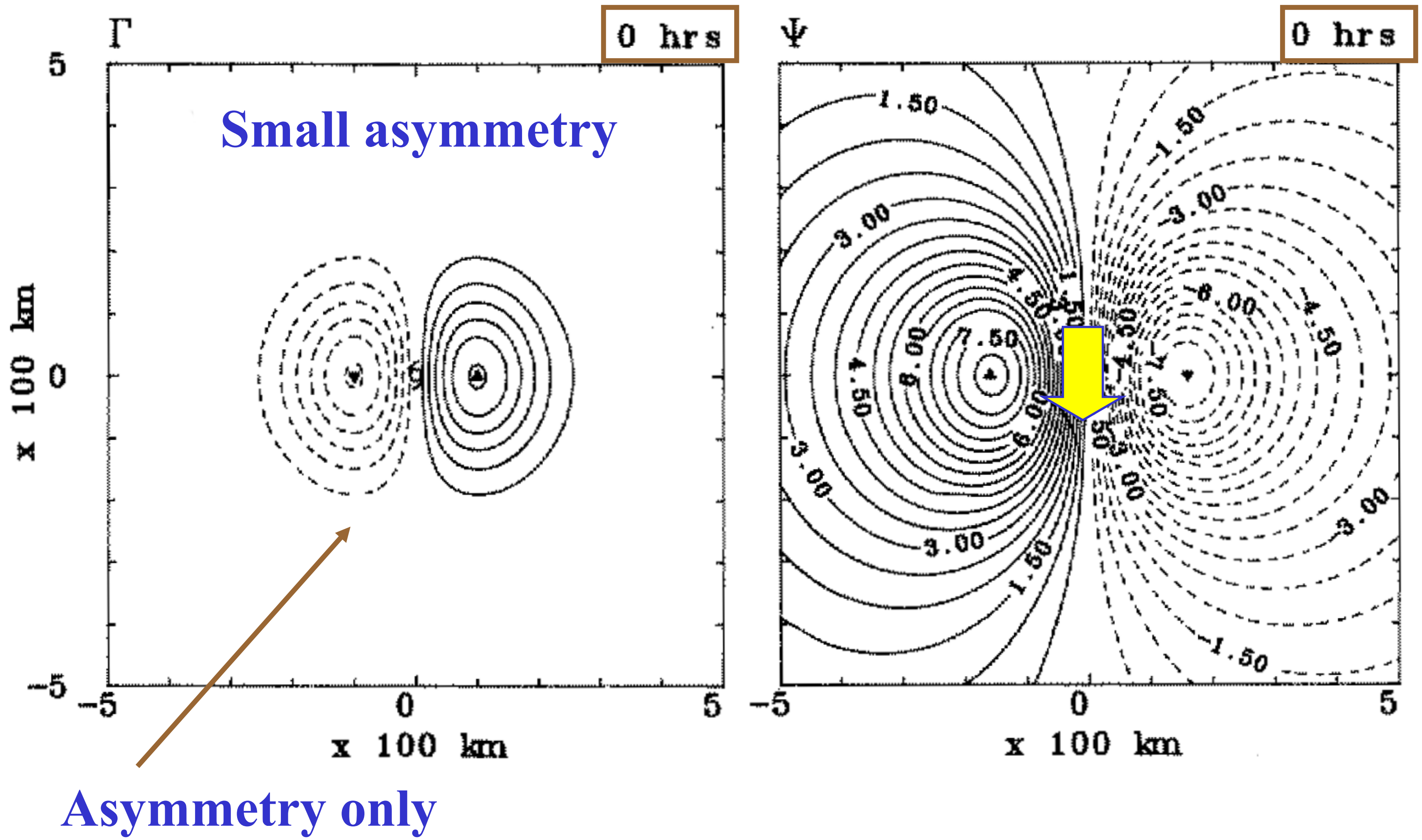
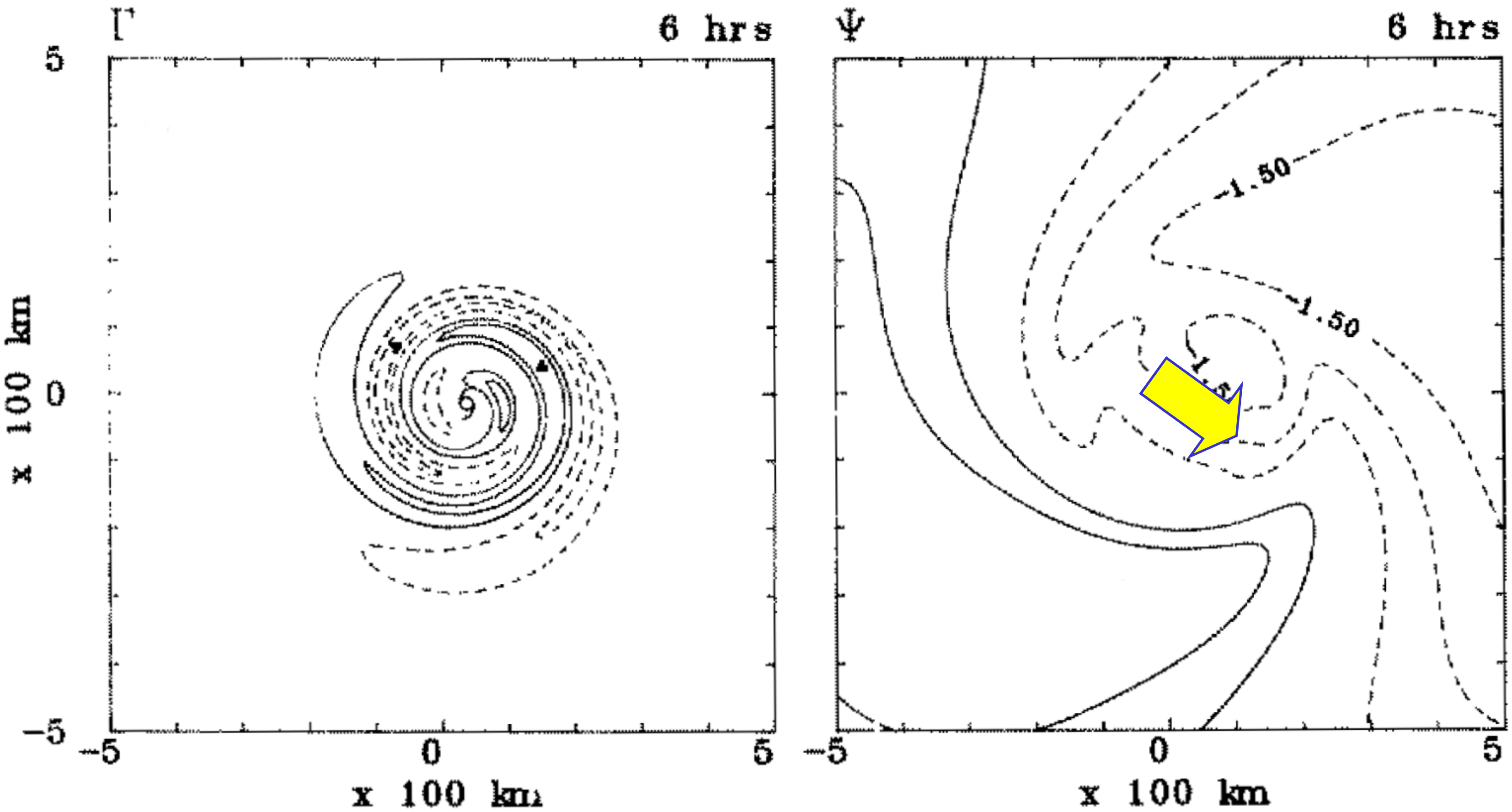
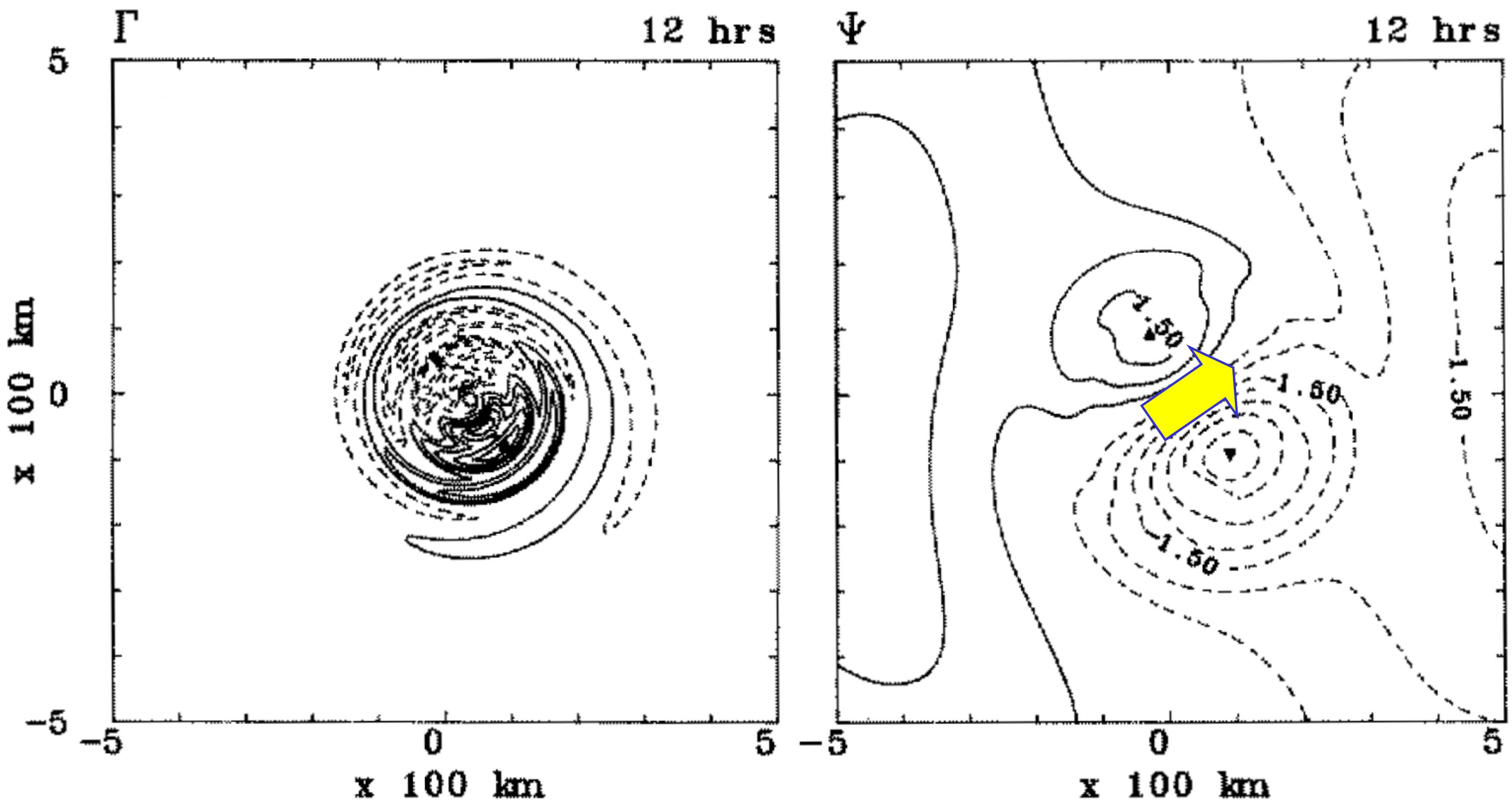


