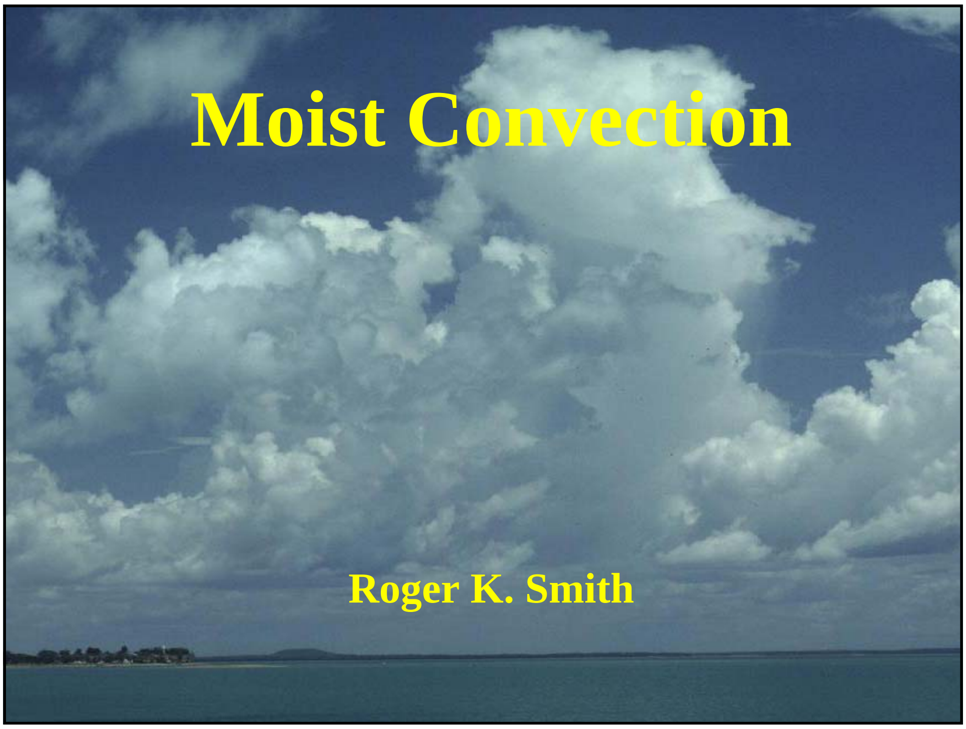
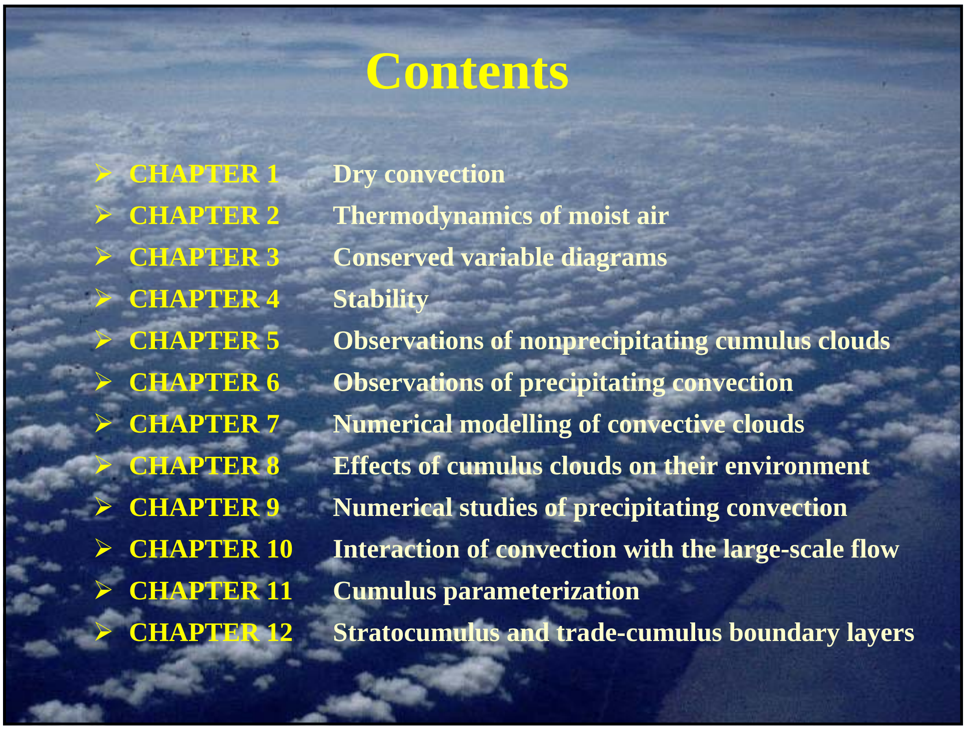

Moist Convection
Roger K. Smith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CHAPTER 10 Interaction of convection with the large-scale flow
CHAPTER 11 Cumulus parameterization
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- R. S. Scorer, (1958): Natural Aerodynamics, Pergamon Press

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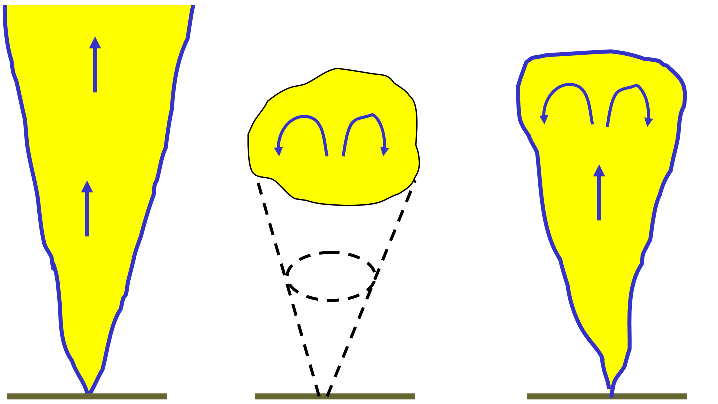
- B. Stevens (2005) Bulk boundary layer concepts for simplified models. Theor. Comp. Fluid Dyn.
- E. J. Zipser (2005) Some views on “hot towers” after 50 years of Tropical Field Programs and two years of TRMM data.

