The tropical cyclone boundary layer

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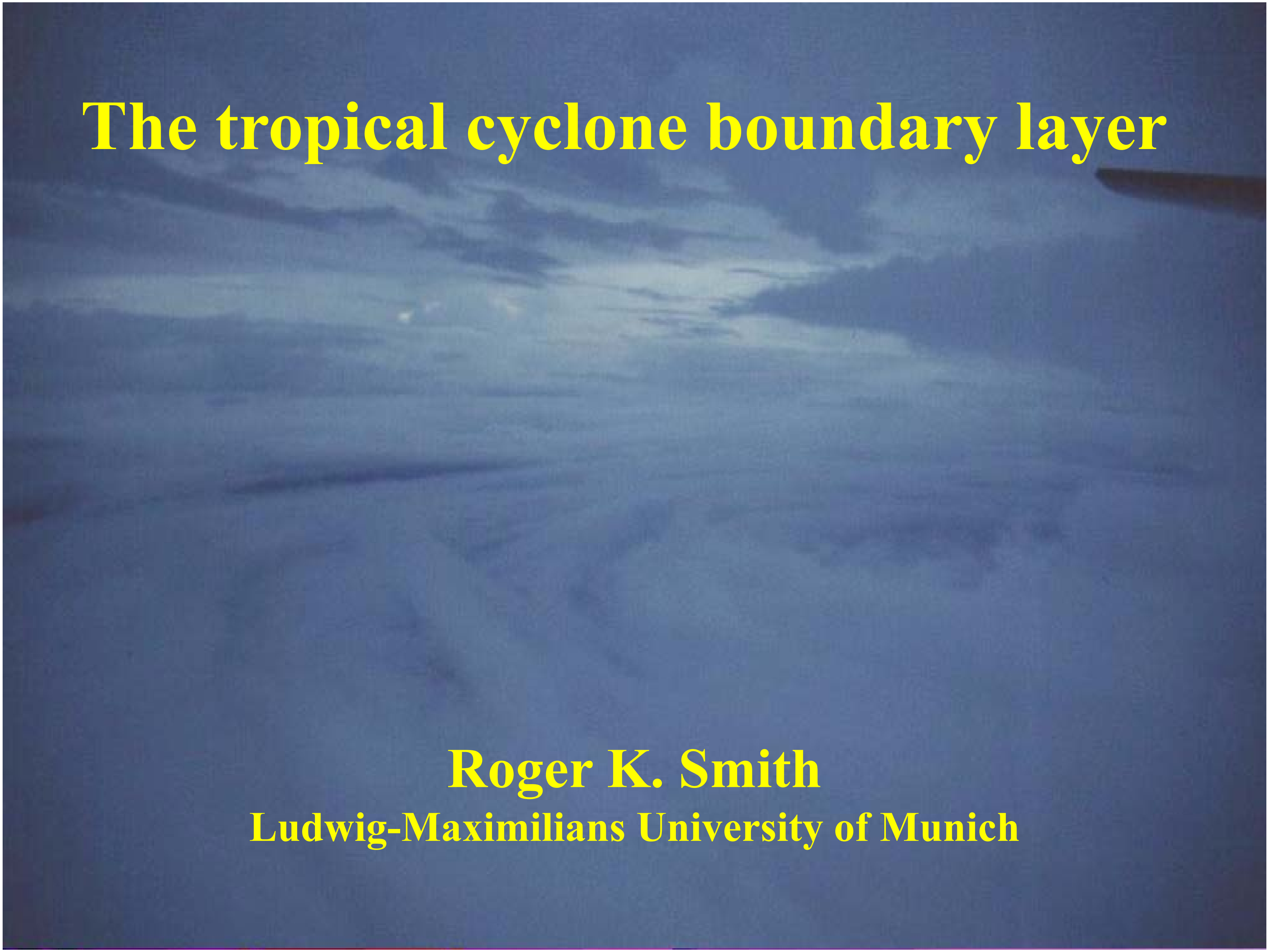
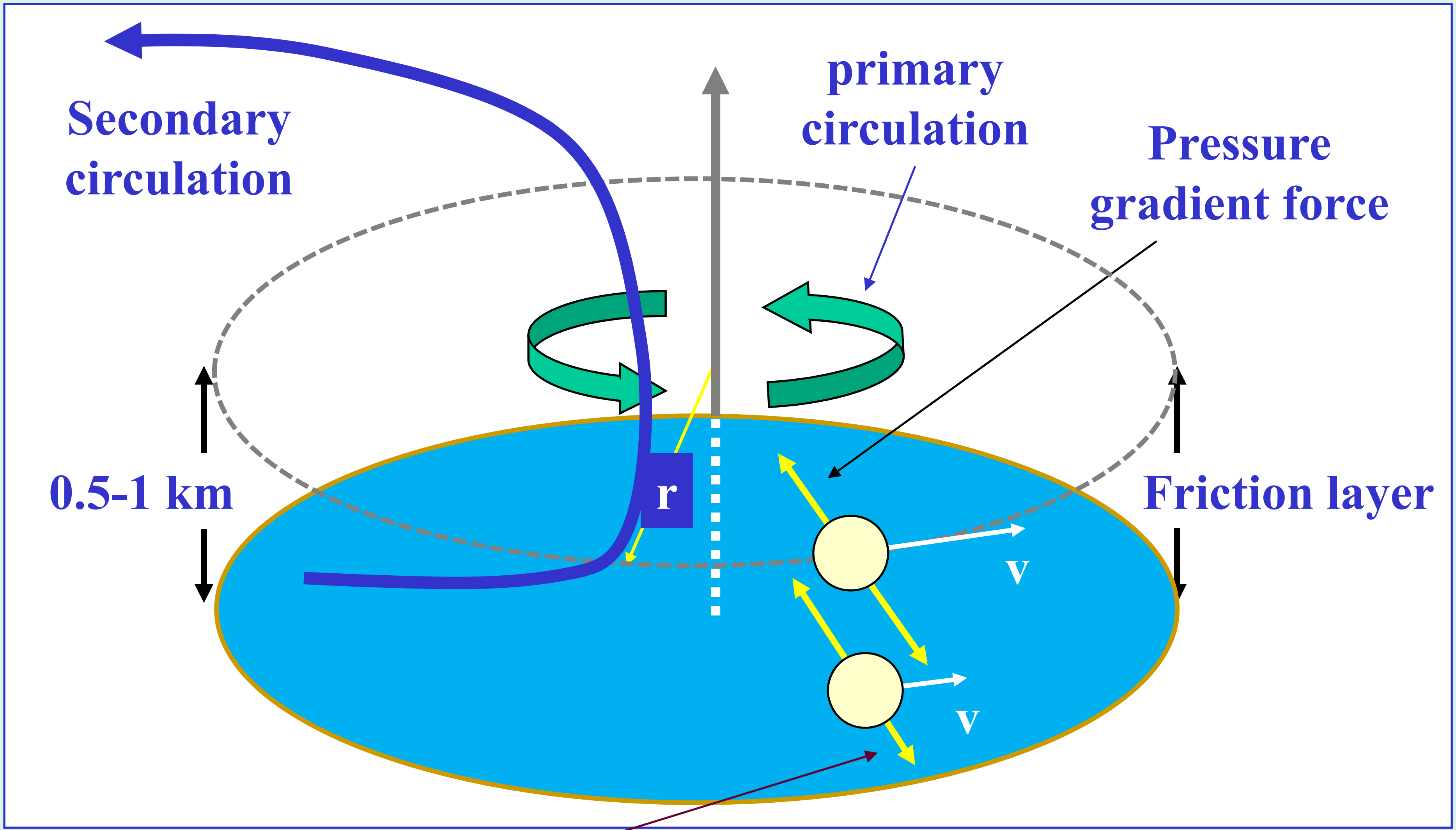


Frictional effects on the secondary circulation



Centrifugal force and Coriolis force are reduced by friction

“Tea cup” Experiment

Boundary-layer scaling

Continuity equation

$$\frac{1}{r}\frac{\partial \rho r u}{\partial r} \qquad \frac{\partial \rho w}{\partial z}$$

$$\rho\frac{U}{R} \qquad \rho\frac{W}{Z}$$

w-momentum

$$\frac{\partial w}{\partial t} \quad + u\frac{\partial w}{\partial r} \quad + w\frac{\partial w}{\partial z} \quad = -\frac{1}{\rho}\frac{\partial p}{\partial z} \quad + K\nabla_h^2 w \quad + K\frac{\partial^2 w}{\partial z^2} \tag{4.3}$$

$$\frac{W}{T} \quad \frac{UW}{R} \quad \frac{WW}{Z} \quad \frac{\Delta p}{\rho Z} \quad K\frac{W}{R^2} \quad K\frac{W}{Z^2} \tag{3a}$$

$$S^2A^2 \quad S^2A^2 \quad S^2A^2 \quad \frac{\Delta p}{\rho V^2} \quad SA^3R_e^{-1} \quad SAR_e^{-1} \tag{3b}$$

