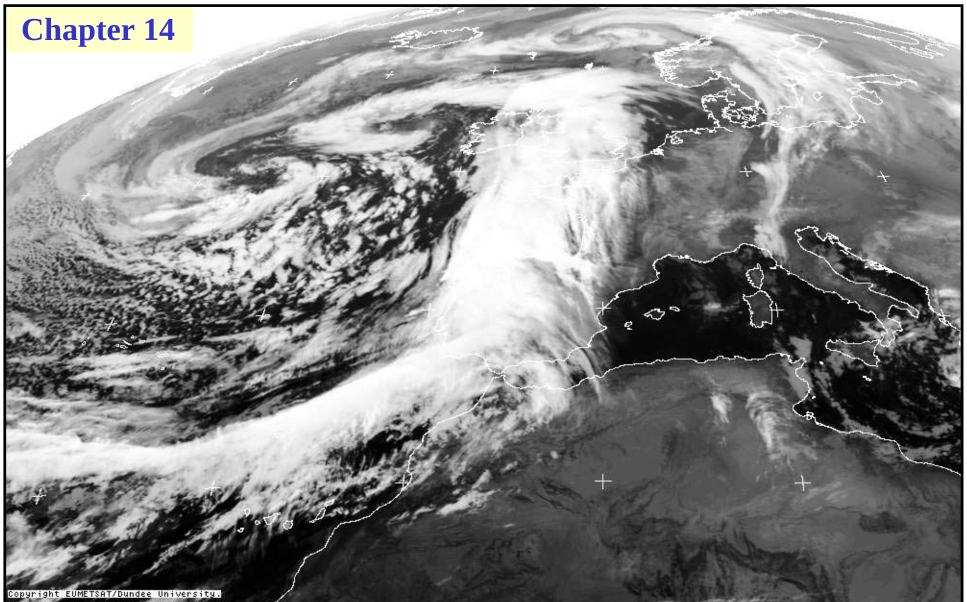
Chapter 14



Fronts and Frontogenesis

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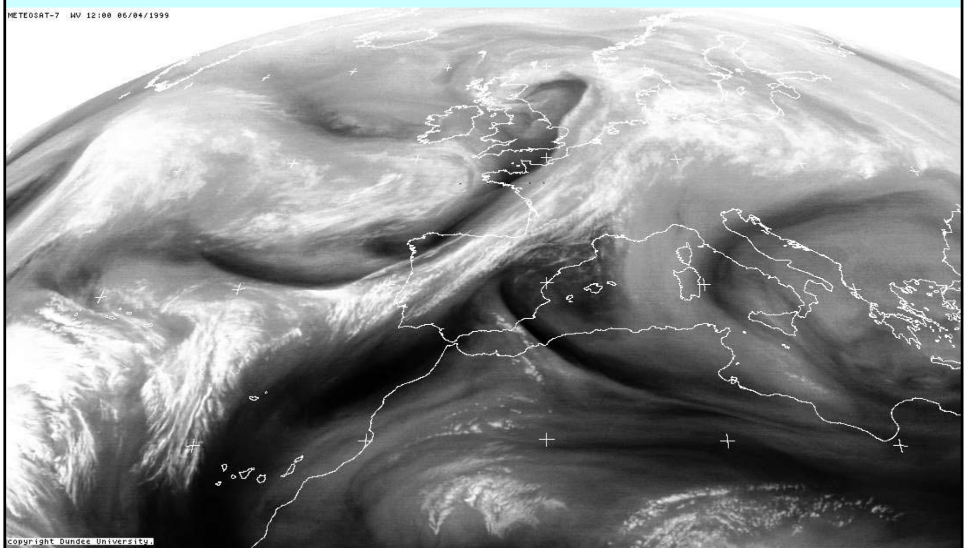

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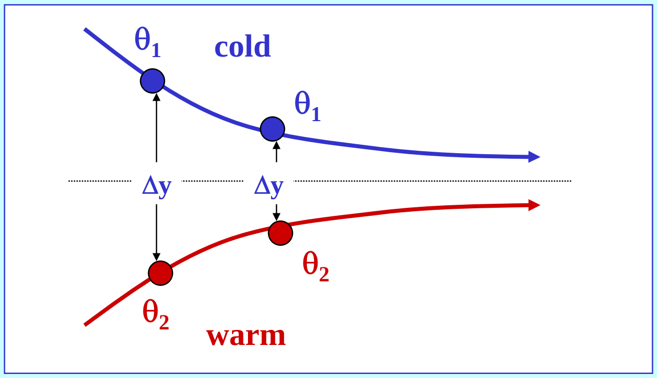
Problems with simple frontal models

- **Chapter 13 examines some simple air mass models of fronts and shows these to have certain deficiencies in relation to observed fronts.**
- **Sawyer (1956) - "although the Norwegian system of frontal analysis has been generally accepted by weather forecasters since the** 1920's**, no satisfactory explanation has been given for the 'up-gliding' motion of the warm air to which is attributed the characteristic frontal cloud and rain. "**
- **"Simple dynamical theory shows that a sloping discontinuity between two air masses with different densities and velocities can exist without vertical movement of either air mass...".**

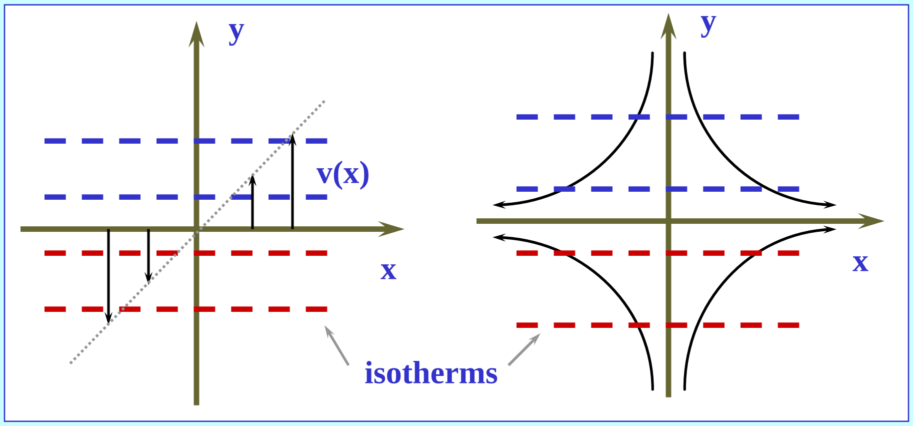
- Sawyer =>
- "A front should be considered not so much as a stable area of strong temperature contrast between two air masses, but as an area into which active confluence of air currents of different temperature is taking place".



- Several processes including friction, turbulence and vertical motion (ascent in warm air leads to cooling, subsidence in cold air leads to warming) might be expected to destroy the sharp temperature contrast of a front within a day or two of formation.
- Clearly defined fronts are likely to be found only where active frontogenesis is in progress; i.e., in an area where the horizontal air movements are such as to intensify the horizontal temperature gradients.
- These ideas are supported by observations.