See my homepage manuscripts

Dry Convection

- **K. A. Emanuel (1994): Chapter 2 of Atmospheric Convection entitled "Convection from local sources"**
- **B. R. Morton (1997): Chapter 6 of The Physics and Parameterization of Moist Atmospheric Convection (Ed. R. K. Smith) entitled "Discrete dry convective entities: I Review"**
- **J. S. Turner (1973): Buoyancy Effects in Fluids**
- **R. S. Scorer (1958): Natural Aerodynamics, Chapter 7**

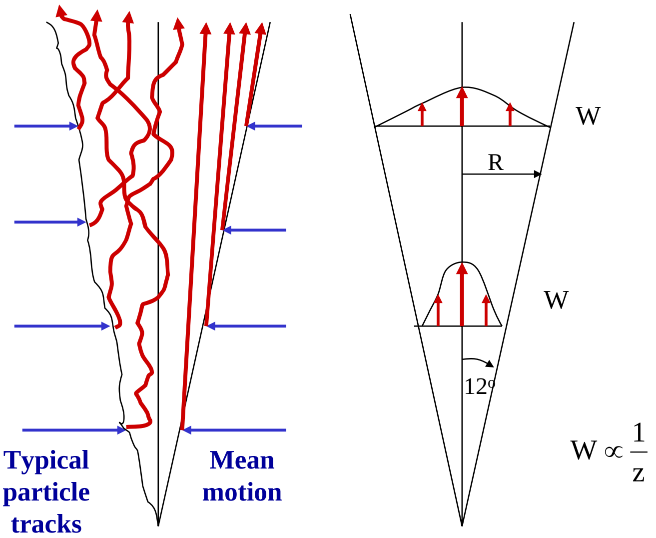
Convection from isolated sources



The arrows show the direction of mean motion. [From Turner, 1973].

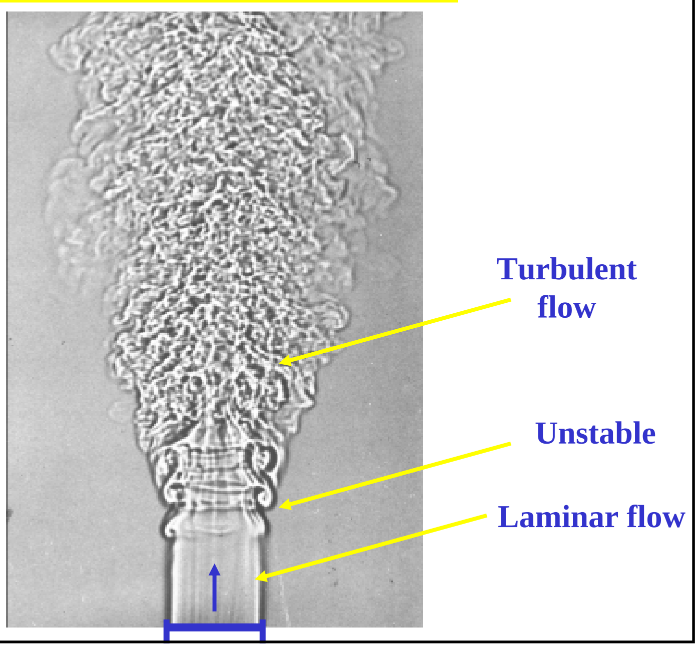
- **Plumes** are steady flows in which the buoyancy is supplied steadily so that the buoyant region is continuous.
- **A pure jet** is a steady flow from maintained source of momentum.
- **Forced plumes** have a source of momentum and buoyancy.
- **Thermals** are discrete buoyant elements in which the buoyancy is confined to a limited volume of fluid.
- **Starting** plumes are plumes with a well-defined, advancing upper edge.
- **Puffs** are analogous to thermals, but originate from pure momentum sources.

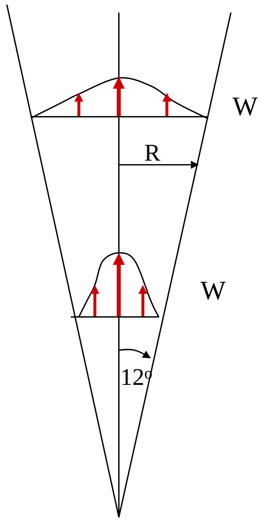
Pure jets, or momentum plumes



The entrainment process and the evolution of the near vertical velocity profile as a function of radius in a jet.