S = U/V A = Z/R Re = VZ/K

$$\nabla_h^2 = (\partial/\partial r)(r\partial/\partial r)$$

u-momentum

$$\frac{\partial u}{\partial t} \quad +u\frac{\partial u}{\partial r} \quad +w\frac{\partial u}{\partial z} \quad -\frac{v^2}{r} \quad -fv \quad = \quad -\frac{1}{\rho}\frac{\partial p}{\partial r} \quad +K\left(\nabla_h^2 u - \frac{u}{r^2}\right) \quad +K\frac{\partial^2 u}{\partial z^2} \qquad (4.1)$$

$$\frac{U}{T} \quad \frac{U^2}{R} \quad W\frac{U}{Z} \quad \frac{V^2}{R} \quad fV \quad \frac{\Delta p}{\rho R} \quad K\frac{U}{R^2} \quad K\frac{U}{Z^2} \qquad (1a)$$

$$S^2 \quad S^2 \quad S^2 \quad 1 \quad \frac{1}{Ro} \quad \frac{\Delta p}{\rho V^2} \quad SA^2R_e^{-1} \quad SR_e^{-1} \qquad (1b)$$

S = U/V A = Z/R Re = VZ/K

v-momentum

$$\frac{\partial v}{\partial t} \quad +u\frac{\partial v}{\partial r} \quad +w\frac{\partial v}{\partial z} \quad +\frac{uv}{r} \quad +fu \quad = \quad +K\left(\nabla_h^2 v - \frac{v}{r^2}\right) \quad +K\frac{\partial^2 v}{\partial z^2} \qquad (4.2)$$

$$\frac{V}{T} \quad U\frac{V}{R} \quad W\frac{V}{Z} \quad U\frac{V}{R} \quad fU \quad K\frac{V}{R^2} \quad K\frac{V}{Z^2} \qquad (2a)$$

$$S \quad S \quad S \quad S \quad \frac{S}{Ro} \quad A^2R_e^{-1} \quad R_e^{-1} \qquad (2b)$$

Full BL equations

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} + \frac{V^2 - v^2}{r} + f(V - v) = K\frac{\partial^2 u}{\partial z^2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r} + w\frac{\partial v}{\partial z} + \frac{uv}{r} + fu = K\frac{\partial^2 v}{\partial z^2}$$

Put $v = V(r) + v'$. Then the equations become:

$$\underline{\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z}} - \left(\frac{2V}{r} + f\right) v' - \underline{\frac{v'^2}{r}} = K\frac{\partial^2 u}{\partial z^2}$$

$$\underline{\frac{\partial v'}{\partial t} + u\frac{\partial v'}{\partial r} + w\frac{\partial v'}{\partial z} + \frac{uv'}{r}} + \left(\frac{dV}{dr} + \frac{V}{r} + f\right) u = K\frac{\partial^2 v'}{\partial z^2}$$

— Nonlinear terms

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} - \left(\frac{2V}{r} + f\right)v' - \frac{v'^2}{r} = K\frac{\partial^2 u}{\partial z^2}$$

$$\underline{\frac{U^2}{R} \quad \frac{U^2}{R} \quad \frac{U^2}{R}} \quad \frac{VV'}{R} \quad fV' \quad \underline{\frac{V'^2}{R}} \quad \frac{KU}{Z^2}$$

$$\frac{\partial v'}{\partial t} + u\frac{\partial v'}{\partial r} + w\frac{\partial v'}{\partial z} + \frac{uv'}{r} + \left(\frac{dV}{dr} + \frac{V}{r} + f\right)u = K\frac{\partial^2 v'}{\partial z^2}$$

$$\underline{\frac{UV'}{R} \quad \frac{UV'}{R} \quad \frac{UV'}{R} \quad \frac{UV'}{R}} \quad \frac{VU}{R} \quad \frac{VU}{R} \quad fU \quad \frac{KV'}{Z^2}$$

Assume $U \approx V'$

$$\frac{Nonlinear\ terms}{Linear\ terms} \approx \frac{U}{V} \approx 0.3$$

(underlined) Nonlinear terms

Linearized equations

$$-v\left(f+\frac{2V}{r}\right)=K\frac{\partial^2 u}{\partial z^2}$$

$$u\left(f+\frac{V}{r}+\frac{\partial V}{\partial r}\right)=K\frac{\partial^2 v}{\partial z^2}$$

Ekman equations

$$-v\left(f+\frac{2V}{r}\right)=K\frac{\partial^2 u}{\partial z^2}$$

$$u\left(f+\frac{V}{r}+\frac{\partial V}{\partial r}\right)=K\frac{\partial^2 v}{\partial z^2}$$

Justification?
Scale analysis

4.3 The Ekman boundary layer

The scale analysis of the u- and v-momentum equations in Table 4.1 show that for small Rossby numbers ($Ro << 1$), there is an approximate balance between the net Coriolis force and the diffusion of momentum, expressed by the equations:

$$f(v_g - v) = K\frac{\partial^2 u}{\partial z^2} \tag{4.1}$$

and

$$fu = K\frac{\partial^2 v}{\partial z^2}, \tag{4.2}$$

Equations (4.1) and (4.2) are linear in u and v and may be readily solved by setting $V = v + iu$, where $i = \sqrt{(-1)}$. Then they reduce to the single differential equation