Figure 11. Evolution of the asymmetric vorticity field (Γ) and corresponding streamfunction field for the initially asymmetric vortex on an *f* plane in the case of small-scale asymmetry (simulation S1). Shown are (a) the initial fields, and the fields at (b) 6 hours and (c) 12 hours. Note that only one quarter of the total flow domain is shown. Contour intervals are $2\times10^{-5}s^{-1}$ for Γ and $5\times10^{4}m^{2}s^{-1}$ for Ψ. Zero contours have been excluded.


Γ
6 hrs
Ψ
6 hrs
-1.50
-1.50
5
0
-5
-5
0
5
x 100 km
x 100 km
x 100 km



Γ
12 hrs
Ψ
12 hrs
5
0
−5
x 100 km
−5
0
5
x 100 km
−5
0
5
x 100 km
1.50
-1.50
-1.50
-1.50

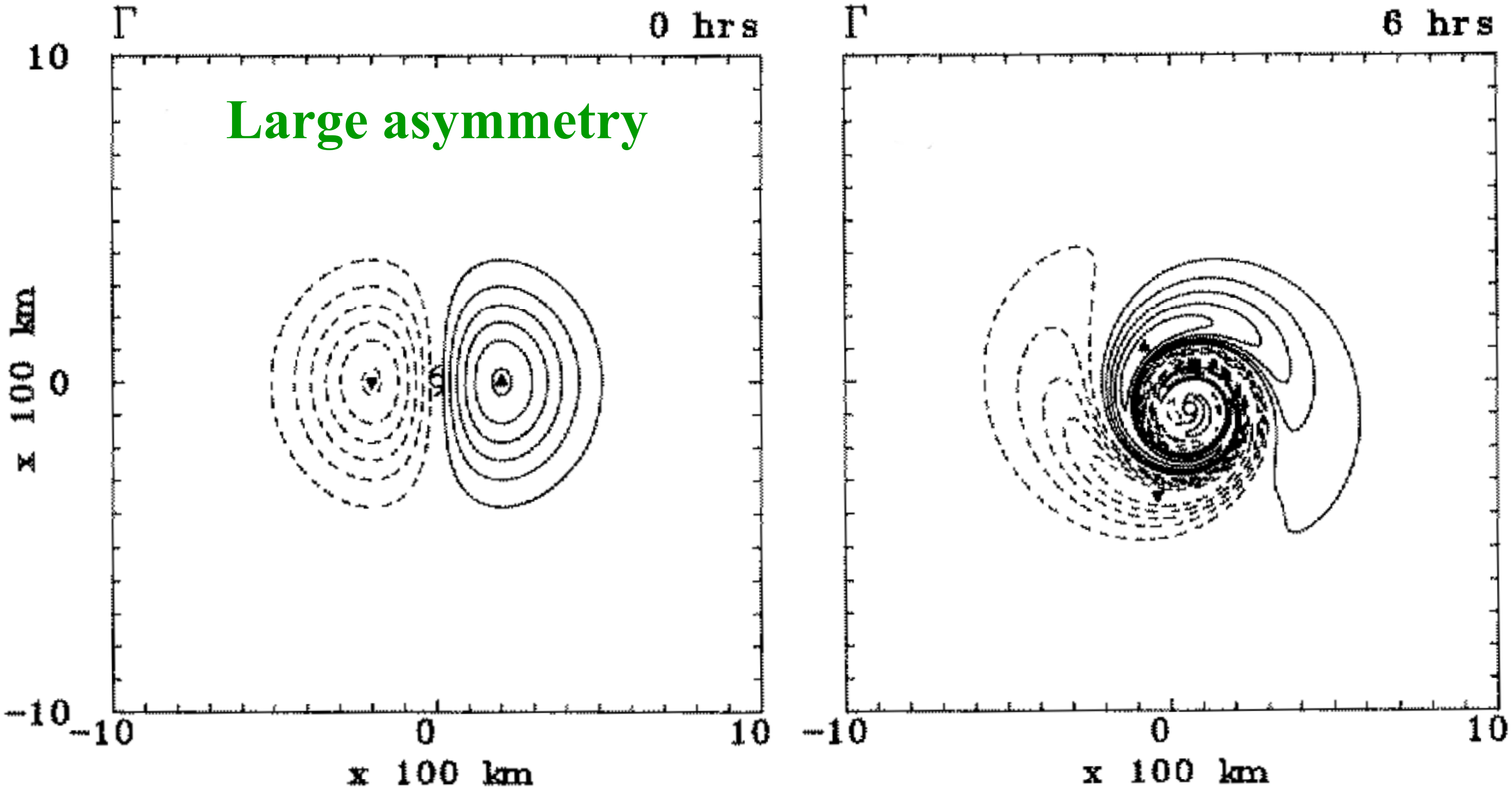


Figure 13. Evolution of the asymmetric vorticity field (Γ) for the initially asymmetric vortex on an f plane in the case of large-scale asymmetry (simulation S2).

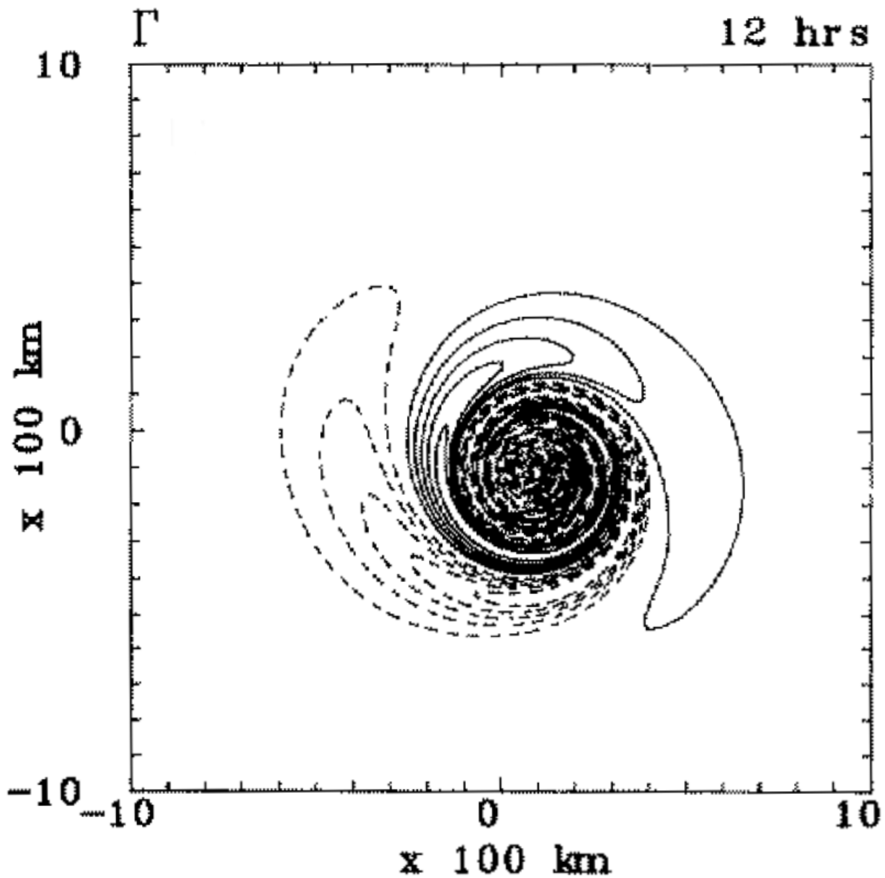


Figure 13. Evolution of the asymmetric vorticity field (Γ) for the initially asymmetric vortex on an f plane in the case of large-scale asymmetry (simulation S2). Shown are (a) the initial field, and the fields at (b) 6 hours and (c) 12 hours. Contour interval is $10^{-5}s^{-1}$. Note: the domain size is twice that shown in Fig. 11. Zero contours have been excluded.