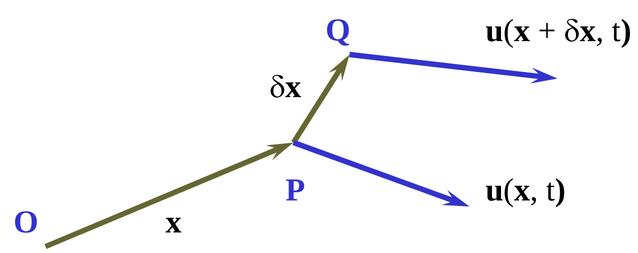
The kinematics of frontogenesis

Two basic horizontal flow configurations which can lead to frontogenesis:



The intensification of a horizontal temperature gradient by (a) horizontal shear, and (b) a pure horizontal deformation field.

Relative motion near a point in a fluid



In tensor notation

$$\delta u_i = \underbrace{\frac{\partial u_i}{\partial x_j}}_{\text{call } u_{i,j}} \delta x_j = [\underbrace{\tfrac{1}{2}(u_{i,j} + u_{j,i})}_{\text{call } e_{ij}} + \underbrace{\tfrac{1}{2}(u_{i,j} - u_{j,i})}_{\text{call } \eta_{ij}}]\delta x_j$$

summation over the suffix j is implied

This decomposition is standard (see e.g. Batchelor, 1970, § 2.3)

$$\delta u_i = \underbrace{\frac{\partial u_i}{\partial x_j}}_{\text{call } u_{i,j}} \delta x_j = [\underbrace{\tfrac{1}{2}(u_{i,j} + u_{j,i})}_{\text{call } e_{ij}} + \underbrace{\tfrac{1}{2}(u_{i,j} - u_{j,i})}_{\text{call } \eta_{ij}}]\delta x_j$$

It can be shown that e_{ij} **and** η_{ij} **are second order tensors**

e_{ij} **is symmetric** ($e_{ji} = e_{ij}$)

η_{ij} **antisymmetric** ($\eta_{ji} = -\eta_{ij}$).

η_{ij} **has only three non zero components and it can be shown that these form the components of the vorticity vector.**

Consider the case of two-dimensional motion

$$\delta u_1 = e_{11}\delta x_1 + e_{12}\delta x_2 + \eta_{12}\delta x_2$$

$$\delta u_2 = e_{21}\delta x_1 + e_{22}\delta x_2 + \eta_{21}\delta x_1$$

Note: $\eta_{11} = \eta_{22} = 0$

Write $(x, y) = (x_1, x_2)$ **and** $(\delta u, \delta v) = (u_1, u_2)$ **and take the origin of coordinates at the point P =>** $(\delta x_1, \delta x_2) = (x, y)$.

$$\eta_{21} = u_{21} - u_{12} = v_x - u_y = -\eta_{12} = \tfrac{1}{2}\zeta$$

ζ **is the vertical component of vorticity**

$$\begin{bmatrix} \delta u \\ \delta v \end{bmatrix} = \begin{bmatrix} u_x & \frac{1}{2}(v_x + u_y) - \frac{1}{2}\zeta \\ \frac{1}{2}(v_x - u_y) + \frac{1}{2}\zeta & v_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

- In preference to the four derivatives u_x, u_y, v_x, v_y, define the equivalent four combinations of these derivatives:

$D = u_x + v_y$, called the divergence

$E = u_x - v_y$ called the stretching deformation

$F = v_x + u_y$ called the shearing deformation

$\zeta = v_x - u_y$ the vorticity

- E is called the stretching deformation because the velocity components are differentiated in the direction of the component.
- F is called the shearing deformation because each velocity component is differentiated at right angles to its direction.

Obviously, we can solve for u_x, v_y, v_x, v_y as functions of D, E, F, ζ.

Then

$$\begin{bmatrix} \delta u \\ \delta v \end{bmatrix} = \begin{bmatrix} u_x & \frac{1}{2}(v_x + u_y) - \frac{1}{2}\zeta \\ \frac{1}{2}(v_x - u_y) + \frac{1}{2}\zeta & v_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

may be written in matrix form as

$$\begin{pmatrix} \delta u \\ \delta v \end{pmatrix} = \frac{1}{2}\left[\begin{pmatrix} D & 0 \\ 0 & D \end{pmatrix} + \begin{pmatrix} E & F \\ F & -E \end{pmatrix} + \begin{pmatrix} 0 & -\zeta \\ \zeta & 0 \end{pmatrix}\right]\begin{pmatrix} x \\ y \end{pmatrix}$$

or in component form as

$$u = u_0 + \tfrac{1}{2}Dx + \tfrac{1}{2}Ex + \tfrac{1}{2}Fy - \tfrac{1}{2}\zeta y + 0(|x|^2)$$

$$v = v_0 + \tfrac{1}{2}Dy - \tfrac{1}{2}Ey + \tfrac{1}{2}Fx + \tfrac{1}{2}\zeta x + 0(|x^2|)$$

$$u = u_0 + \tfrac{1}{2}Dx + \tfrac{1}{2}Ex + \tfrac{1}{2}Fy - \tfrac{1}{2}\zeta y + 0(|x|^2)$$

$$v = v_0 + \tfrac{1}{2}Dy - \tfrac{1}{2}Ey + \tfrac{1}{2}Fx + \tfrac{1}{2}\zeta x + 0(|x^2|)$$

$\delta u = u - u_o$, $\delta v = v - v_o$, **and** (u_o, v_o) **is the translation velocity at the point P itself (now the origin).**

Choose the frame of reference so that $u_o = v_o = 0 \Rightarrow$ $\delta u = u$, $\delta v = v$.

- **The relative motion near the point P can be decomposed into four basic components as follows:**

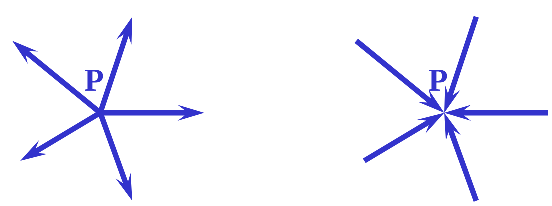
  (I) Pure divergence (only D **nonzero)**
  (II) Pure rotation (only ζ **nonzero)**
  (III) Pure stretching deformation (only E **nonzero)**
  (IV) Pure shearing deformation (only F **nonzero).**

(I) Pure divergence (only D nonzero)

$$u = \tfrac{1}{2}Dx, \quad v = \tfrac{1}{2}Dy$$

$$\mathbf{u} = \tfrac{1}{2}Dr(\cos\theta, \sin\theta) = \tfrac{1}{2}D\mathbf{r}$$