$$K\frac{d^2 V}{dz^2} - ifV = -ifV_g, \tag{4.3}$$

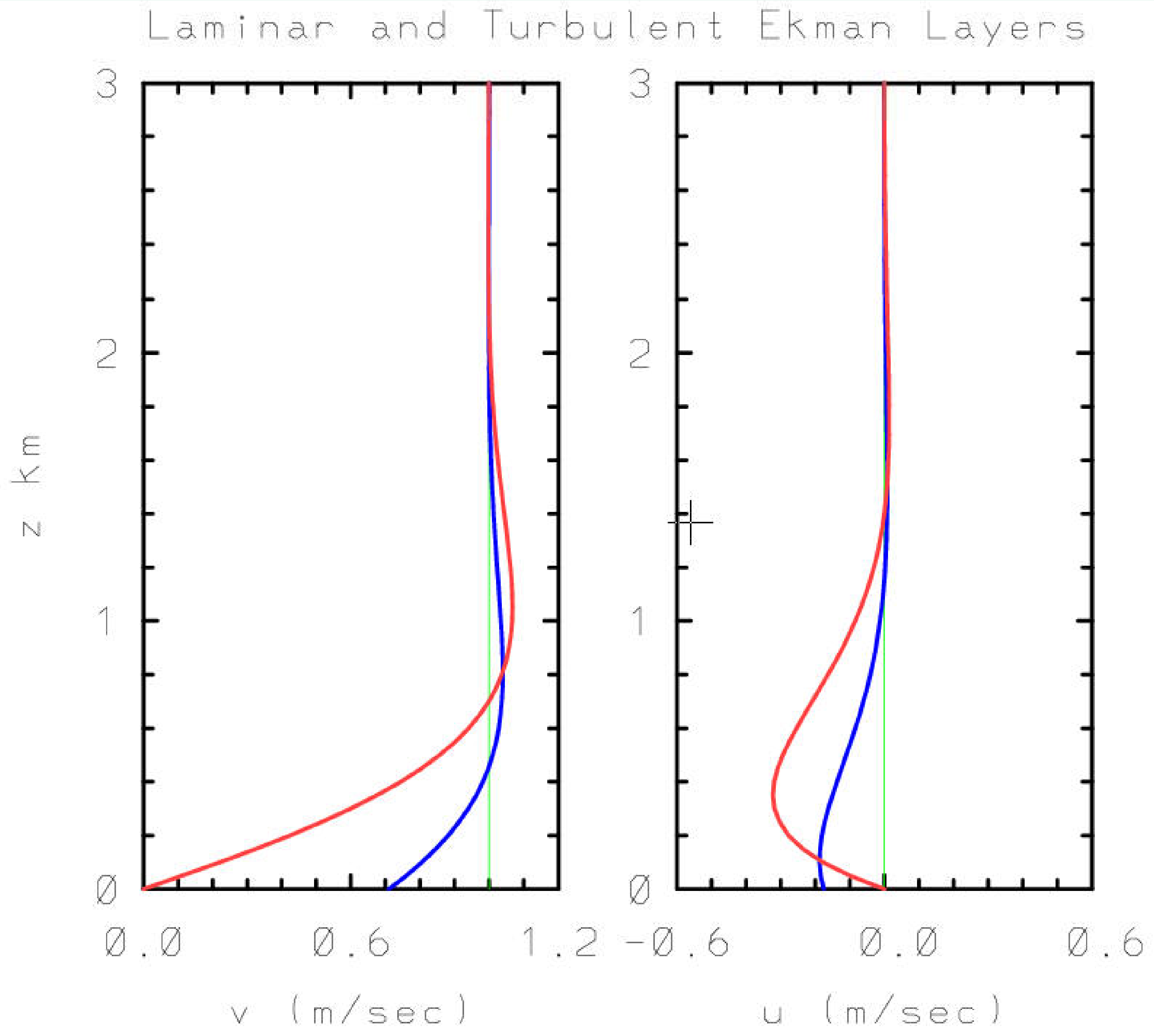
the solution has the form

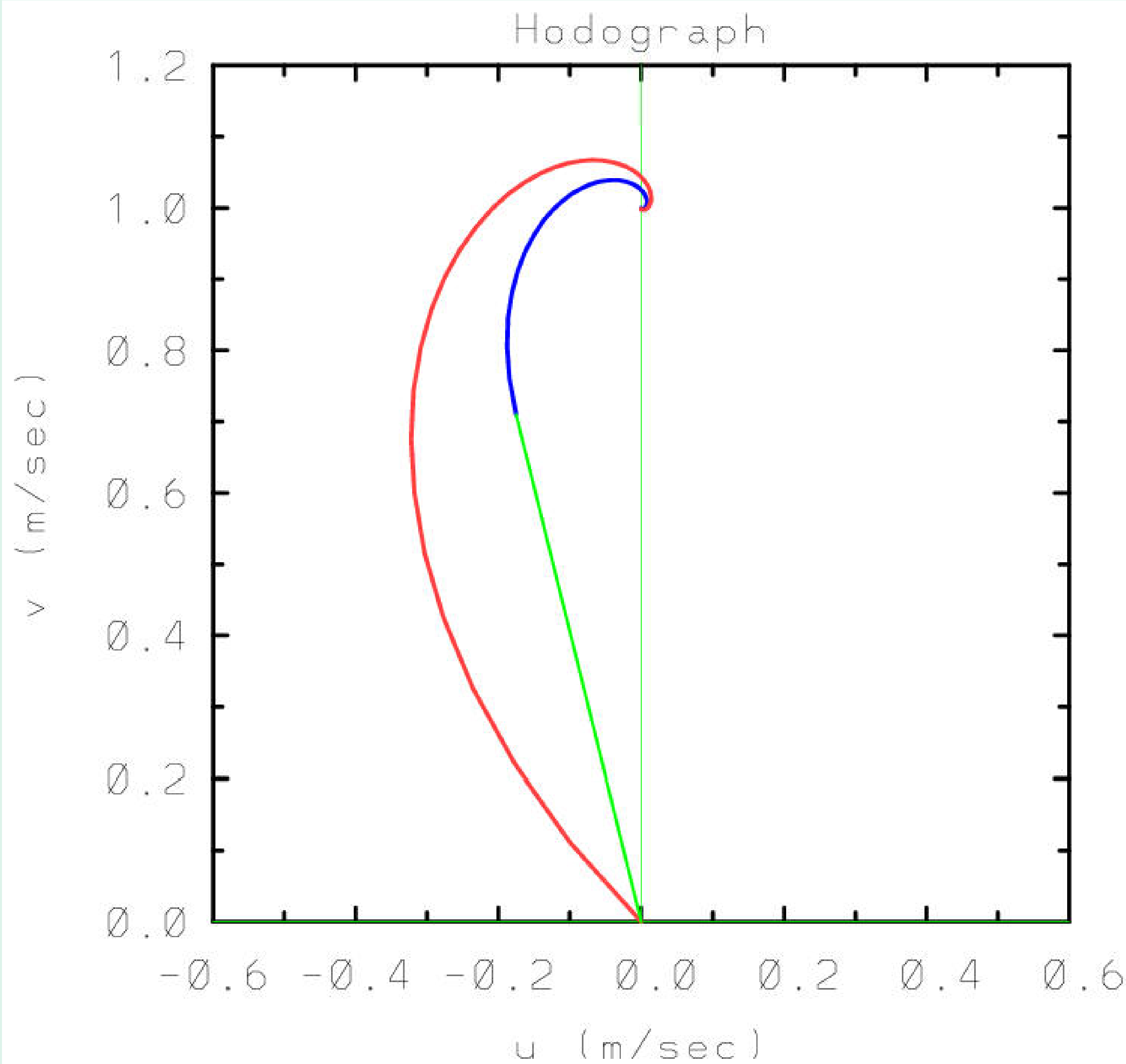
$$V = V_g[1 - A\exp(-(1-i)z/\delta),] \tag{4.4}$$

Representation of frictional stress

$$K \left. \frac{\partial \mathbf{u}}{\partial z} \right|_{z=0} = C_D |\mathbf{u_b}| \mathbf{u_b}$$

$$\mathbf{u_b} = (u_b, v_b)$$


Laminar and Turbulent Ekman Layers
z km
3
2
1
0
0.0
0.6
1.2
-0.6
0.0
0.6
v (m/sec)
u (m/sec)

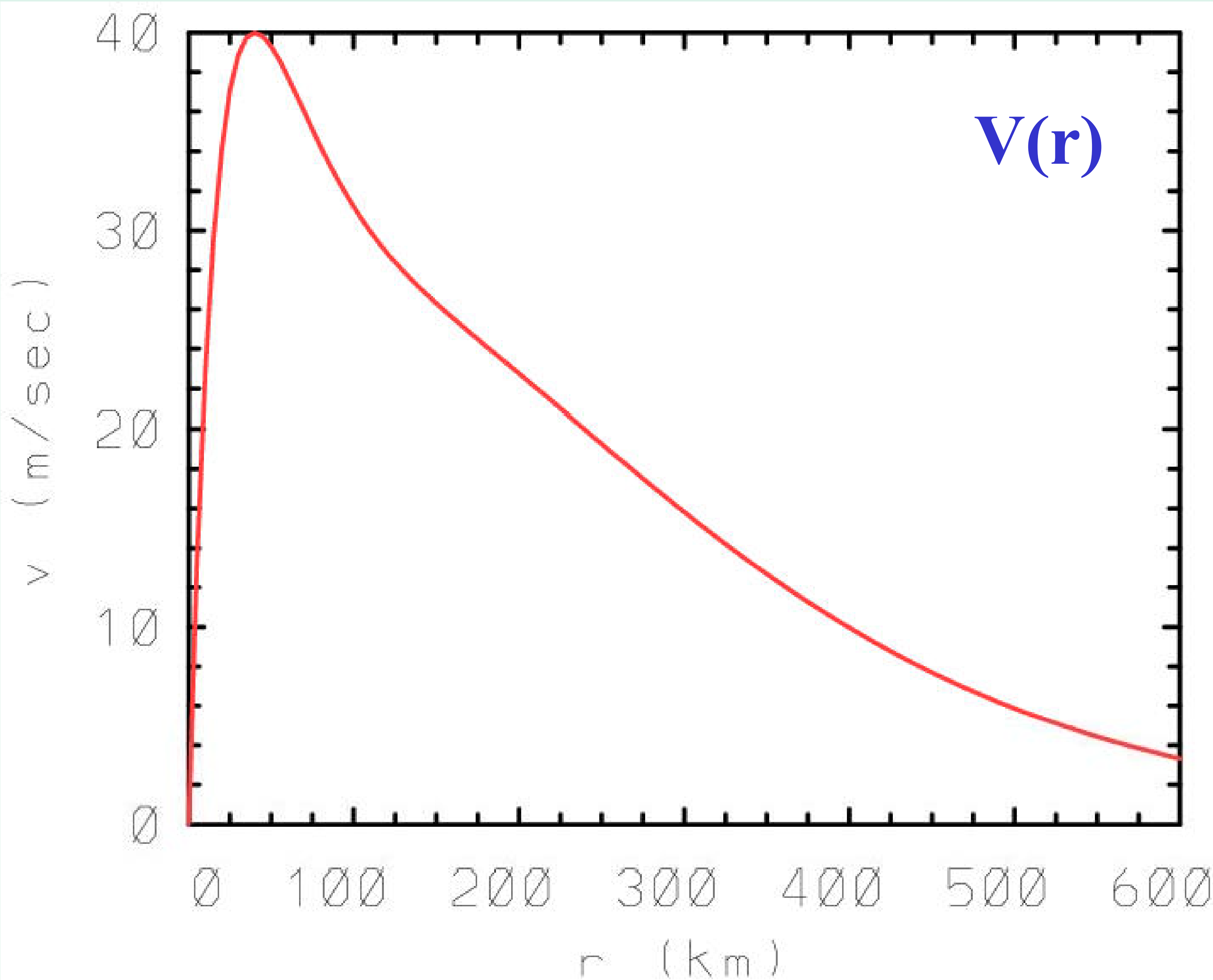

Hodograph
1.2
1.0
0.8
0.6
0.4
0.2
0.0
-0.6
-0.4
-0.2
0.0
0.2
0.4
0.6
v (m/sec)
u (m/sec)