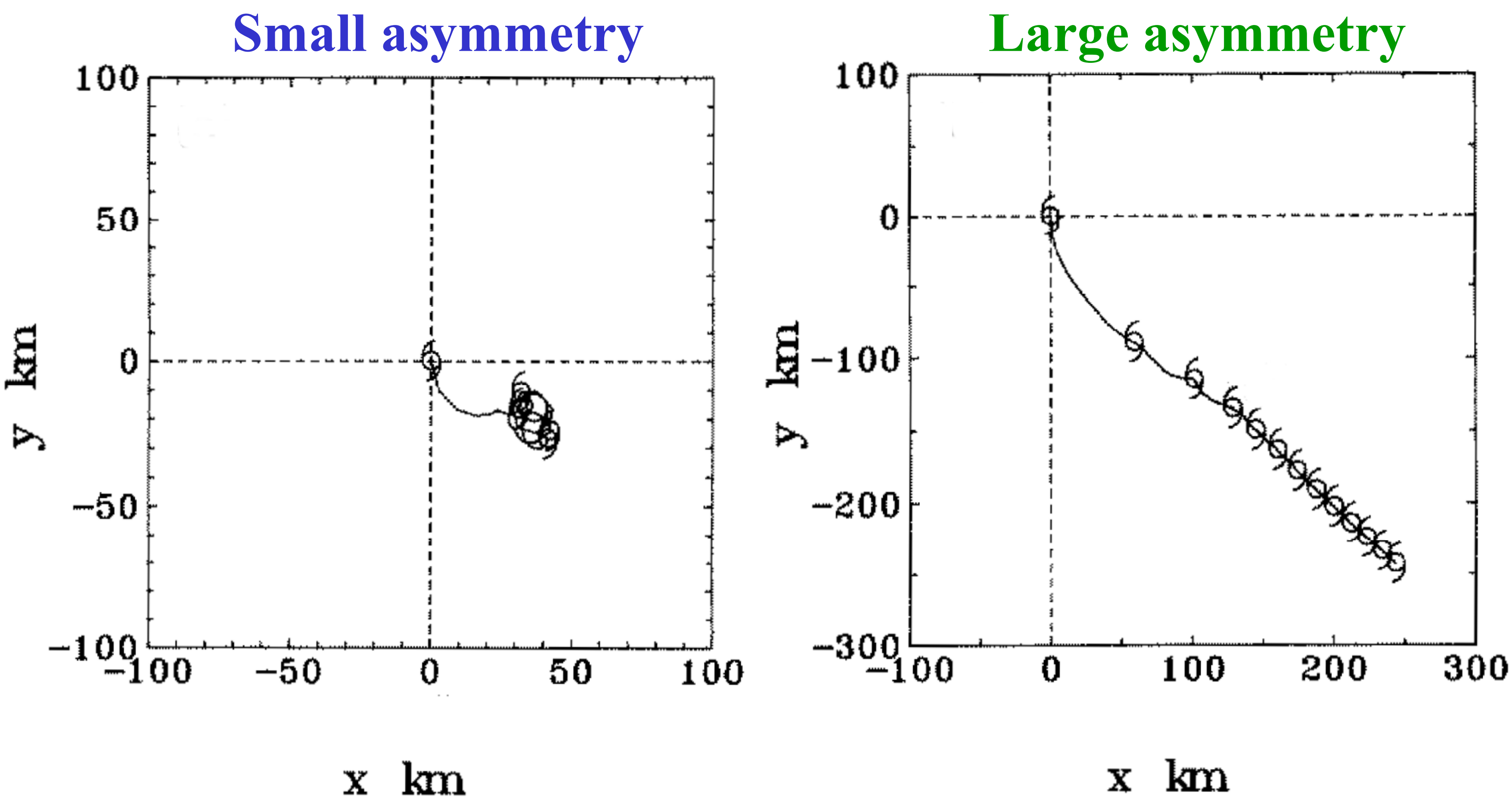


Figure 12. Tracks of initially asymmetric vortices in the calculations S1 to S4 defined in the text. (a) Small asymmetry, f plane; (b) large asymmetry, f plane; (c) small asymmetry, β plane; and (d) large asymmetry, β plane.

Small asymmetry **β-plane** **Large asymmetry**

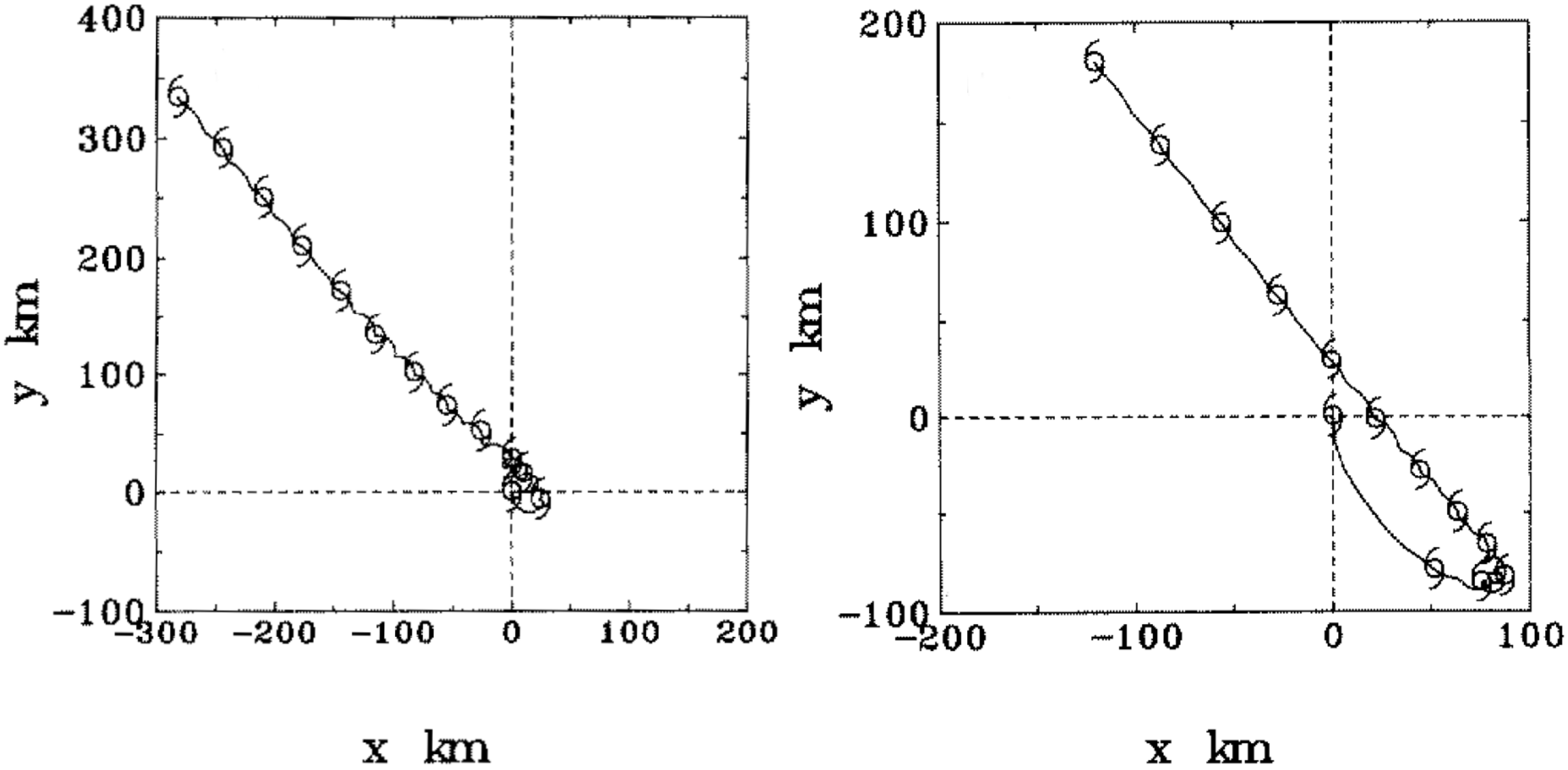


Figure 12. Tracks of initially asymmetric vortices in the calculations S1 to S4 defined in the text. (a) Small asymmetry, f plane; (b) large asymmetry, f plane; (c) small asymmetry, β plane; and (d) large asymmetry, β plane.