$\mathbf{r}$ **is the position vector from P.**



D > 0 **D < 0**

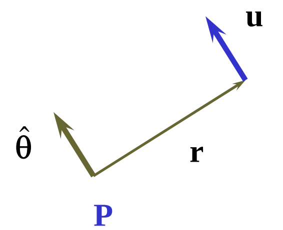
The motion is purely radial and is from or to the point P according to the sign of D**.**

(II) Pure rotation (only ζ nonzero).

$$u = -\tfrac{1}{2}\zeta y, \; v = \tfrac{1}{2}\zeta x$$

$$\mathbf{u} = \tfrac{1}{2}\zeta r(-\sin\theta, \cos\theta) = \tfrac{1}{2}\zeta r\hat{\theta}$$

the unit normal vector to r



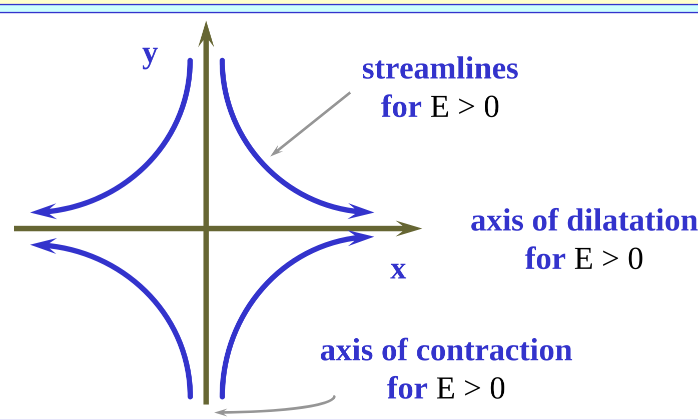
The motion corresponds with solid body rotation with angular velocity $\tfrac{1}{2}\zeta$.

(III) Pure stretching deformation (only E nonzero)

$$u = \tfrac{1}{2}Ex, \qquad v = -\tfrac{1}{2}Ey$$

On a streamline, dy/dx = v/u = –y/x **, or** xdy + ydx = d(xy) = 0**.**

The streamlines are rectangular hyperbolae xy = constant**.**



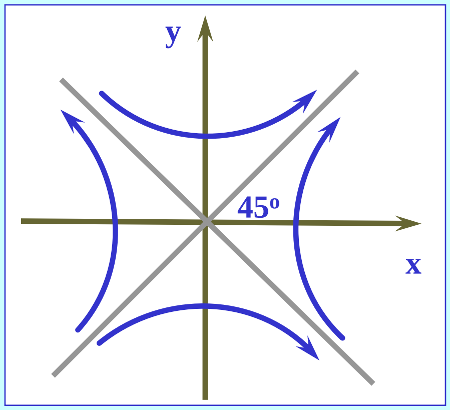
(IV) Pure shearing deformation (only F nonzero)

$$u = \tfrac{1}{2}Fy, \qquad v = \tfrac{1}{2}Fx$$

The streamlines are given now by $dy/dx = x/y$

$\Rightarrow$ $y^2 - x^2 =$ constant.

The streamlines are again rectangular hyperbolae, but with their axes of dilatation and contraction at 45 degrees to the coordinate axes.

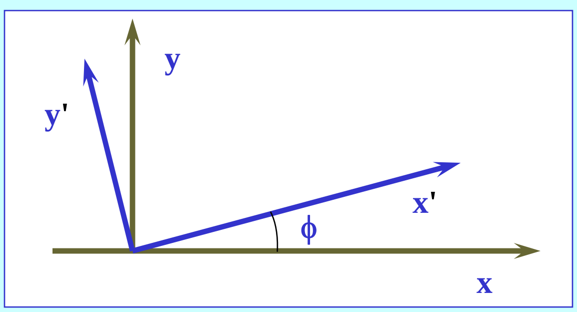


The flow directions are for $F > 0$.

(V) Total deformation (only E and F nonzero)

$$\begin{pmatrix} \delta u \\ \delta v \end{pmatrix} = \frac{1}{2}\begin{bmatrix} E & F \\ F & -E \end{bmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

By rotating the axes (x, y) **to** (x', y') **we can choose** ϕ **so that the two deformation fields together reduce to a single deformation field with the axis of dilatation at angle** ϕ **to the x axis.**



- **Let the components of any vector** (a, b) **in the** (x, y) **coordinates be** (a', b') **in the** (x', y') **coordinates:**

$$\Rightarrow \quad \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} a' \\ b' \end{pmatrix}$$

and

$$\begin{pmatrix} a' \\ b' \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\begin{pmatrix} \delta u \\ \delta v \end{pmatrix} = \tfrac{1}{2} \begin{bmatrix} E & F \\ F & -E \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \quad \Rightarrow \quad \begin{pmatrix} \delta u' \\ \delta v' \end{pmatrix} = \tfrac{1}{2} \begin{pmatrix} E' & F' \\ F' & -E' \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$$

where

$$E' = E\cos 2\phi + F\sin 2\phi$$
$$F' = F\cos 2\phi - E\sin 2\phi$$

$$\begin{pmatrix} \delta u \\ \delta v \end{pmatrix} = \tfrac{1}{2} \begin{bmatrix} E & F \\ F & -E \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \qquad \begin{pmatrix} \delta u' \\ \delta v' \end{pmatrix} = \tfrac{1}{2} \begin{pmatrix} E' & F' \\ F' & -E' \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$$

where

$$E' = E\cos 2\phi + F\sin 2\phi$$
$$F' = F\cos 2\phi - E\sin 2\phi$$

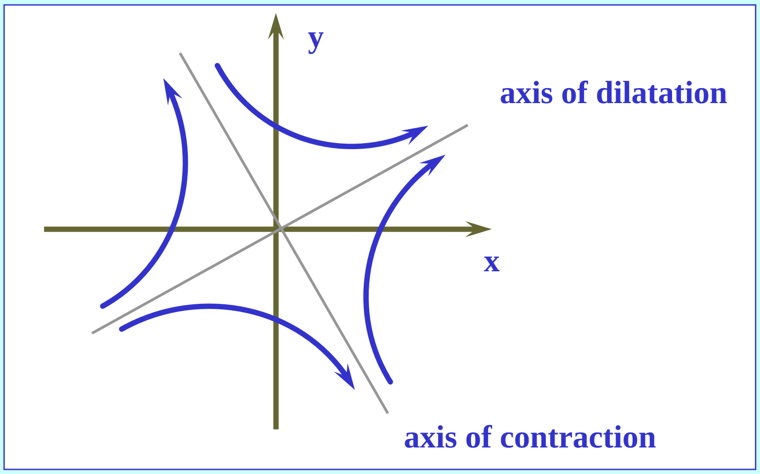
$\Rightarrow$ E **and** F**, and also the total deformation matrices are not invariant under rotation of axes, unlike, for example, the matrices representing divergence and vorticity**

$E'^2 + F'^2 = E^2 + F^2$ **is invariant under rotation of axes.**

We can rotate the coordinate axes in such a way that F' = 0**; then** E' **is the sole deformation in this set of axes.**

$\Rightarrow$ $\tan 2\phi = F/E$ **and** $E' = (E^2 + F^2)^{1/2}$

The stretching and shearing deformation fields may be combined to give a **total deformation field** with strength E' and with **the axis of dilatation inclined at an angle** ϕ to the x- axis.