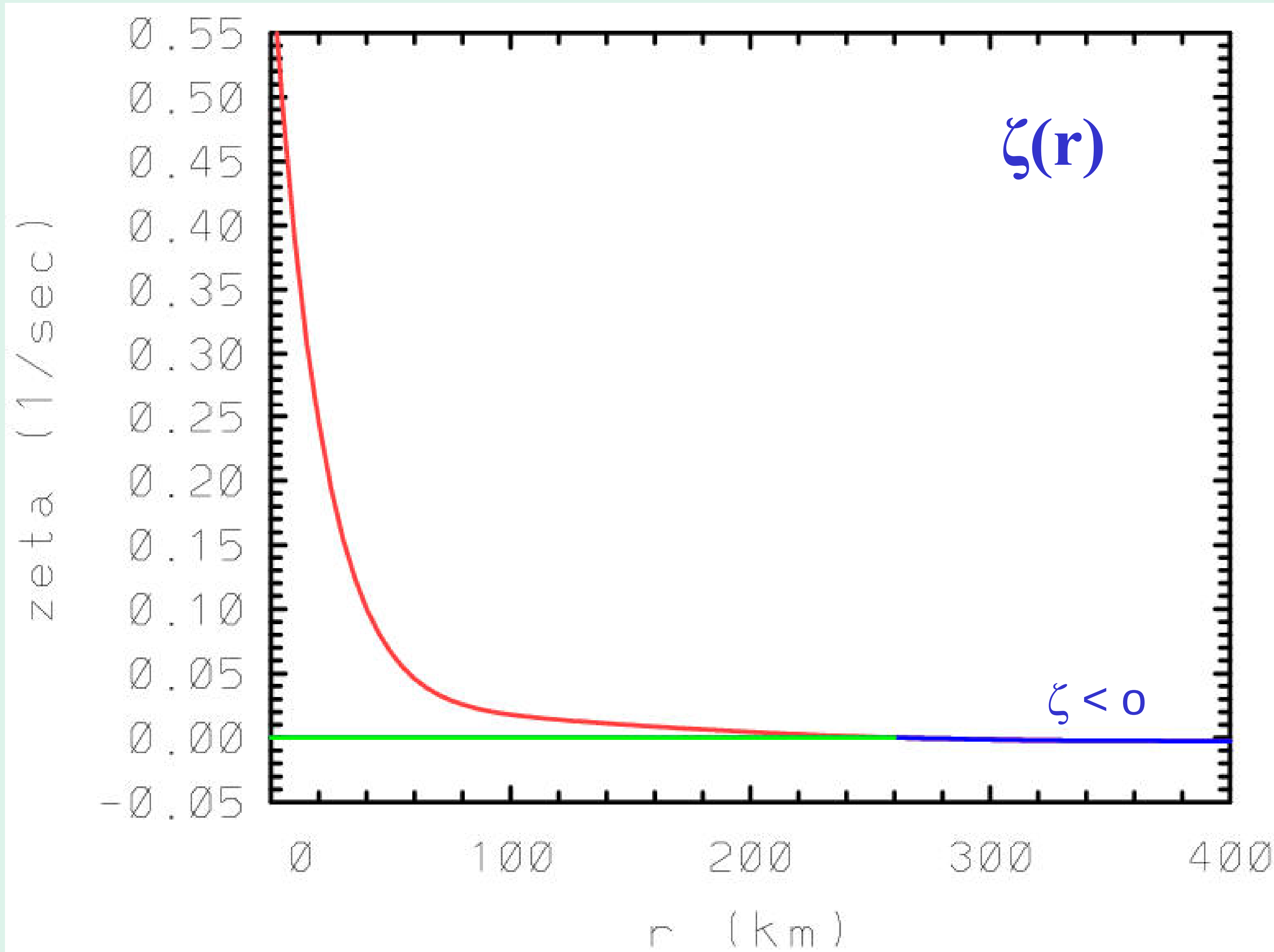
Linear solution

$$-v\left(f+\frac{2V}{r}\right)=K\frac{\partial^2 u}{\partial z^2}$$

$$u\left(f+\frac{V}{r}+\frac{\partial V}{\partial r}\right)=K\frac{\partial^2 v}{\partial z^2}$$

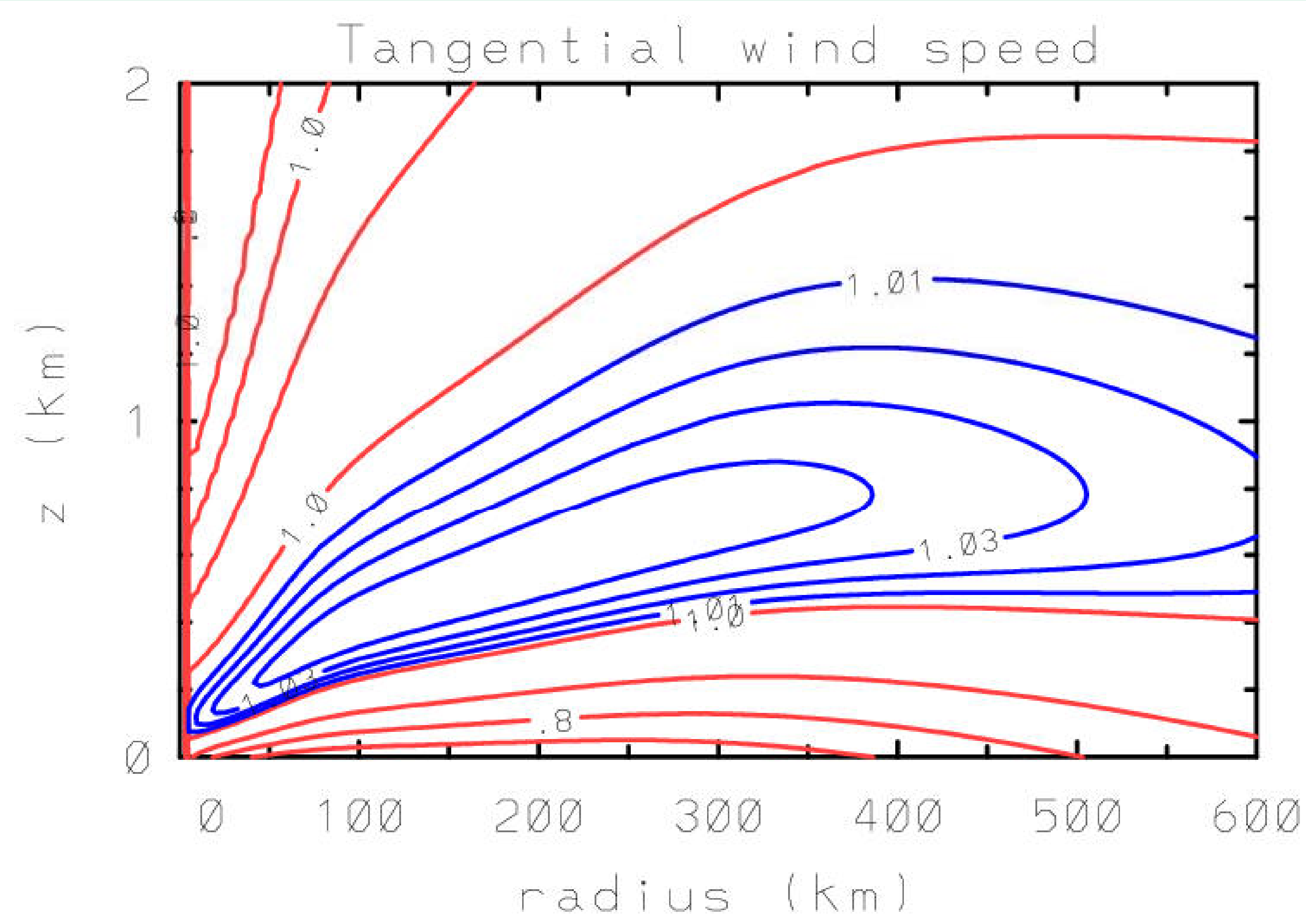
Can solve analytically


V(r)
40
30
20
10
0
v (m/sec)
0
100
200
300
400
500
600
r (km)