Two-vortex interaction

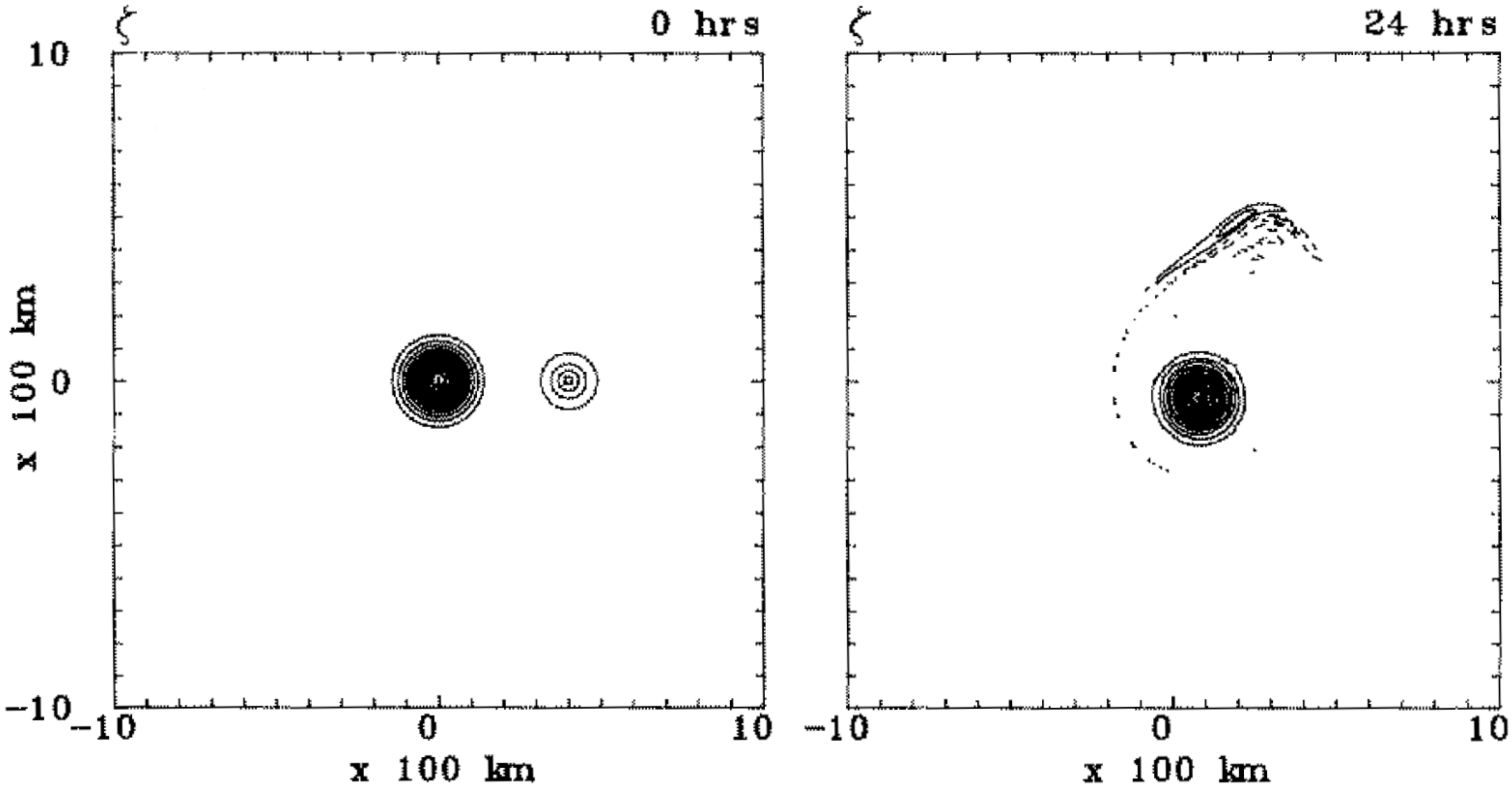


Figure 15. Tracks of the large vortex in the case of a strong-vortex, weak-vortex interaction, (a) on an f plane, and (b) on a β plane. Cyclone symbols denote successive six-hour positions, the initial position being at the origin in each case.