Instability of a pure jet





Volume flux F(z) **proportional to** WR^2

Momentum flux M(z) **proportional to** W^2R^2

Dimensional analysis

The mean properties of the jet such as the average vertical velocity and mean radius can depend only on M **and** z**, there being no other parameters in the problem.**

$$W = c_1 M^{1/2} / z$$
$$R = c_2 z$$

$$F = c_3 W R^2 = c_1 c_3 c_2^2 M^{1/2} z$$

The rate at which fluid is entrained is $\frac{dF}{dz}$

Define an entrainment velocity u_E **such that** $2\pi R u_E = \frac{dF}{dz}$

$$u_E = \frac{1}{2\pi R}\frac{dF}{dz} = \frac{1}{2\pi R} c_2 c_3 c_2^2 M^{1/2} = (c_2 c_3 / 2\pi) W$$

⇒ u_E **is inversely proportional to** R **and directly proportional to** W.

- **We have tacitly assumed that the orifice of the jet is small compared with the distance from it that we are considering.**
- **Close to the orifice the motion is determined by how the fluid is ejected, but it soon takes up the configuration described.**
- **Of course, the orifice will always have a finite size, but one can think of the motion as being due to a virtual point source situated some distance below the orifice.**

Pure buoyant plumes

In this case, the momentum flux is no longer conserved because momentum is generated by the buoyancy force in the plume.

Then the buoyancy flux, Σ, **is a constant** (see later)

$$\Sigma = 2\pi \int_0^\infty \sigma(r,z) w(r,z) r dr$$

The buoyancy flux has dimensions:

(force/unit mass) × **velocity** × **area =** L^4T^{-3}.

Dimensional analysis

The mean properties of the plume, which include the average vertical velocity w**, the average buoyancy** b**, and mean radius** R**, can depend only on** Σ **and** z:

$$w = c_1 \Sigma^{1/3} z^{-1/3}$$
$$b = c_2 \Sigma^{2/3} z^{-5/3}$$
$$R = c_3 z$$

We can say something also about the dependence of the time-averaged quantities on radius within the plume:

These quantities must depend on r/R**, where** r **is the distance from the plume axis and** R **is a radial scale for the plume width**

$$w = \frac{\Sigma^{1/3}}{z^{1/3}} F_1\left(\frac{r}{R}\right)$$
$$b = \frac{\Sigma^{2/3}}{z^{5/3}} F_2\left(\frac{r}{R}\right)$$

$R = \alpha z$, **α is a constant**

F_1 **and** F_2 **are functions of scaled radius.**

The mass flux F**, which is proportional to** WR^2**, increases with height as** $z^{5/3}$.

Entrainment velocity: $$2\pi R u_E = \frac{dF}{dz}$$

Again, this implies a turbulent entrainment of mass in which the mean inflow velocity is proportional to w.

$$\frac{dF}{dz} \propto z^{2/3} \quad \Rightarrow \quad u \propto z^{1/3} \propto w$$

Yih (1951) **determined the functions** F_1 **and** F_2 **experimentally for turbulent plumes in air contained in a large closed room.**

He found that

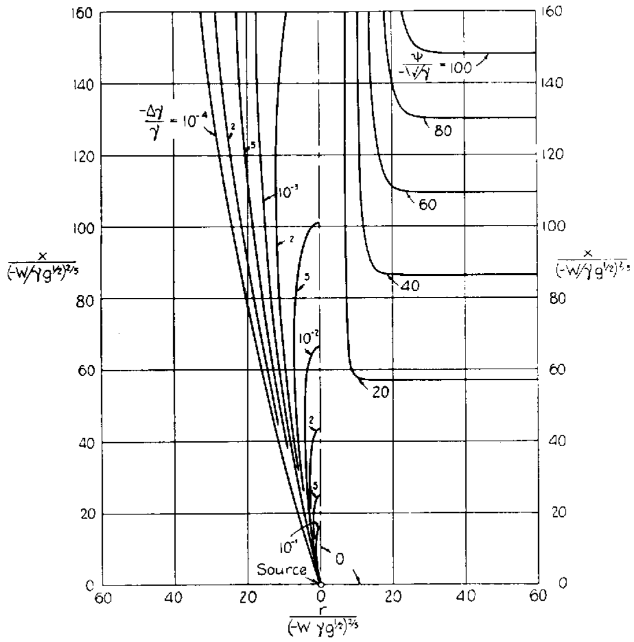
$$w = 4.7\frac{\Sigma^{1/3}}{z^{1/3}}\exp\left(-\frac{96r^2}{z^2}\right)$$

$$b = 11.0\frac{\Sigma^{2/3}}{z^{5/3}}\exp\left(-\frac{71r^2}{z^2}\right)$$

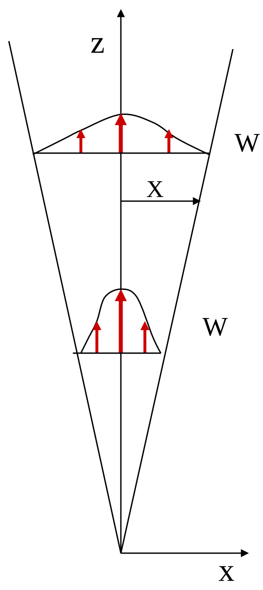
$$r = 0.12z$$

The mean plume has a conical cross-section with about a 7° angle of spread.

The streamlines and isotherms corresponding to these expressions are shown in the next figure =>



Plumes originating from line sources



Suppose that plume originates from a maintained line source of buoyancy.

Volume flux F(z) **proportional to** WX^2

Momentum flux M(z) **proportional to** W^2X^2

The mean flow structure would be expected to have the form of a wedge and the buoyancy flux will be defined per unit with length along the source:

$$\Sigma = \int_{-\infty}^{\infty} w\sigma dx$$

Again, the turbulent flow can depend only on Σ **and on the variables** x **and** z.