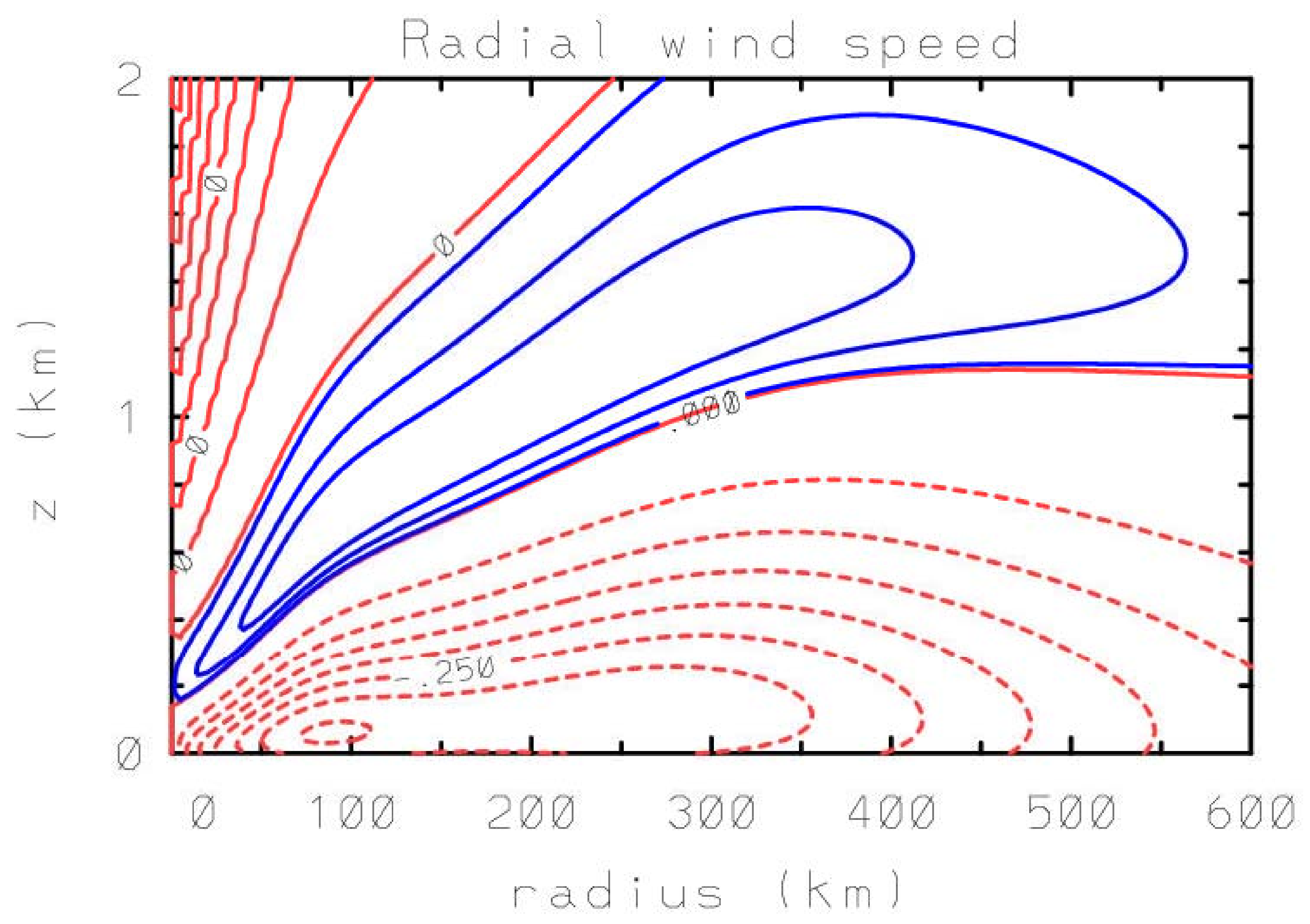


ζ(r)
ζ < o
zeta (1/sec)
0.55
0.50
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
-0.05
0
100
200
300
400
r (km)