General two-dimensional motion near a point

- In summary, the general two-dimensional motion in the neighbourhood of a point can be broken up into **a field of divergence**, a field of **solid body rotation**, and **a single field of total deformation**, characterized by its magnitude E' (> 0) and the orientation of the axis of dilatation, ϕ.
- We consider now how these flow field components act to change horizontal temperature gradients.

The frontogenesis function

- One measure of the frontogenetic or frontolytic tendency in a flow is the **frontogenesis function**:

$$\boxed{D|\nabla_h \theta| / Dt}$$

$$D/Dt \equiv \partial/\partial t + u\partial/\partial x + v\partial/\partial y + w\partial/\partial z$$

- Start with the thermodynamic equation

$$\frac{D\theta}{Dt} = q$$

diabatic heat sources and sinks

- Differentiating with respect to x and y in turn

$$\frac{D}{Dt}\left(\frac{\partial\theta}{\partial x}\right) + \frac{\partial u}{\partial x}\frac{\partial\theta}{\partial x} + \frac{\partial v}{\partial x}\frac{\partial\theta}{\partial y} + \frac{\partial w}{\partial x}\frac{\partial\theta}{\partial z} = \frac{\partial\dot{q}}{\partial x}$$

and

$$\frac{D}{Dt}\left(\frac{\partial\theta}{\partial y}\right) + \frac{\partial u}{\partial y}\frac{\partial\theta}{\partial x} + \frac{\partial v}{\partial y}\frac{\partial\theta}{\partial y} + \frac{\partial w}{\partial y}\frac{\partial\theta}{\partial z} = \frac{\partial\dot{q}}{\partial y}$$

Now

$$\frac{D}{Dt}|\nabla_h\theta|^2 = 2\left(\frac{\partial\theta}{\partial x}, \frac{\partial\theta}{\partial y}\right)\cdot\left[\frac{D}{Dt}\left(\frac{\partial\theta}{\partial x}\right), \frac{D}{Dt}\left(\frac{\partial\theta}{\partial y}\right)\right]$$

Use

$$\left.\begin{aligned} u_x &= \tfrac{1}{2}(D+E), \quad & v_x &= \tfrac{1}{2}(F+\zeta), \\ v_y &= \tfrac{1}{2}(D-E), \quad & u_y &= \tfrac{1}{2}(F-\zeta), \end{aligned}\right\}$$

Note that ζ does not appear on the right-hand-side!

$$\begin{aligned} \frac{D}{Dt}|\nabla_h\theta|^2 &= 2\theta_x\dot{q}_x + 2\theta_y\dot{q}_y - 2(w_x\theta_x + w_y\theta_y)\theta_z \\ &\quad - D|\nabla_h\theta|^2 - [E\theta_x^2 + 2F\theta_x\theta_y - E\theta_y^2] \end{aligned}$$

There are four separate effects contributing to frontogenesis (or frontolysis):

$$\frac{D}{Dt}\left|\nabla_h \theta\right| = T_1 + T_2 + T_3 + T_4$$

where

$$T_1 = (\theta_x \dot{q}_x + \theta_y \dot{q}_y) / \left|\nabla_h \theta\right| = \hat{\mathbf{n}} \cdot \nabla_h \dot{q}$$

$$T_2 = -(w_x \theta_x + w_y \theta_y)\theta_z / \left|\nabla_h \theta\right| = -\theta_z \hat{\mathbf{n}} \cdot \nabla_h w$$

$$T_3 = -\tfrac{1}{2} D \left|\nabla_h \theta\right|$$

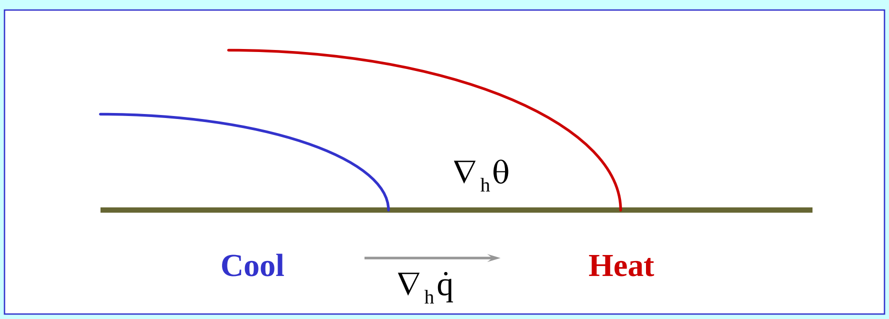
$$T_4 = -\tfrac{1}{2}[E\theta_x^2 + 2F\theta_x\theta_y - E\theta_y^2] / \left|\nabla_h \theta\right|$$

unit vector in the direction of $\left|\nabla_h \theta\right|$

Interpretation

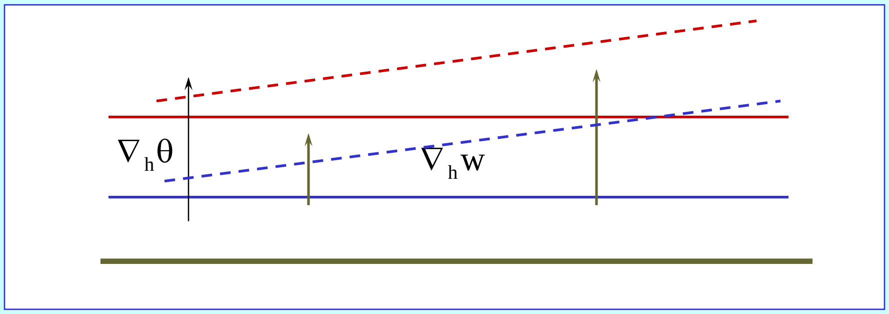
$$T_1 = (\theta_x \dot{q}_x + \theta_y \dot{q}_y) / \left|\nabla_h \theta\right| = \hat{\mathbf{n}} \cdot \nabla_h \dot{q}$$

T_1 : represents the rate of frontogenesis due to a gradient of diabatic heating in the direction of the existing temperature gradient