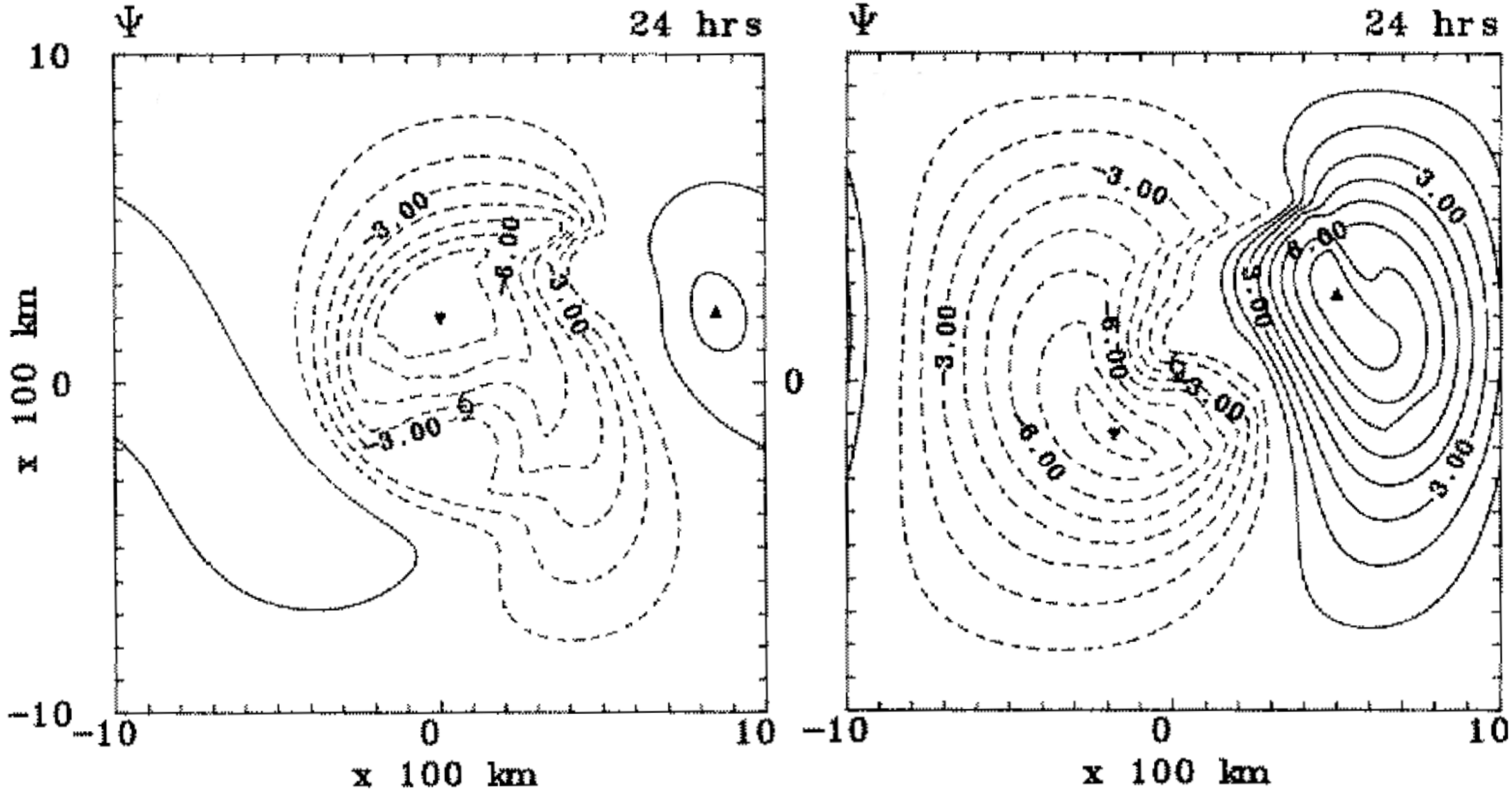


Figure 15. Tracks of the large vortex in the case of a strong-vortex, weak-vortex interaction, (a) on an *f* plane, and (b) on a β plane. Cyclone symbols denote