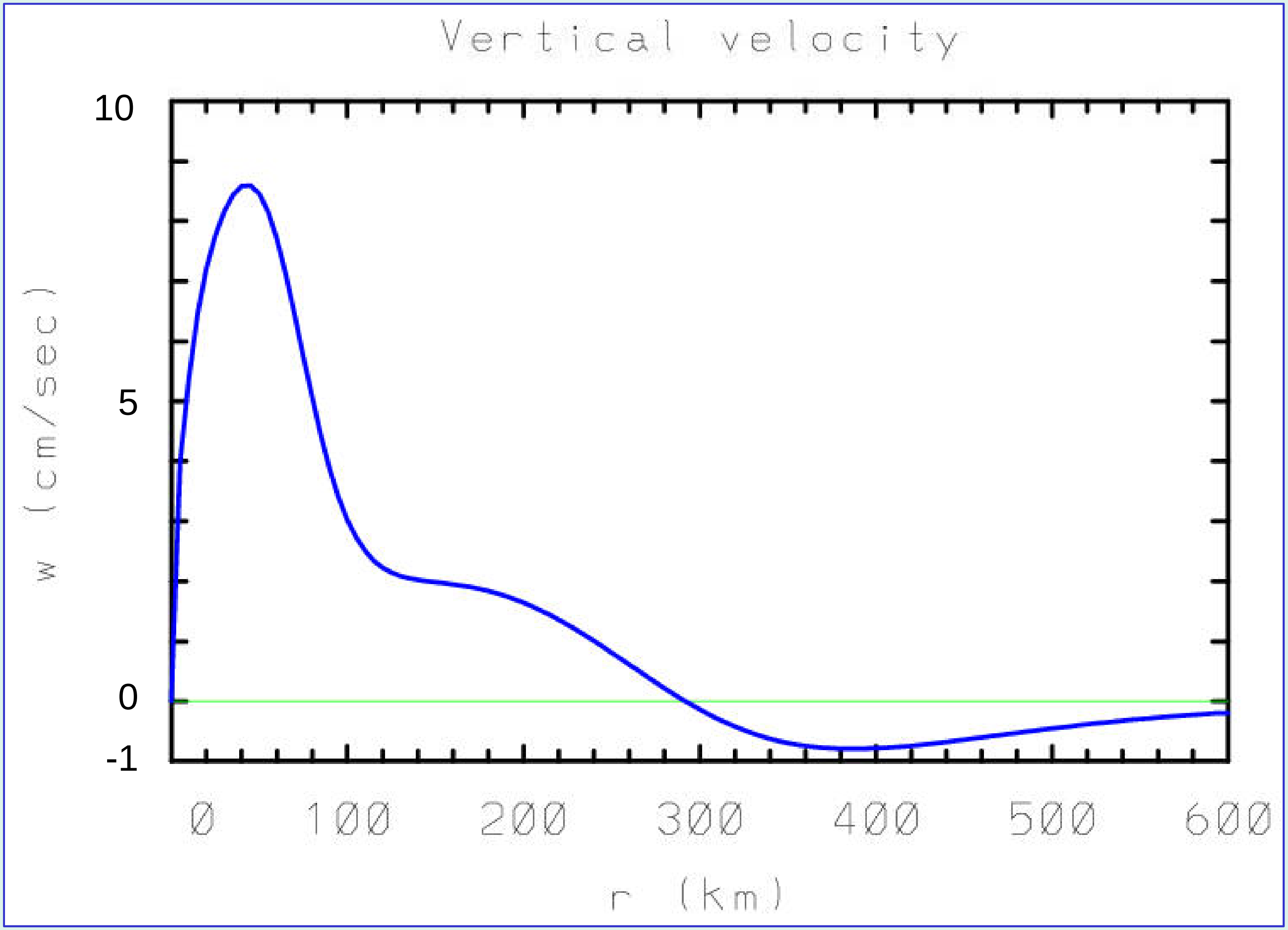
Linear theory



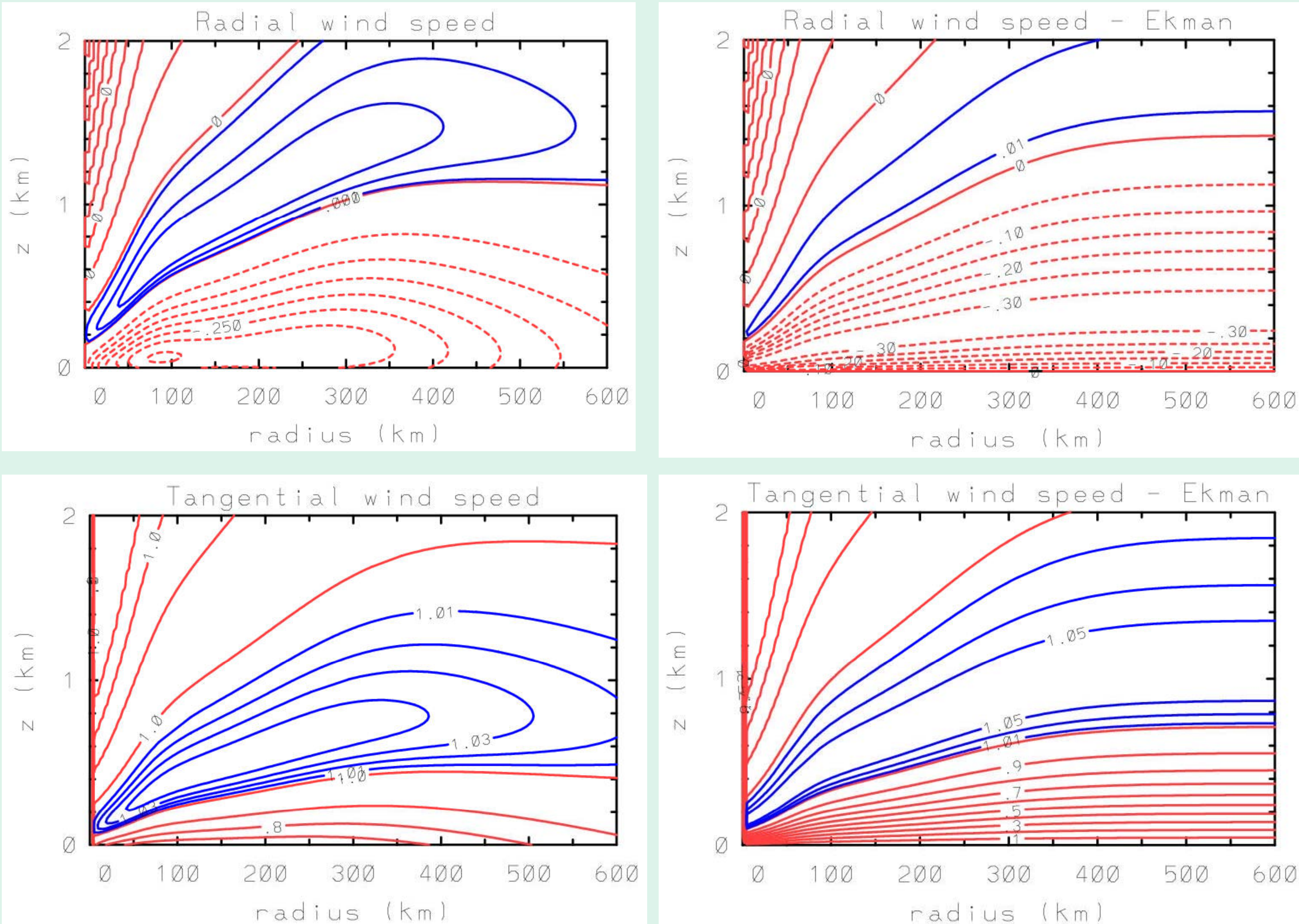
Linear theory

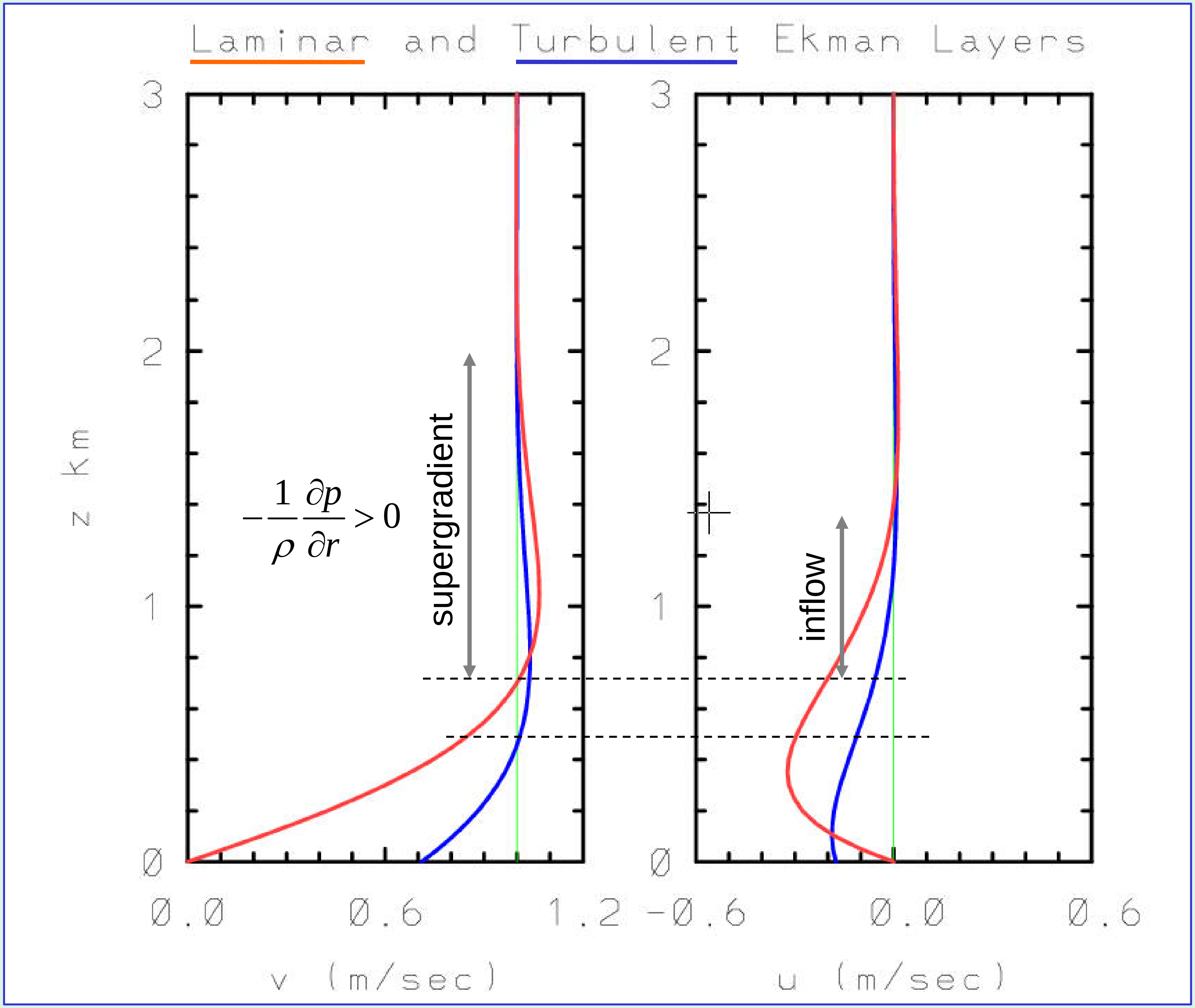


Linear theory



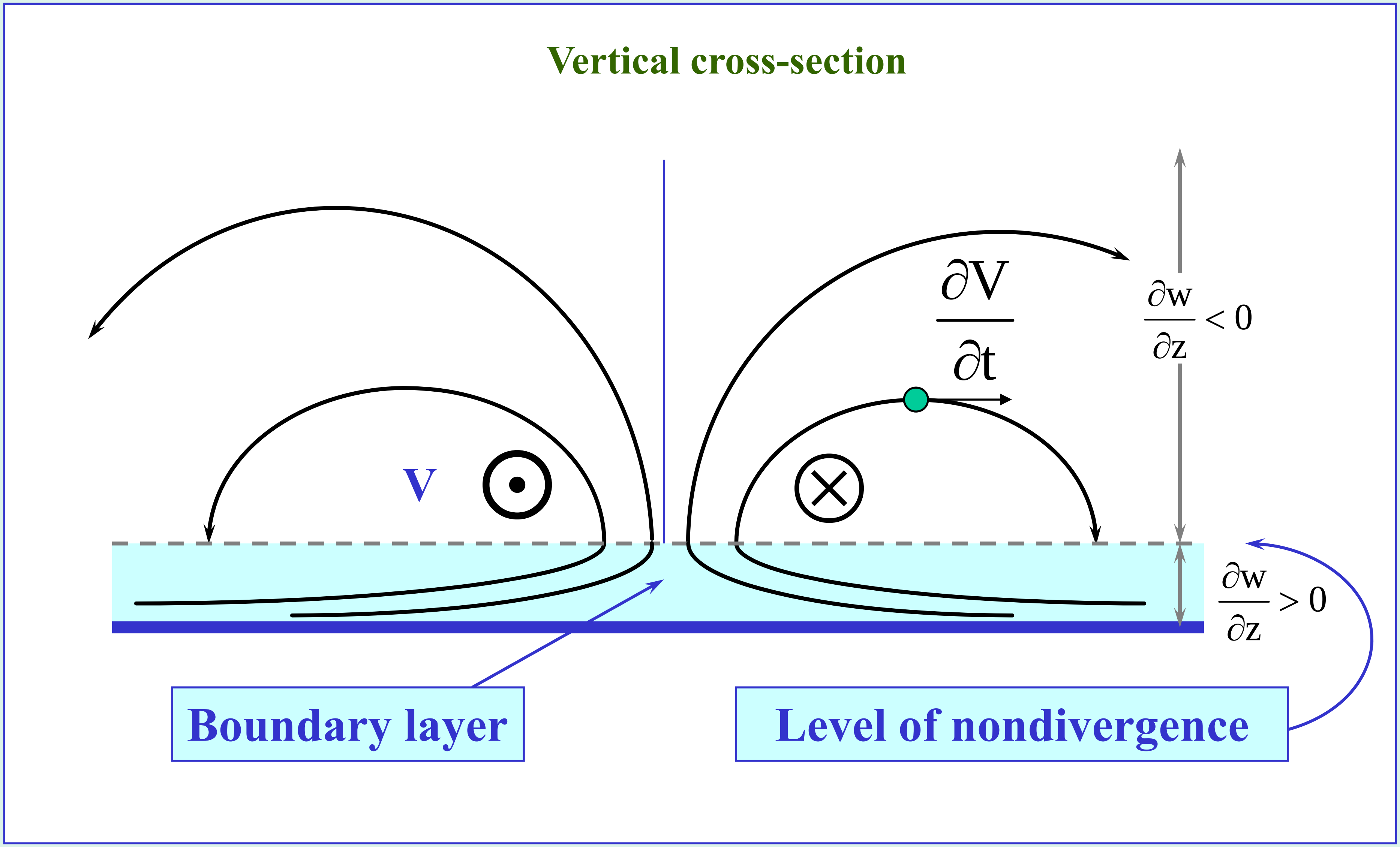
Linear theory versus Ekman theory



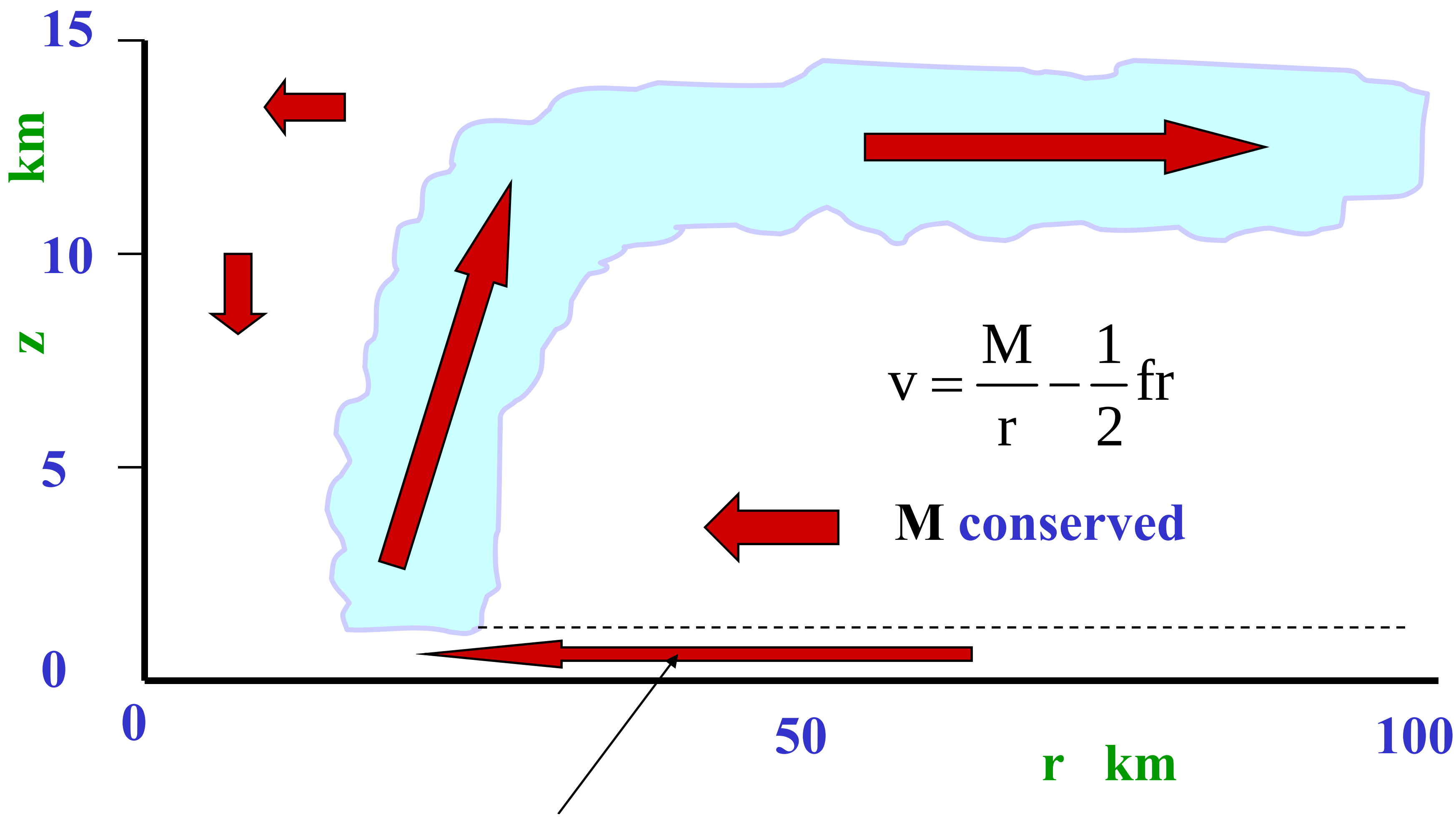


Laminar and Turbulent Ekman Layers
z km
$-\frac{1}{\rho}\frac{\partial p}{\partial r} > 0$
supergradient
inflow
3
2
1
0
0.0
0.6
1.2