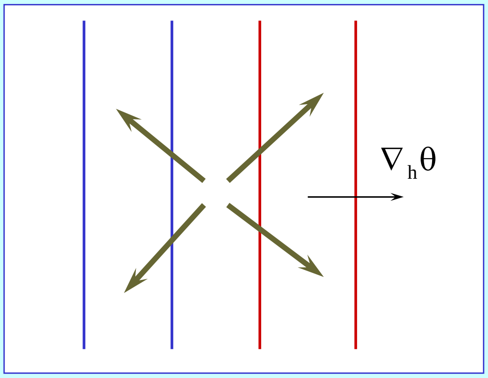
$$T_2 = -(w_x\theta_x + w_y\theta_y)\theta_z / \left|\nabla_h\theta\right| = -\theta_z \hat{\mathbf{n}} \cdot \nabla_h w$$

T_2 : **represents the conversion of vertical temperature gradient to horizontal gradient by a component of differential vertical motion in the direction of the existing temperature gradient**



$$T_3 = -\tfrac{1}{2} D \left|\nabla_h\theta\right|$$

T_3 : **represents the rate of increase of horizontal temperature gradient due to horizontal convergence (i.e., negative divergence) in the presence of an existing gradient**



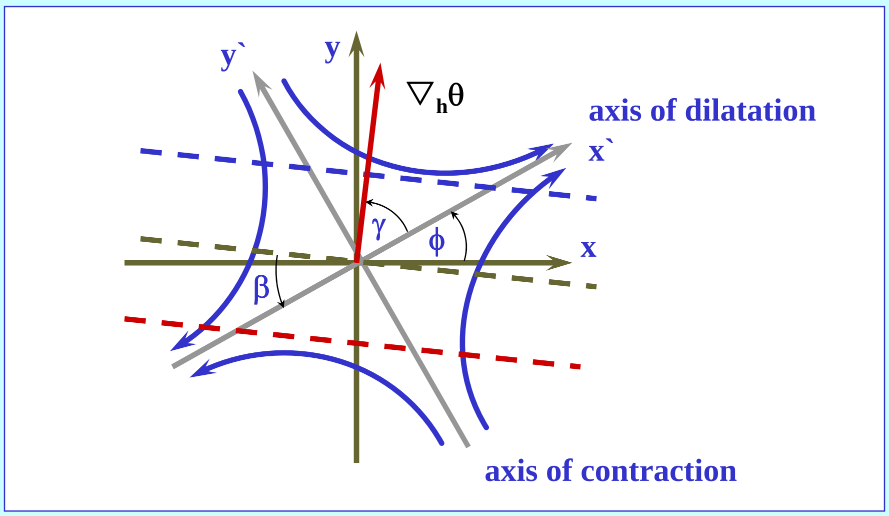
$$T_4 = -\tfrac{1}{2}[E\theta_x^2 + 2F\theta_x\theta_y - E\theta_y^2] / |\nabla_h\theta|$$

T_4 : **represents the frontogenetic effect of a (total) horizontal deformation field.**

- **Further insight into this term may be obtained by a rotation of axes to those of the deformation field.**
- **Let** $\theta'_{x'}$ **denote** $\partial\theta' / \partial x'$ **and relate** $\nabla'_h\theta'$ **to** $\nabla_h\theta$

Solve for E **and** F **in terms of** E' **and** ϕ **(remember** ϕ **is such that** F' = 0**)**

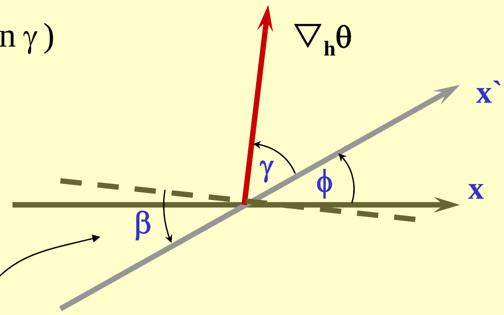
$$T_4 = -\tfrac{1}{2}|\nabla_h\theta|^{-1}[E' \cos\, 2\phi\{(\theta'^2_{x'} - \theta'^2_{y'})\cos\, 2\phi - 2\theta'_{x'}\theta'_{y'} \sin\, 2\phi\}$$
$$+ E' \sin\, 2\phi\{(\theta'^2_{x'} - \theta'^2_{y'})\sin\, 2\phi + 2\theta'_{x'}\theta'_{y'} \cos\, 2\phi\}] .$$



Schematic frontogenetic effect of a horizontal deformation field on a horizontal temperature field.

Set $\nabla'_h\theta = |\nabla_h\theta|(\cos\gamma, \sin\gamma)$

a few lines of algebra



$$T_4 = -\tfrac{1}{2}E'|\nabla_h\theta|\cos 2\gamma$$
$$= \tfrac{1}{2}E'|\nabla_h\theta|\cos 2\beta$$

angle between the axis of dilatation and the potential-temperature isotherms (isentropes)

The frontogenetic effect of deformation is a maximum when the isentropes are parallel with the dilatation axis ($\beta = 0$)**, reducing to zero as the angle** β **between the isentropes and the dilatation axis increases to** 45 **deg.**

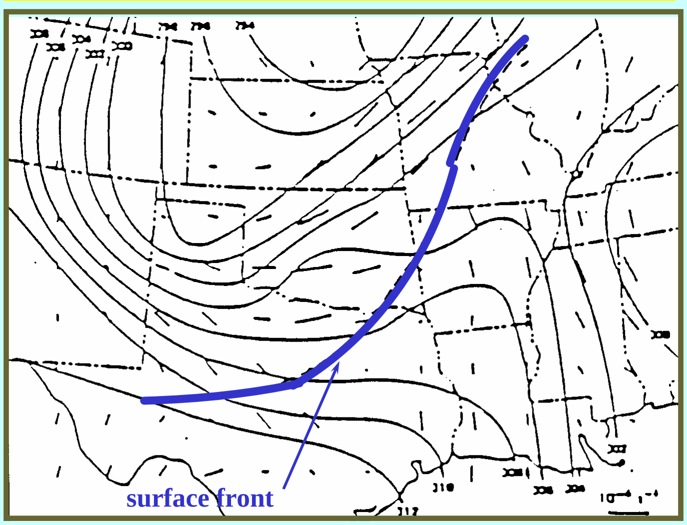
When the angle β **is between** 45 **and** 90 **deg., deformation has a frontolytic effect, i.e.,** $T_4 < 0$.