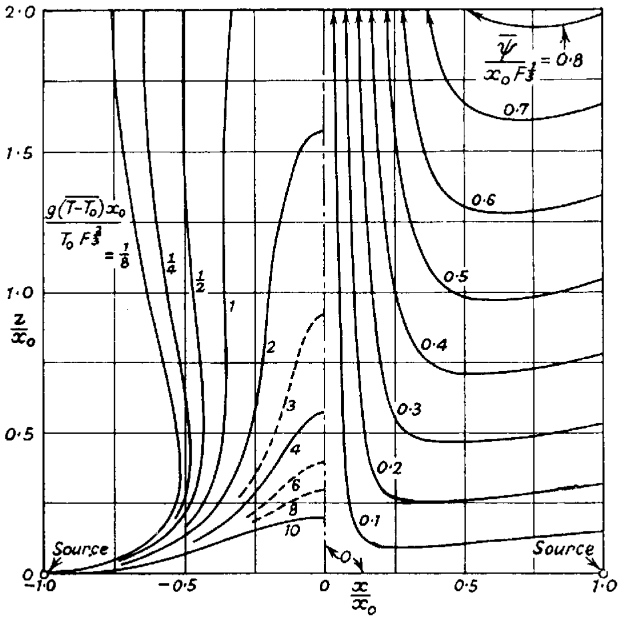
In this case the similarity solution is:

$$w = 1.80G(\Sigma', z)\exp\left(-\frac{32x^2}{z^2}\right)$$

$$b = 2.6H(\Sigma', z)\exp\left(-\frac{41x^2}{z^2}\right)$$

$$R = 0.16z$$

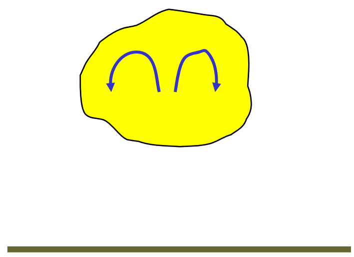
Maintained plumes originating from **two** parallel line sources



After Rouse, *et al*. (1952)

Thermals

- Thermals are a result of turbulent convection from an instantaneous isolated (or point) source.
- When buoyancy is created instantaneously in an isolated region of fluid, the buoyant fluid will rise through its environment as a turbulent thermal, entraining ambient air as it does so.



- If we regard time rather than height as the key independent variable, many assumptions concerning the behaviour of plumes may be applied also to thermals, i.e.,

  1. The radial profiles of velocity and buoyancy are geometrically similar at all times,
  2. The mean entrainment velocity is proportional to the mean vertical velocity, and
  3. The density perturbation in the thermal is small compared to the mean density (the Boussinesq approximation).

For thermals in a neutrally-stratified fluid, the only external parameter is the amount of buoyancy released by the source:

$$Q = \iiint_{V_o} b_o dV$$

the initial volume of the source (V_o)

the initial buoyancy distribution (b_o)

Let z(t) = the height of some centre in the rising thermal at time t. Then using dimensional analysis (Batchelor, 1954):

vertical velocity $$w = \frac{Q^{1/2}}{z} G\left(\frac{r}{R}\right)$$

buoyancy $$b = \frac{Q}{z^3} H\left(\frac{r}{R}\right)$$

characteristic radius $$R = \gamma z$$

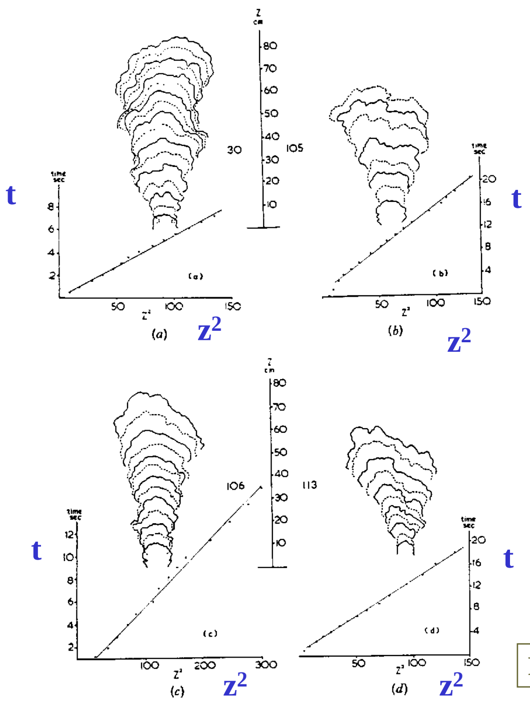
γ: a constant

Evolution of thermals

$$w = \frac{Q^{1/2}}{z} G\left(\frac{r}{R}\right) \qquad b = \frac{Q}{z^3} H\left(\frac{r}{R}\right) \qquad R = \gamma z$$

Note that $z(dz/dt)$ **is independent of** t **so that** $z \propto t^{1/2}$, **while** w **and** b **vary as** $t^{-1/2}$ **and** $t^{-3/2}$**, respectively.**