-0.6
0.0
0.6
v (m/sec)
u (m/sec)

Dynamics of vortex spindown



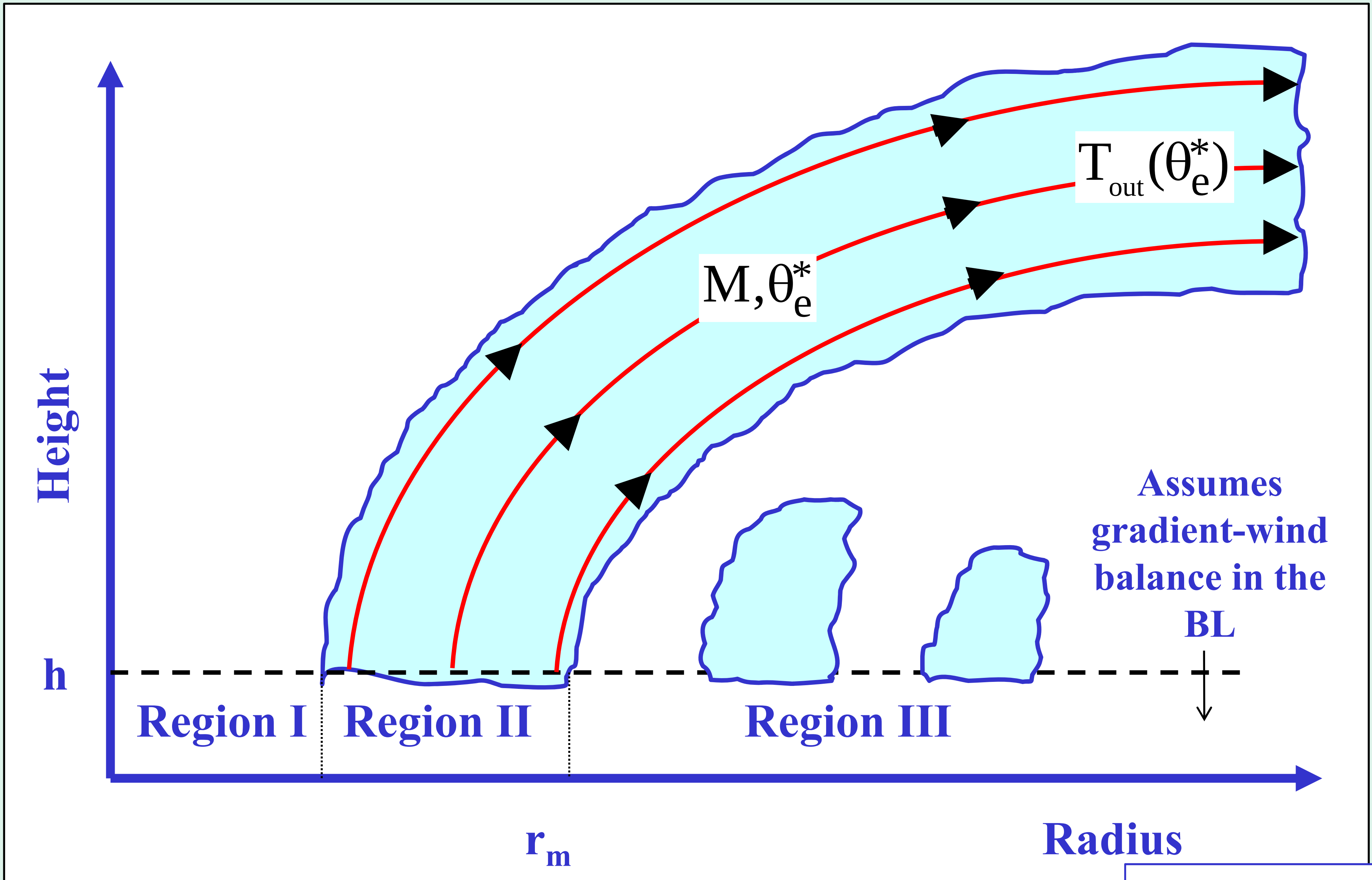
Conventional view of tropical cyclone spin up



$$v = \frac{M}{r} - \frac{1}{2}fr$$

M not conserved, boundary layer supplies clouds with moisture

Emanuel's 1986 steady-state TC model



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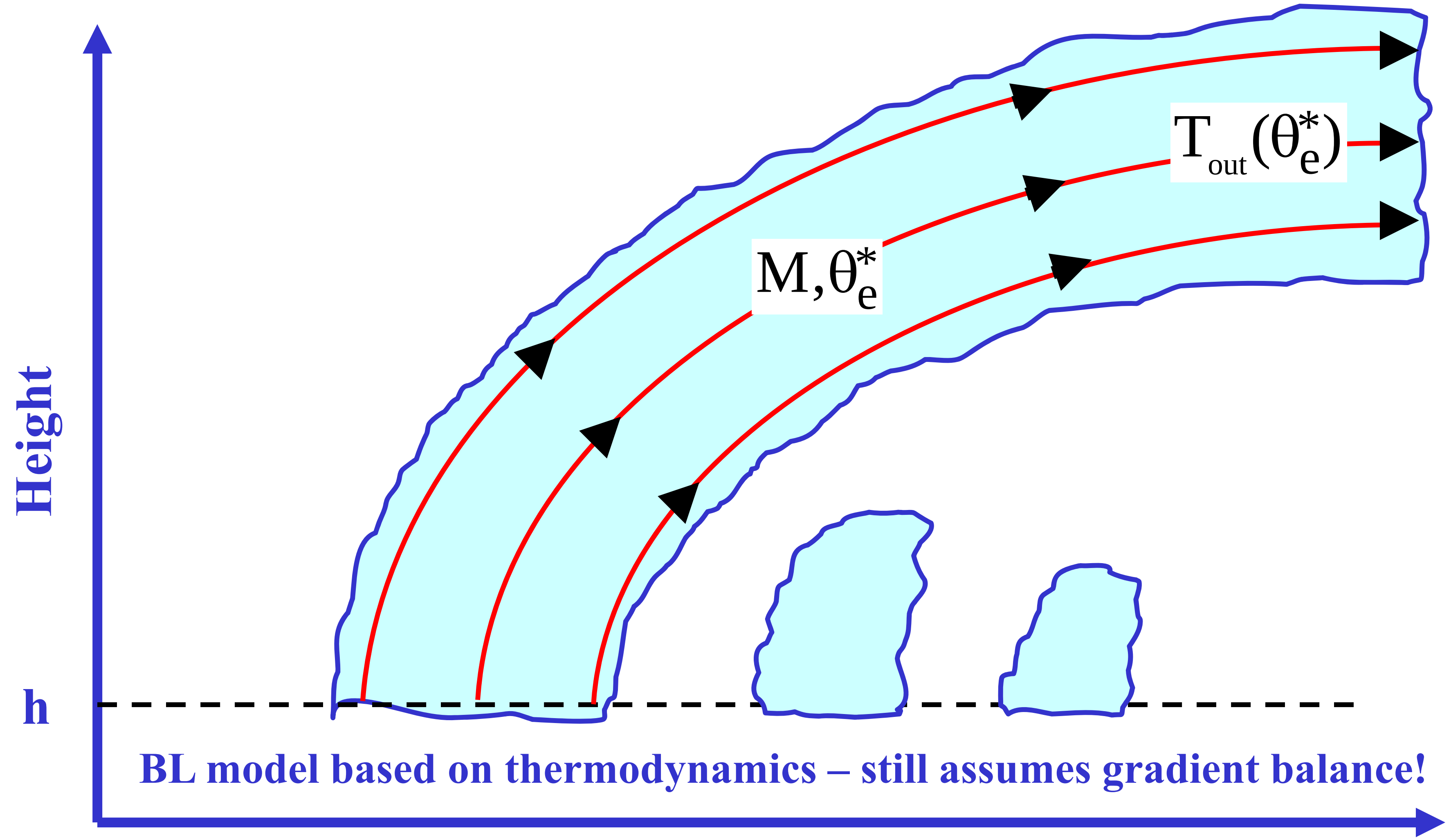
Emanuel's 1997 model for TC spin up