Observational studies

- **A number of observational studies have tried to determine the relative importance of the contributions** T_n **to the frontogenesis function.**
- **Unfortunately, observational estimates of** T_2 **are "noisy", since estimates for** w **tend to be noisy, let alone for** $\nabla_h w$.
- T_4 **is also extremely difficult to estimate from observational data currently available.**
- **A case study by Ogura and Portis (1982, see their Fig. 25) shows that** T_2, T_3 **and** T_4 **are all important in the immediate vicinity of the front, whereas this and other investigations suggest that horizontal deformation (including horizontal shear) plays a primary role on the synoptic scale.**

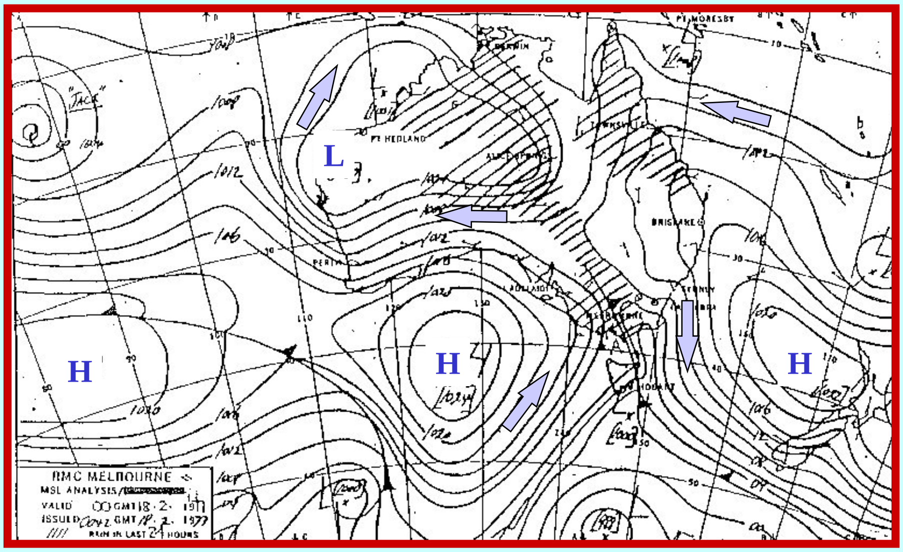
- This importance is illustrated in Fig. 14.7, which is taken from a case study by Ogura and Portis (1982), and in Figs. 4.2 and 4.12, which show a typical summertime synoptic situation in the Australian region.

From a case study by Ogura and Portis (1982)

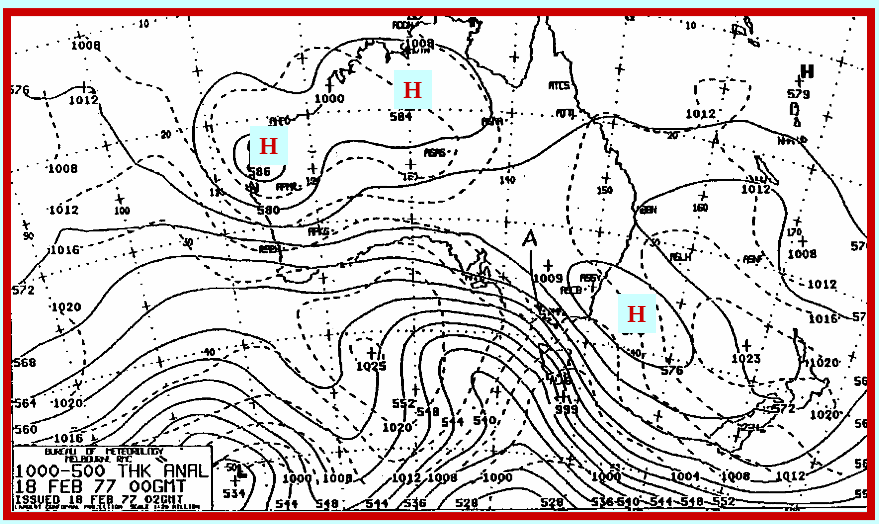


The direction of the dilatation axis and the resultant deformation on the 800 mb surface at 0200 GMT, 26 April 1979 with the contours of the 800 mb potential temperature field at the same time superimposed.

A mean sea level isobaric chart over Australia

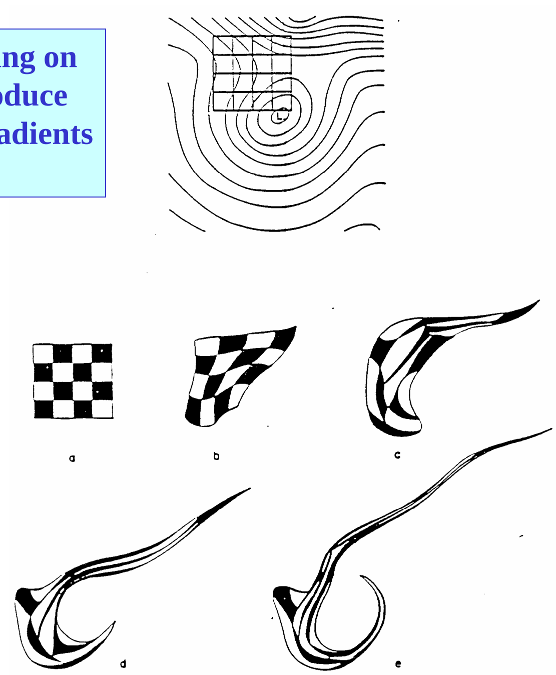


A 1000-500 mb thickness chart over Australia



- In a study of many fronts over the British Isles, Sawyer (1956) found that 'active' fronts are associated with a deformation field which leads to an intensification of the horizontal temperature gradient.
- He found also that the effect is most clearly defined at the 700 mb level at which the rate of contraction of fluid elements in the direction of the temperature gradient usually has a well-defined maximum near the front.

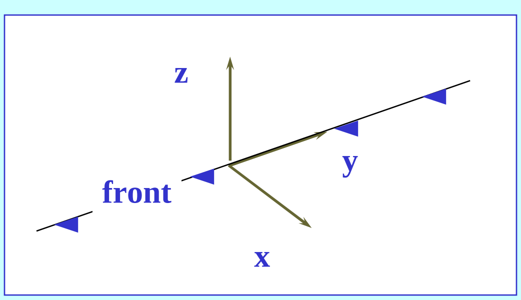
Flow deformation acting on a passive tracer to produce locally large tracer gradients from Welander 1955