successive six-hour positions, the initial position being at the origin in each case.

Track of the large vortex

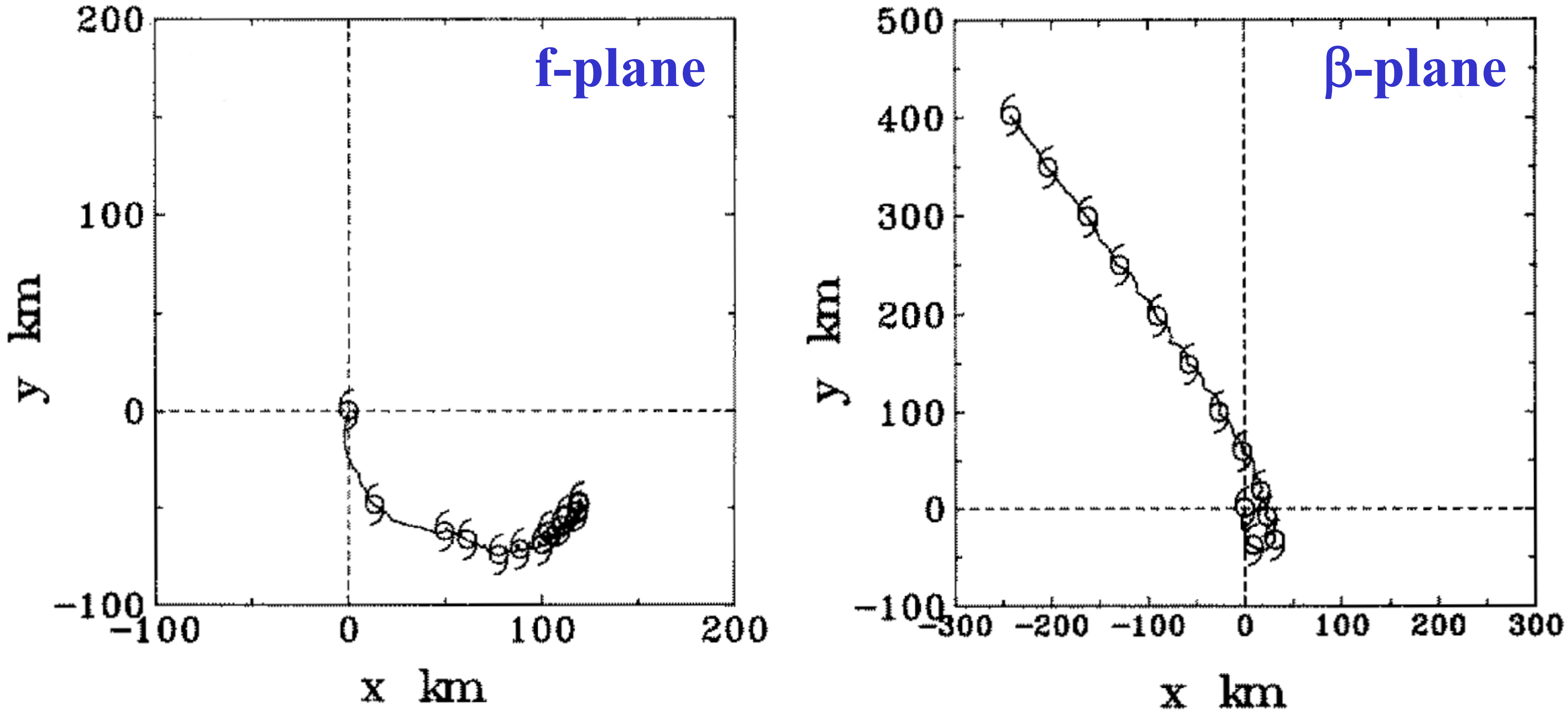
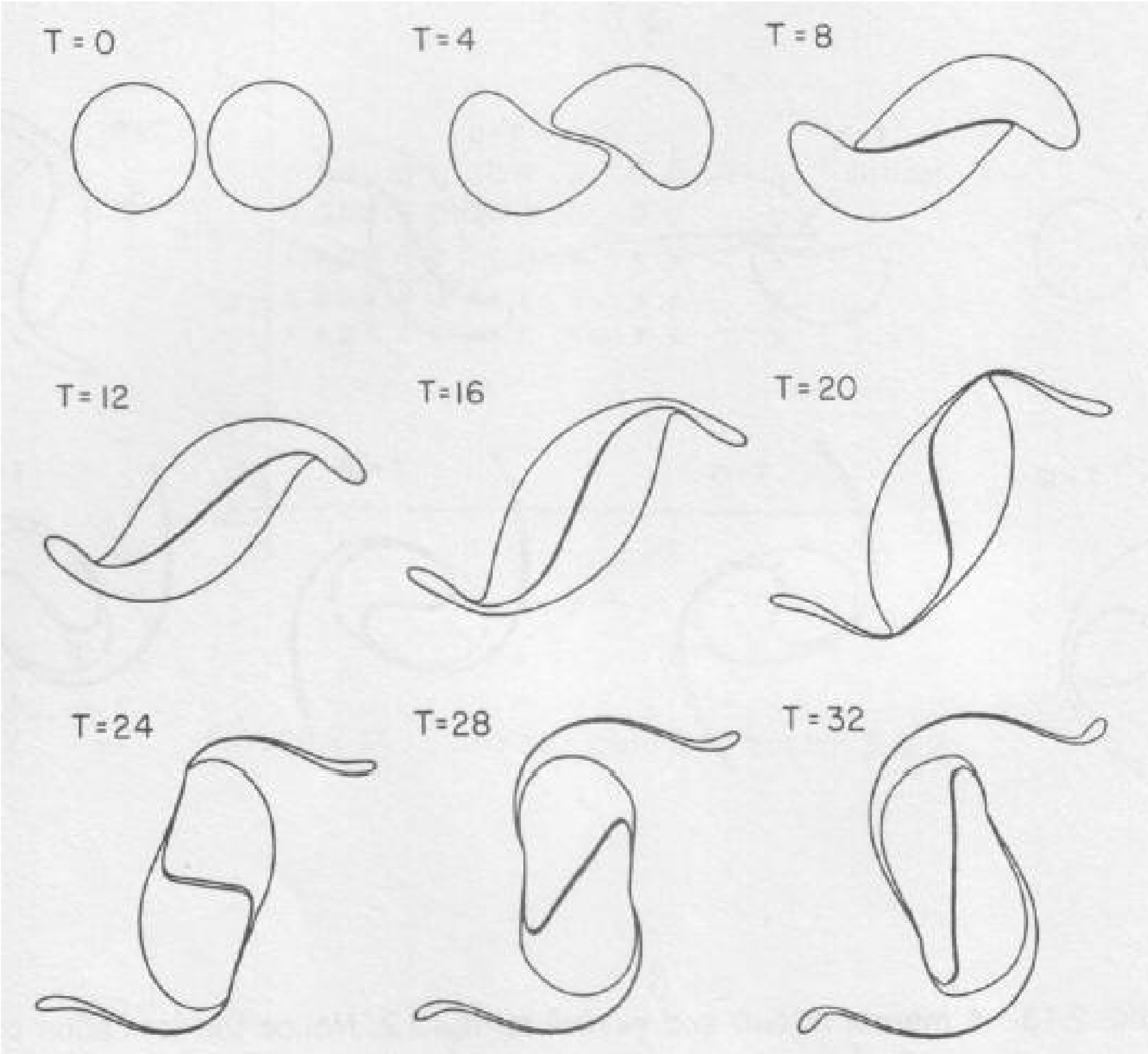