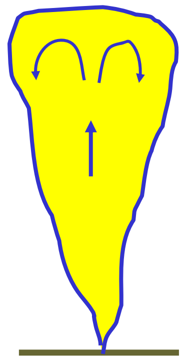
z^2 **proportional to** t



From Scorer, 1957

Starting plumes

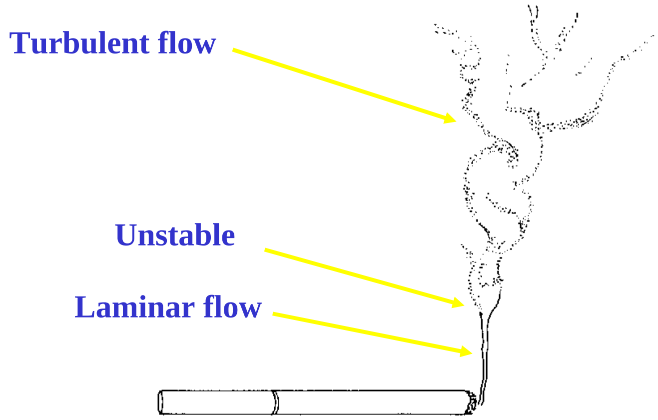
- One can obtain solutions for a turbulent starting plume in a neutrally-stratified fluid.
- One assumes that the advancing cap of the plume behaves like a thermal, while the body of the plume is similar to a full plume (Turner, 1962).



a starting plume

- The solutions for a pure thermal and a pure plume are matched across the interface between them, taking into account the fact that the rate of ascent of the cap is slower than the vertical motion within the centre of the cap.
- Turner's results, supported by laboratory experiments, show that the rate of ascent of the cap is intermediate between the ascent rate of a pure thermal and vertical velocity in a pure plume.
- He found also that approximately half of the total entrainment of ambient fluid is through the advancing cap.

Laminar plumes originating from a maintained point source



Behaviour of a rising smoke plume over a cigarette

Transition to turbulence

- Laminar flows become unstable when the Reynolds number exceeds a critical value, and usually they become turbulent.
- The local Reynolds number, Re, for a laminar plume is defined by

$$\mathrm{Re} \equiv \frac{wR}{\nu} \approx \frac{z^{1/2}\Sigma^{1/4}}{\nu^{3/4}} \qquad \Sigma = 2\pi\int_o^\infty wbrdr$$

ν: kinematic viscosity

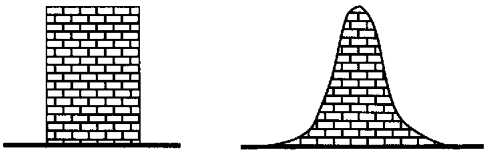
- This increases as the square root of the height so that we would expect the plume to become turbulent beyond a certain height (witness again the cigarette!)

Turbulent convection in a stably-stratified fluid

- If the environment of a plume is stably-stratified, the ambient density variation will affect the buoyancy of the plume and one would expect the plume to become negatively buoyant beyond a certain height.
- If the environment is unstably-stratified, it is likely to be convecting and the ambient turbulence will be a factor in the plume dynamics. Neglecting this effect, one would expect the plume to ascend more rapidly than if the environment were neutral.
- Stably-stratified case: an additional parameter characterizing the stratification is required to describe the system and dimensional analysis is more limited. Therefore we make more explicit use of the governing equations.

- Following Morton *et al.* (1956), we *assume* a particular radial dependence of the velocity and buoyancy, and integrate the Boussinesq equations over a horizontal plane.
- We end up with a set of ordinary differential equations for the evolution of quantities along plume.
- The basic assumptions are borrowed from the self-similar solutions in unstratified flow:
  - The flow is steady,
  - The radial profiles of mean vertical velocity and mean buoyancy are similar at all heights,
  - The mean turbulent entrainment velocity u is proportional to w, i.e., $u = -\alpha w$, and
  - The Boussinesq approximation is valid.

- Note that $u = -\alpha w$ is exactly true when $N = 0$.
- Two simple choices for the radial dependence are: ‘top-hat’ or Gaussian profile.
  - The particular form we choose will affect only the numerical value of the coefficients in the resulting equations for w and σ, but not their dependence on z or the buoyancy flux at the source level.



Mathematics =>

- Choose a ‘top-hat’ profile.

$$\frac{1}{r}\frac{\partial}{\partial r}(ru) + \frac{\partial w}{\partial z} = 0$$

- Integrate the mass continuity equation expressed in cylindrical coordinates over the horizontal area of the plume

$$\int_o^{2\pi} d\theta \int_o^R \frac{1}{r}\frac{\partial}{\partial r}(ru) r dr + \frac{\partial}{\partial z}\int_o^{2\pi} d\theta \int_o^R w r dr = 0$$

$$2\pi u(R) + \frac{\partial}{\partial z}(\pi R^2 w) = 0$$

entrainment relation =>

$$\frac{d}{dz}(\pi R^2 w) = 2\pi\alpha R w$$

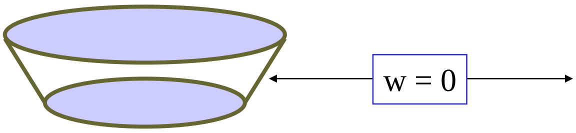
The increase of volume flux with height is proportional to the volume entrainment through the boundary of the plume (and similarly for the mass flux).

- In flux from, the vertical momentum equation is:

$$\nabla\cdot(\mathbf{u}w)=\frac{1}{r}\frac{\partial}{\partial r}(ruw)+\frac{\partial}{\partial z}(w^2)=b+\frac{1}{r}\frac{\partial \tau}{\partial r}$$

The **perturbation pressure gradient** is neglected in comparison with the buoyancy force, but a term representing the effect of the **turbulent frictional stress**, τ is included.