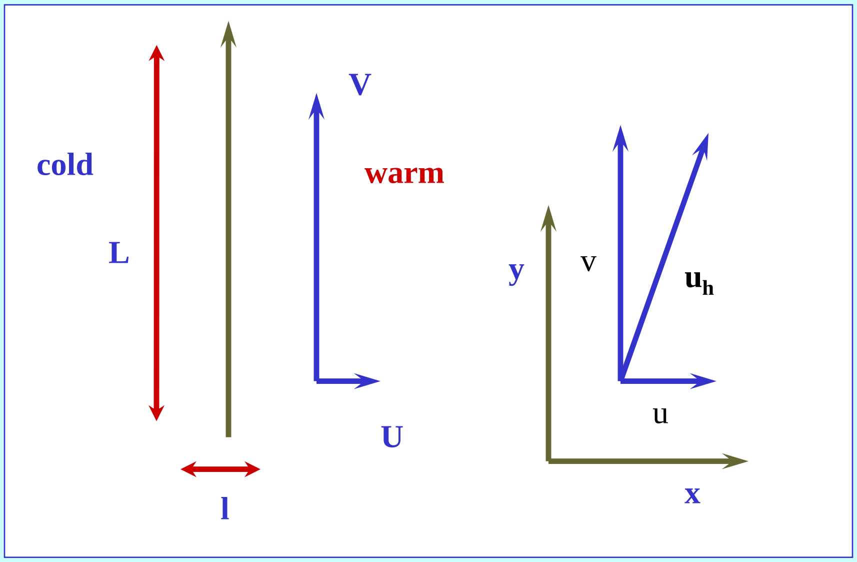
Dynamics of frontogenesis

- The foregoing theory is concerned solely with the kinematics of frontogenesis and shows how particular flow patterns can lead to the intensification of horizontal temperature gradients.
- We consider now the dynamical consequences of increased horizontal temperature gradients
- We know that if the flow is quasi-geostrophic, these increased gradients must be associated with increased vertical shear through the thermal wind equation.
- We show now by scale analysis that the quasi-geostrophic approximation is not wholly valid when frontal gradients become large, but the equations can still be simplified.

- The following theory is based on the review article by Hoskins (1982).
- It is observed, *inter alia*, that atmospheric fronts are marked by large cross-front gradients of velocity and temperature.
- Assume that the curvature of the front is locally unimportant and choose axes with x in the cross-front direction, y in the along-front direction and z upwards:



Frontal scales and coordinates



Observations show that typically,

U ~ 2 ms $^{-1}$

V ~ 20 ms $^{-1}$

L 1000 km

ℓ ~ 200 km

=> V >> U **and** L >> ℓ**.**

The Rossby number for the front, defined as

$$Ro = V / f\ell \sim 20 \div (10^{-4} \times 2 \times 10^{5})$$

is typically of order unity.

The relative vorticity (~V/ℓ) is comparable with f **and the motion is not quasi-geostrophic.**

The ratio of inertial to Coriolis accelerations in the x and y directions =>

$$\frac{Du}{Dt} / fv \sim \frac{U^2/\ell}{fV} = \left(\frac{U}{V}\right)^2 \frac{V}{f\ell} << 1$$

and

$$\frac{Dv}{Dt} / fu \sim \frac{UV/\ell}{fU} = \frac{V}{f\ell} \sim 1$$

The motion is quasi -geostrophic across the front, but not along it.

A more detailed scale analysis is presented by Hoskins and Bretherton (1972, p15), starting with the equations in orthogonal curvilinear coordinates orientated along and normal to the surface front.

- The scale analysis, the result of Exercise (14.3), and making the Boussinesq approximation, the equations of motion for a front are

$$-fv = -\partial_x P$$

$$\frac{Dv}{Dt} + fu = -\partial_y P$$

$$0 = -\partial_z P + b$$

$$\frac{D\sigma}{Dt} + N_0^2 w = 0$$

$$\partial_x u + \partial_y v + \partial_z w = 0$$

$$P = p / \rho_*$$

buoyancy force per unit mass

$$N_0^2 = (g/\theta_0)(d\theta_0/dz)$$

N_0 = the Brunt-Väisälä frequency of the basic state

- I assume that f and N_0 are constants.

Quasi-geostrophic frontogenesis

- While the scale analysis shows that **frontal motions are not quasi-geostrophic** overall, much insight into frontal dynamics may be acquired from a study of frontogenesis within quasi-geostrophic theory.
- Such a study provides also a framework in which later modifications, relaxing the quasi-geostrophic assumption, may be better appreciated.

The quasi-geostrophic approximation involves replacing D/Dt by

$$\frac{D_g}{Dt} \equiv \frac{\partial}{\partial t} + u_g \frac{\partial}{\partial x} + v_g \frac{\partial}{\partial y}$$

where $v_g = v$ is computed from $fv = \partial_x P$ as it stands and

$$u_g = -(1/f)\partial_y P$$

Set $u = u_g + u_a$

$$\frac{Dv}{Dt} + fu_a = 0$$

and

$$\partial_x u_a + \partial_z w = 0$$

$$-fv = -\partial_x P \quad \Longrightarrow \quad -fv = -\partial_x P$$

$$\frac{Dv}{Dt} + fu = -\partial_y P \quad \Longrightarrow \quad \frac{D_g v}{Dt} + f u_a = 0$$

$$0 = -\partial_z P + b \quad \Longrightarrow \quad 0 = -\partial_z P + b$$

$$\frac{Db}{Dt} + N_0^2 w = 0 \quad \Longrightarrow \quad \frac{D_g b}{Dt} + N_0^2 w = 0$$

$$\partial_x u + \partial_y v + \partial_z w = 0 \quad \Longrightarrow \quad \begin{cases} \partial_x u_g + \partial_y v_g = 0 \\ \partial_x u_a + \partial_z w = 0 \end{cases}$$

$$\left.\begin{aligned} -fv &= -\partial_x P \\ 0 &= -\partial_z P + b \end{aligned}\right\} \quad \Longrightarrow \quad f v_z = b_x$$

Let us consider the maintenance of cross-front thermal wind balance expressed by $f v_z = b_x$ **.**

$$\frac{D_g}{Dt}(f v_z) = -Q_1 - f^2 u_{az}$$

Note that $u_{gx} + v_y = 0$

$$Q_1 = u_{gx} b_x - v_x b_y = -\frac{\partial(v,b)}{\partial(x,y)}$$

$$\frac{D_g}{Dt} b_x = Q_1 - N_0^2 w_x$$

These equations describe how the geostrophic velocity field acting through Q_1 **attempts to destroy thermal wind balance by changing** $f v_z$ **and** b_x **by equal and opposite amounts and how ageostrophic motions** (u_a, w) **come to the rescue!**

$$N_0^2 w_x - f^2 u_{az} = 2Q_1$$

Also from $u_{ax} + w_z = 0$, there exists a streamfunction ψ for the cross-frontal circulation satisfying

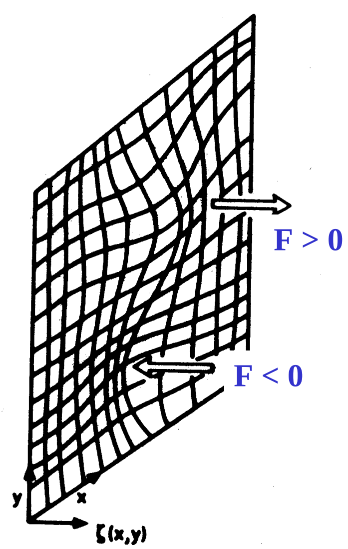
$$(u_a, w) = (\psi_z, -\psi_x)$$

$$\Rightarrow \quad \boxed{N_0^2 \psi_{xx} + f^2 \psi_{zz} = -2Q_1}$$

This is a Poisson-type elliptic partial differential equation for the cross-frontal circulation, a circulation which is forced by Q_1.