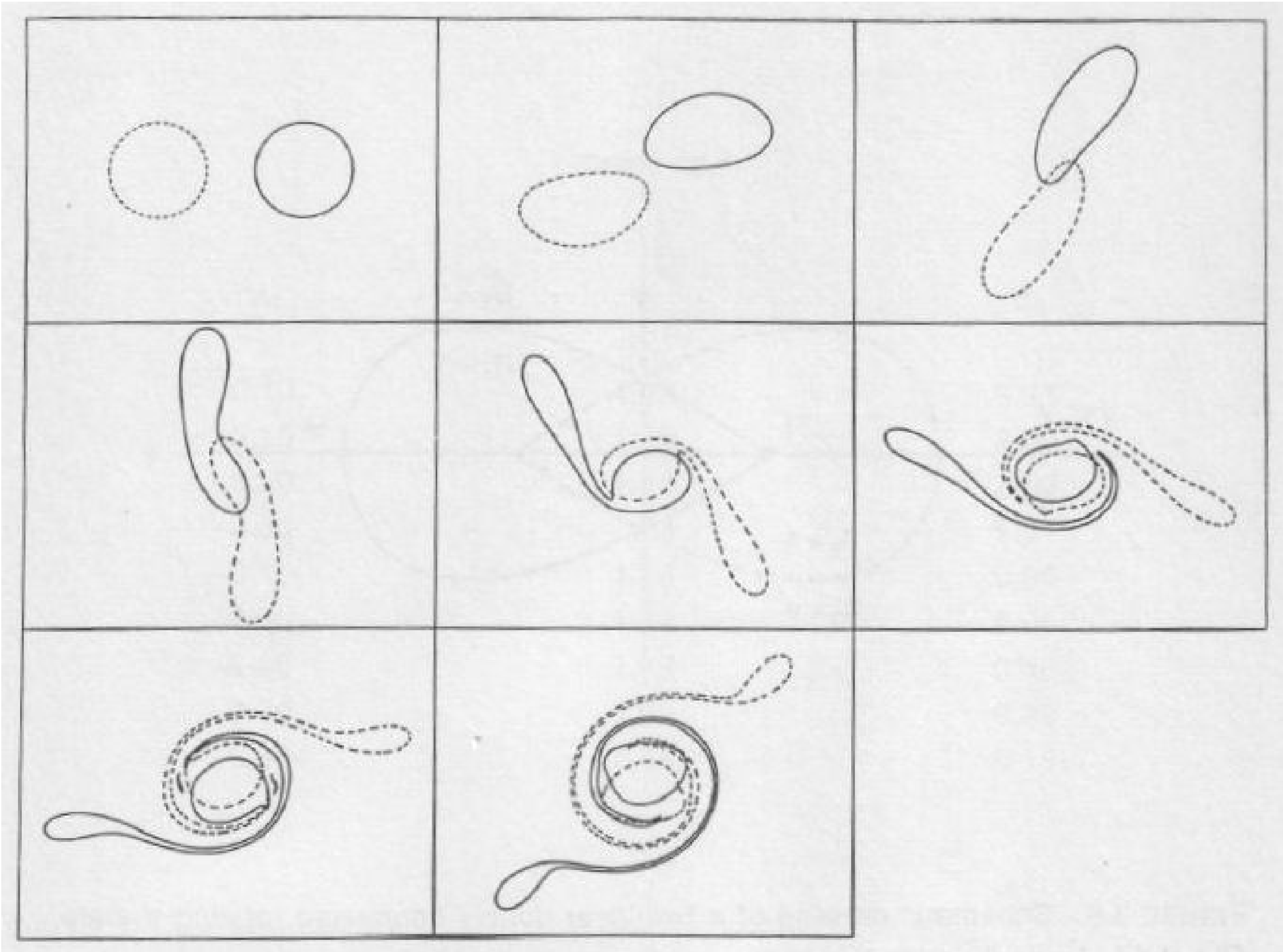
Figure 15. Tracks of the large vortex in the case of a strong-vortex, weak-vortex interaction, (a) on an f plane, and (b) on a β plane. Cyclone symbols denote successive six-hour positions, the initial position being at the origin in each case.