- The trick now is to integrate this equation over the conical-shaped volume:



$$\int_o^{2\pi} d\theta \int_o^R rdr \int_z^{z+\Delta z} \nabla\cdot(\mathbf{u}w)\,dz = \int_o^{2\pi} d\theta \int_o^R rdr \int_z^{z+\Delta z} bdz$$

Apply the divergence theorem ➔

$$\int_o^{2\pi} d\theta \int_o^R rdr \int_z^{z+\Delta z} \nabla\cdot(\mathbf{u}w)\,dz = \int_o^{2\pi} d\theta \int_o^R rdr \int_z^{z+\Delta z} bdz$$

by the **divergence theorem**, this term can be written as a surface integral:

$$\int_S \int w\mathbf{u}\cdot\hat{\mathbf{n}}dS = \int_o^{2\pi} d\theta \int_o^R rdr \int_z^{z+\Delta z} bdz$$

↑ the unit normal vector outwards from the surface S enclosing the volume

$w = 0$ at the lateral boundaries of the plume ➔ the only contributions to the surface integral are from the horizontal disks at z and $z + dz$ where $w\mathbf{u}\cdot\mathbf{n} = w^2$.

$$\left[\pi R^2 w^2 + \frac{d}{dz}\left(\pi R^2 w^2\right)\Delta z\right] - \pi R^2 w^2 = R^2 b^2 \Delta z$$

as $\Delta z \to 0$

$$\frac{d}{dz}\left(\pi R^2 w^2\right) = R^2 b$$

- **In flux from, the steady form of the buoyancy equation is:**

$$\nabla \cdot (\mathbf{u}b) + N^2 w = \frac{1}{r}\frac{\partial F_b}{\partial r} \quad \text{where} \quad N^2 = \frac{1}{\theta_o}\frac{d\theta_o}{dz}$$

We have to include a term N^2w **to represent the effect of the environmental stability and a turbulent heat flux gradient.**

Integrating over the incremental volume and using the divergence theorem gives =>

$$\frac{d}{dz}\left(\pi R^2 wb\right) = -\pi N^2 R^2 w$$

Governing equations

$$\frac{d}{dz}(\pi R^2 w) = 2\pi\alpha R w$$

$$\frac{d}{dz}\left(\pi R^2 w^2\right) = R^2 b$$

$$\frac{d}{dz}\left(\pi R^2 wb\right) = -\pi N^2 R^2 w$$

Unknowns are: **plume radius,** R(z) **vertical velocity,** w(z) **buoyancy,** b(z)

Some notes

- **Note that the special case of neutral stratification is recovered by setting** θ_a = constant**, whereupon** $N = 0$.
- **Then**

$$\frac{d}{dz}\left(\pi R^2 wb\right) = -\pi N^2 R^2 w$$

implies that the buoyancy flux $\pi R^2 wb$ **is independent of** z**.**

- **The above equation shows that when the fluid is stably-stratified (**$N^2 > 0$**), the buoyancy flux** $\Sigma = \pi R^2 wb$ **decreases with height and must eventually change sign.**
- **In such a situation we cannot expect similarity solutions to apply and one must solve the governing equations.**

Transformed equations

Put $V = Rw$, $Y = R^2 w$, $B = R^2 wb$

$$\frac{d}{dz}(\pi R^2 w) = 2\pi\alpha R w \quad \rightarrow \quad \frac{dY}{dz} = 2\alpha V$$

$$\frac{d}{dz}\left(\pi R^2 w^2\right) = R^2 b \quad \rightarrow \quad \frac{dV^4}{dz} = 4BY$$

$$\frac{d}{dz}\left(\pi R^2 wb\right) = -\pi N^2 R^2 w \quad \rightarrow \quad \frac{dB}{dz} = -2YN^2$$

Initial conditions:

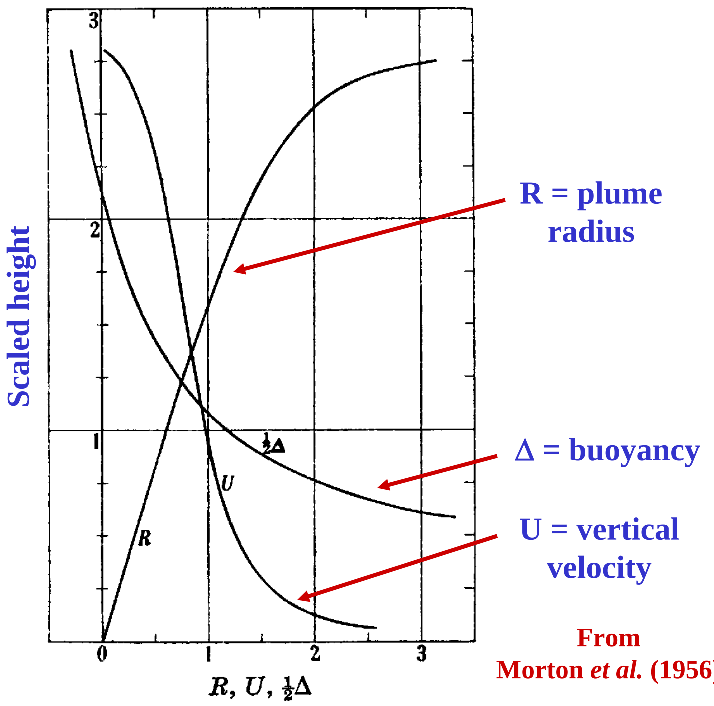
$R = w = 0$, $B = B_o$ at $z = 0$ → $Y = V = 0$, $B = (2/\pi)B_o$ at $z = 0$

Solutions

- There are only two external parameters in the problem B_o and N^2 (for the present we assume that N^2 is a constant).
- These two parameters must determine the character of the plume and they can be removed by suitably nondimensionalizing the dependent and independent variables (for details see Emanuel (1994, p. 30).
- Analytic solutions may be obtained when $N^2 = 0$, or for special vertical variations of $N^2 > 0$ (for details see Emanuel (1994, pp28-29).
- For general variation of N^2 the equations may be solved numerically using, for example, a Runge-Kutta method.