Model sets: $v_g(h) = v_b$

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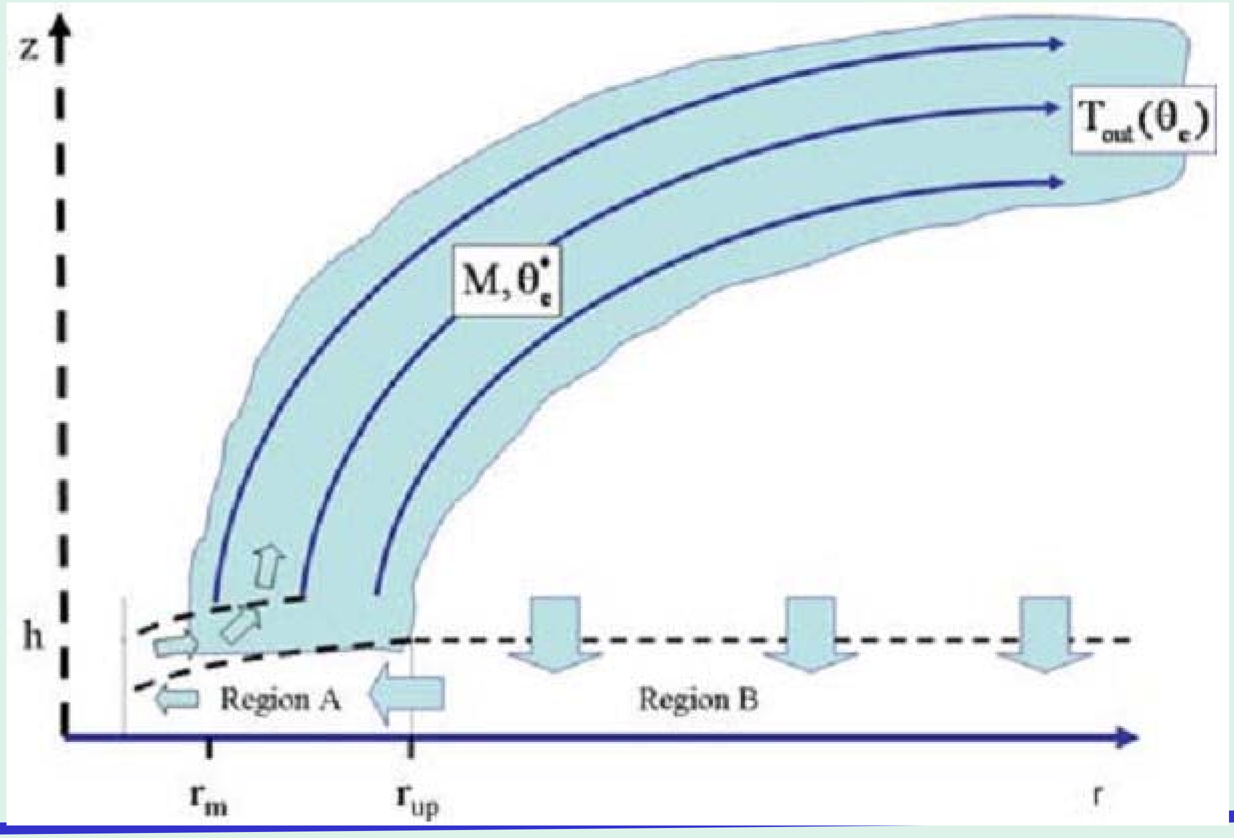
Questions

- **Does the boundary layer determine the tangential wind speed of air that ascends into the updraught?**
- **Consistent only if** $v_g(h) = v_b$**!**
- **Considerations: steady boundary layer equations are parabolic => information travels inwards. Region of ascent out of the boundary layer requires an open boundary condition.**
- **Can one improve the Emanuel model by relaxing the assumption of gradient wind balance in the boundary layer?**

Smith et al. (2008), Smith and Montgomery (2010)



Height

Momentum source

Radius

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Tropical cyclone spin-up revisited

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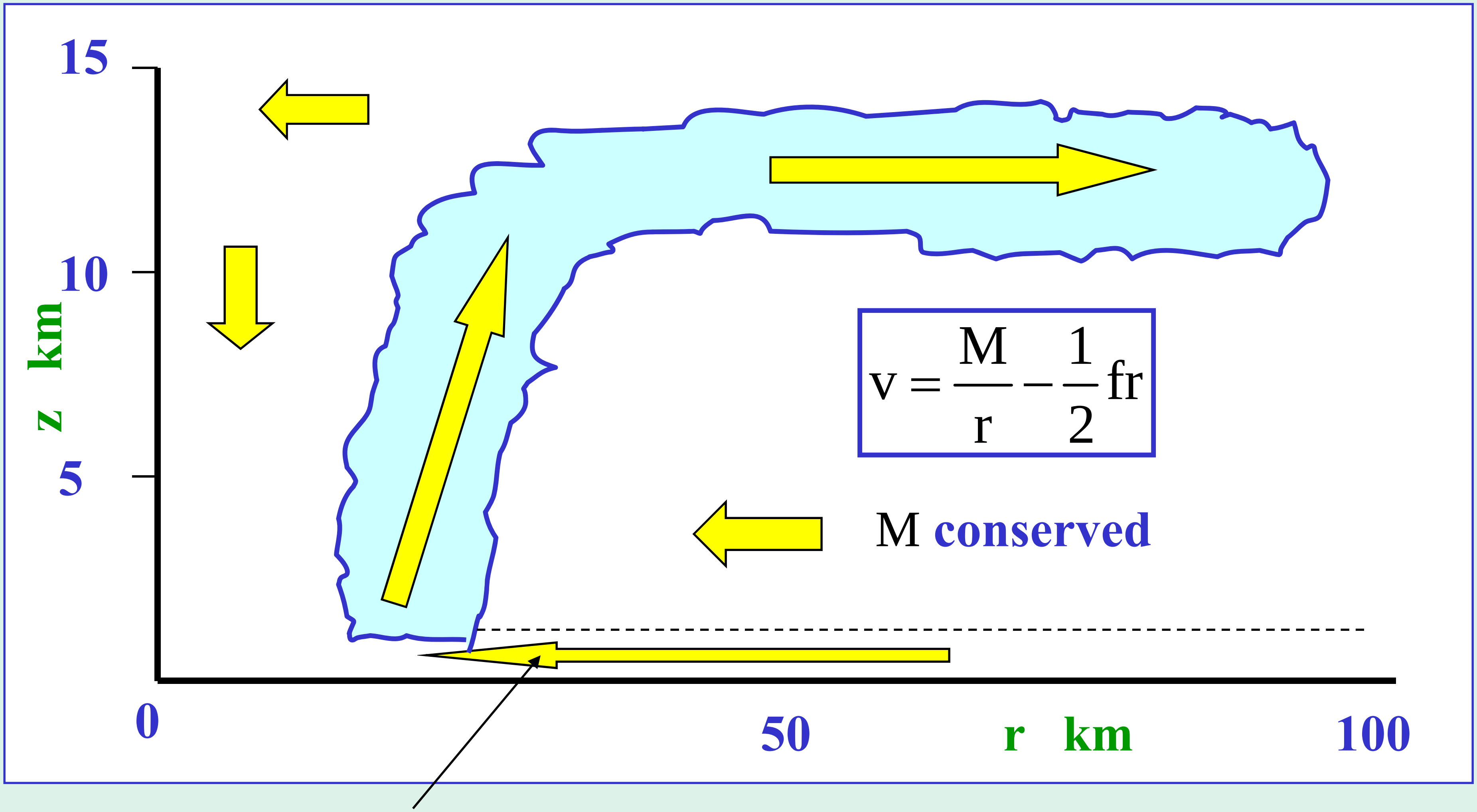
Zhang et al (2001) found that spin up occurred in the BL in Hurricane Andrew.

Revised view of tropical cyclone spin up



M reduced by friction, but strong convergence → small r

From Montgomery, Nguyen & Smith (2009): QJRMS