Vortex interaction

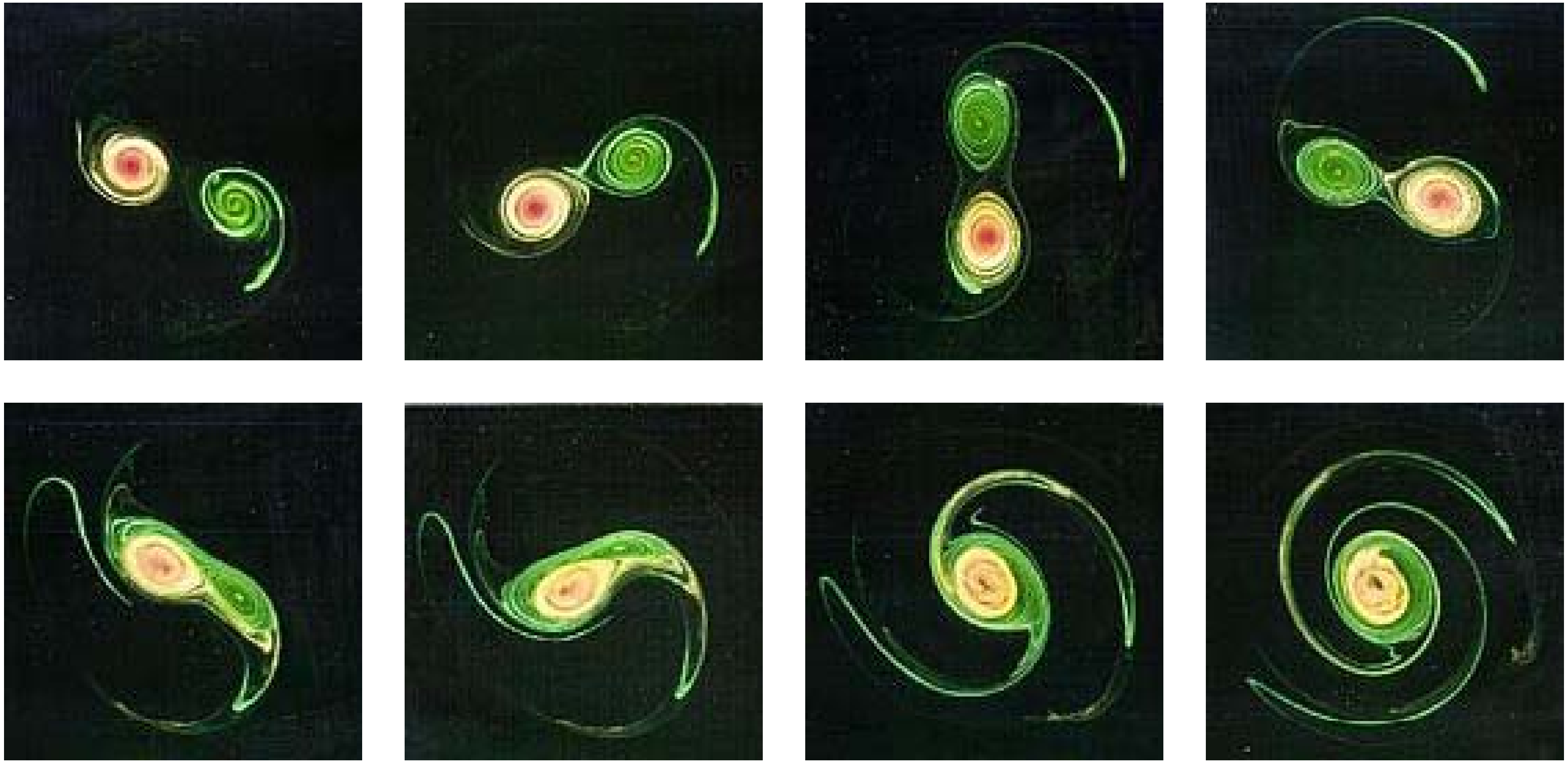
- Two like-signed potential vortices will circle around a common centre without getting closer (Fujiwhara effect).
- Two like-signed vortices with a finite vorticity core will merge when their distance of separation is smaller than some critical value.
- This merger process is the predominant mechanism for the evolution of two-dimensional turbulence, and has for this reason been studied extensively.
- The existence of a critical distance has been confirmed by a number of high-resolution numerical simulations of inviscid two-dimensional flows as well as by laboratory experiments on interacting barotropic vortices in a rotating fluid.



Original study by Polvani (1988-89) : constant PV vortices, no dependence of merger on stratification

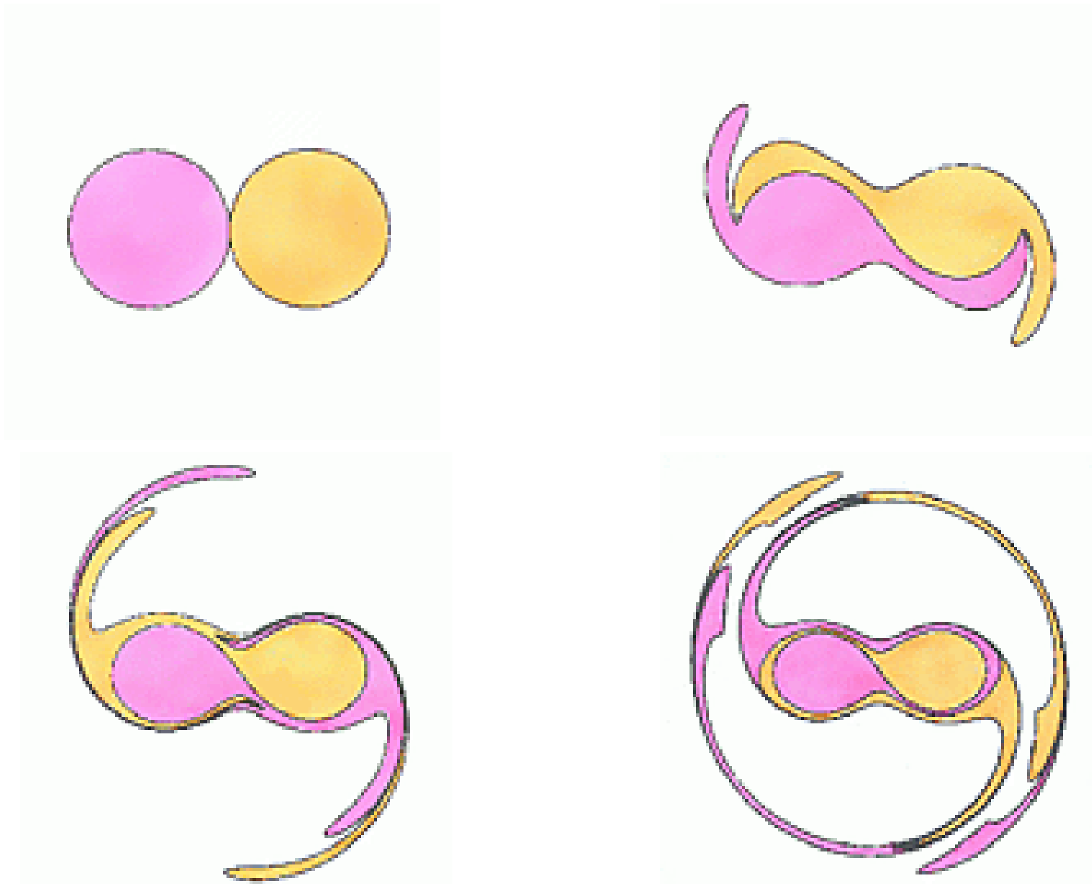


Vortices further apart



Dye visualization of the merger of two cyclonic vortices at successive times.

Dye visualizations of the merger process demonstrate the formation of cusps and the existence of long filaments. These characteristic features of vortex merging can be well captured by simple point-vortex models in which each vortex is represented by a point vortex surrounded by a contour of passive tracers. The method of contour kinematics is used to calculate numerically the evolution of the material contours. A typical calculated evolution of a two-point-vortex configuration is shown in the next figure.



Calculated evolution of initially circular contours of passive tracers which are advected by the co-rotating velocity field induced by the two point vortices (not shown). The distance between the point vortices was artificially decreased.


The End!