Morton *et al.* solutions

- Solutions were obtained by Morton *et al.* (1956), although they assumed Gaussian profiles for the radial distribution of buoyancy and vertical velocity.
- This is a superior approximation to the 'top-hat' profiles assumed here and leads to equations with slightly different numerical coefficients:
- Profiles are shown in the next slide =>



From Morton *et al.* (1956)

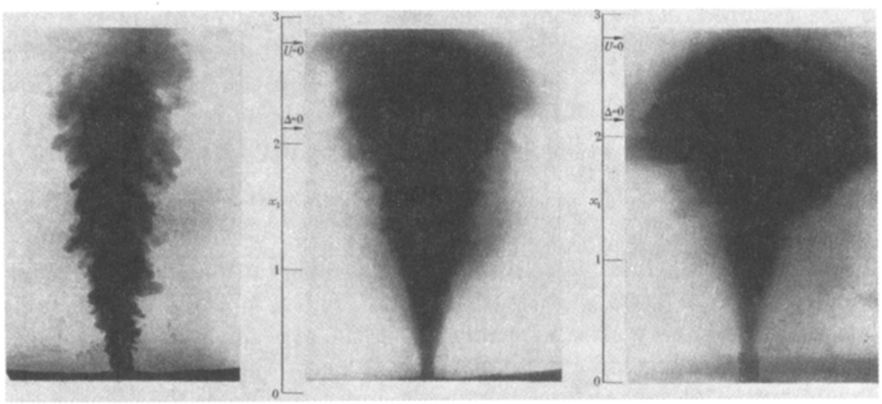
Theory for thermals

- A similar theory can be worked out for the motion of a turbulent thermal in a stably-stratified fluid.
- Under certain conditions, analytic solutions may be obtained.
- For further details, see Morton *et al.* (1956) and the summary in Emanuel (1994, pp. 31-34).

Laboratory experiments

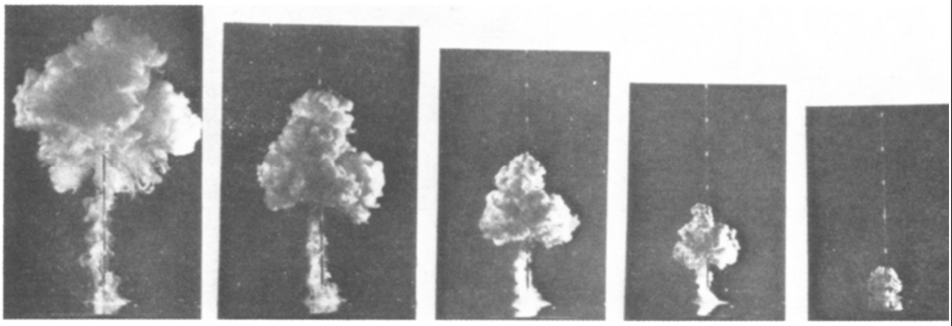
- A range of laboratory experiments have been carried out with the aim of verifying theoretical predictions of the behaviour of plumes and thermals.
- In most of these, positively (negatively) buoyant fluid is released from the bottom (top) of a task containing a large amount of fluid that is either homogenous or stably-stratified.
- A stable stratification can be produced in water by successively adding layers of salt solution of increasing density to the bottom of the tank.
- As the salt diffuses it eventually establishes a smooth density gradient.

Plumes

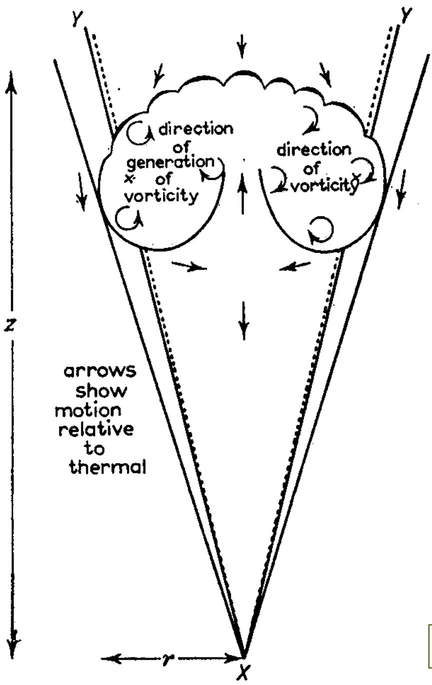


Photographs of plumes in neutrally and stably stratified fluids. At left is a plume in a neutrally-stratified ambient fluid; at right time exposures of a plume in a stably-stratified fluid at early and late stages in its development. (from Morton *et al.*, 1956).

Thermals

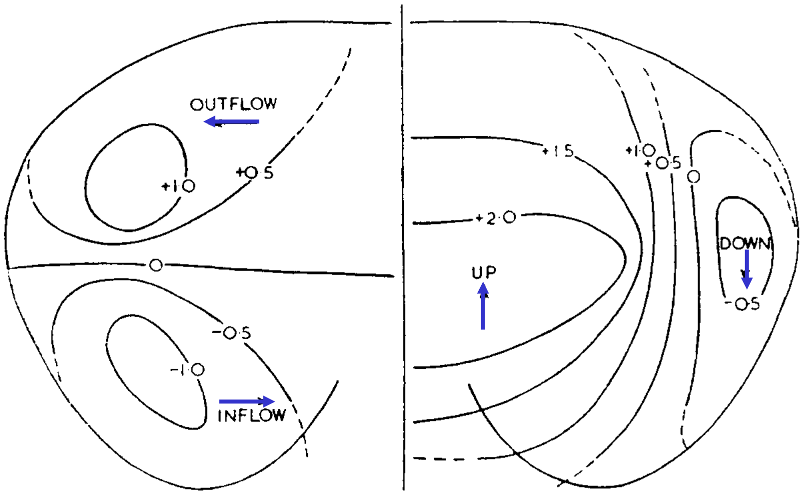


Sequence of photographs showing the decent a cloud of dense fluid in a tank of lighter fluid. The photographs are shown upside down (from Scorer, 1957). Note that the shape of the thermal may persist while the volume increases several times



From Scorer, 1958

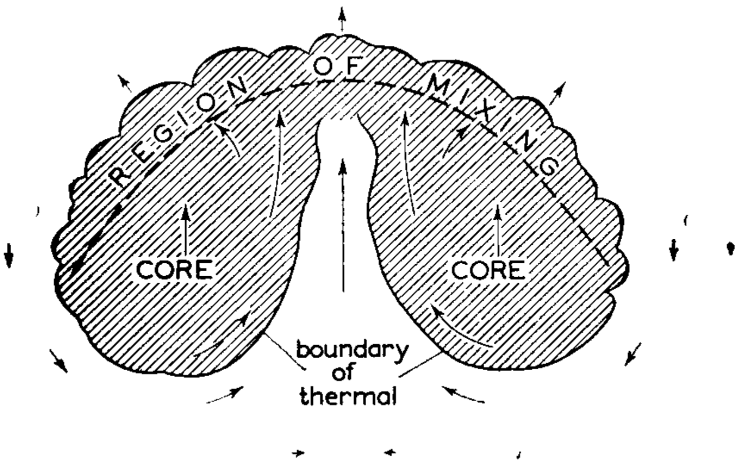
Motion in a thermal



radial motion

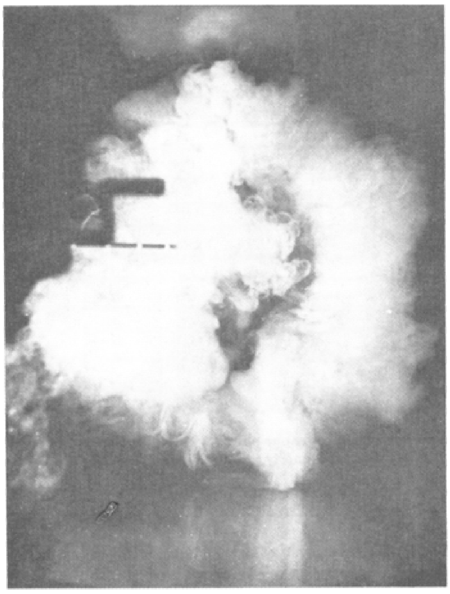
vertical motion

From Scorer, 1957

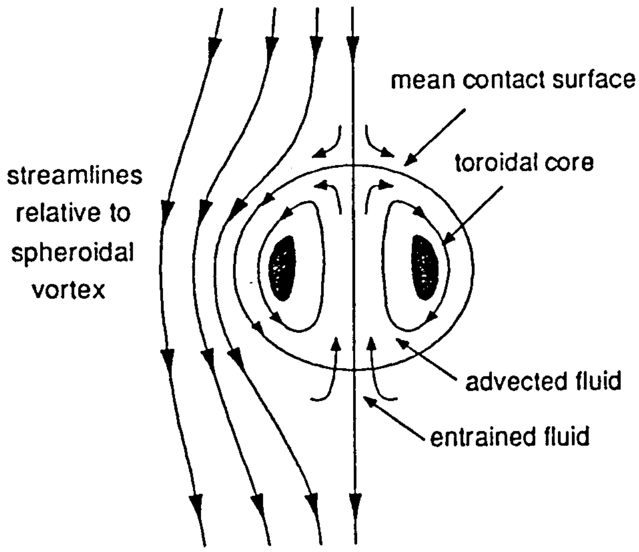


From Scorer, 1958

Sinking thermal from above

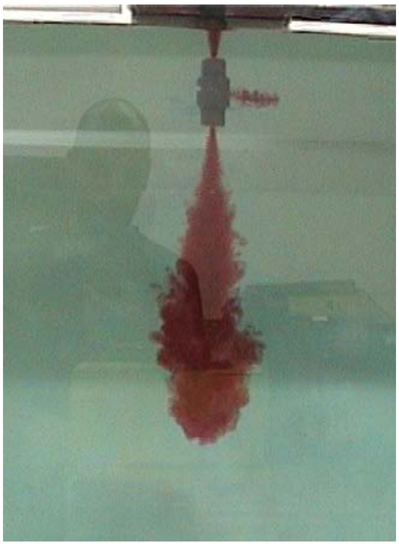


From Scorer, 1957



Sketch of streamlines relative to a spheroidal vortex [From Morton, 1997]

Sinking plume – MIM GFD Laboratory



From Goler, 2005

Sinking plume

From Goler, 2005


The End