$$Q_1 = u_{gx} b_x - v_x b_y$$

Membrane analogy for solving a Poisson Equation

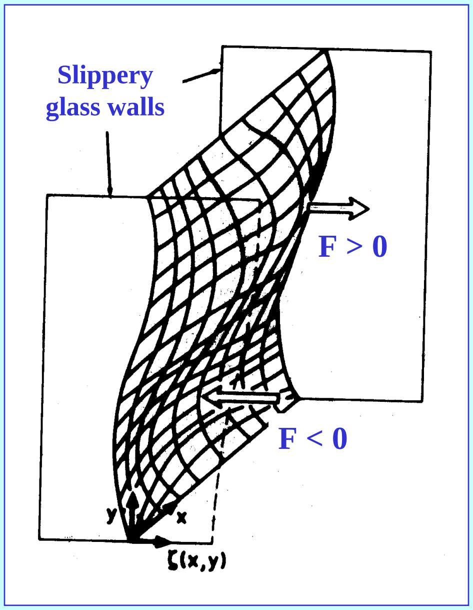


$$\frac{\partial^2 \zeta}{\partial x^2} + \frac{\partial^2 \zeta}{\partial x^2} = -F(x, y)$$

This is an Elliptic PDE

Here z = 0 on the domain boundary

This is called a Dirichlet condition



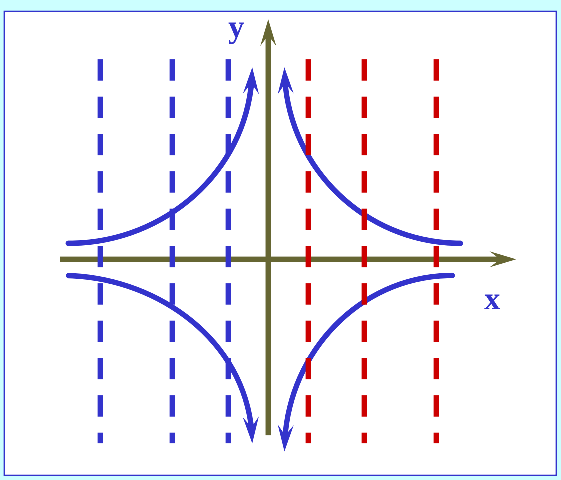
$$\frac{\partial^2\zeta}{\partial x^2} + \frac{\partial^2\zeta}{\partial x^2} = -F(x,y)$$

Here $\zeta = 0$ on parts of the domain boundary and $\partial\zeta/\partial n = 0$ on other parts of the boundary

$$\frac{\partial\zeta}{\partial n} = 0$$

prescribed on a boundary is called a Neumann condition.

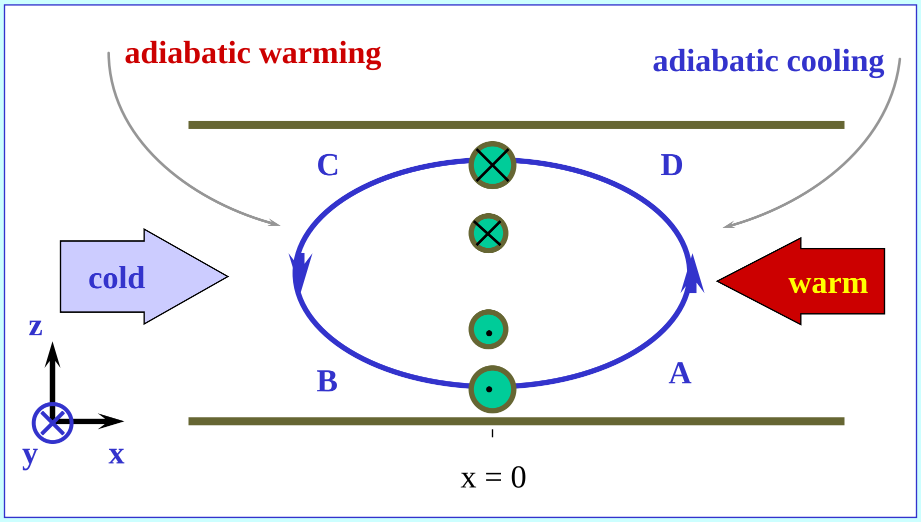
Frontogenesis in a deformation field



$$u_g = -\alpha x$$
$$v = \alpha y$$

$$Q_1 = u_{gx} b_x - \cancel{v_x b_y} = -\frac{\partial(v,b)}{\partial(x,y)} = -\alpha b_x$$

Frontogenesis in a field of geostrophic confluence



(northern hemisphere case)

$$Q_1 = u_{gx} b_x - v_x b_y = -\frac{\partial(v, b)}{\partial(x, y)}$$

- **If** w = 0, Q_1 **is simply the rate at which the buoyancy (or temperature) gradient increases in the cross-front direction following a fluid parcel, due to advective rearrangement of the buoyancy field by the horizontal motion.**

$$\frac{D_g}{Dt} b_x = Q_1 - N_0^2 w_x$$

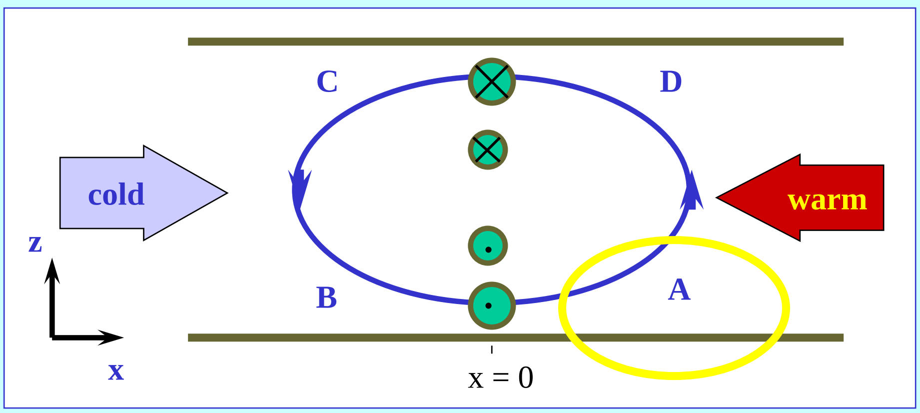
$$Q_1 = u_{gx} b_x - v_x b_y$$

- b_x **increases due to confluence (**$u_x < 0$**) acting on this component of buoyancy gradient and due to along-front horizontal shear** v_x **acting on any along-front buoyancy gradient** b_y**.**
- Db_x/Dt **is an alternative measure of frontogenesis to the Boussinesq form of the frontogenesis function** $D|\nabla_h b|/Dt$ **analogous to the left hand side of this, i.e.,** $T_1 + T_2 + T_3 + T_4$**.**

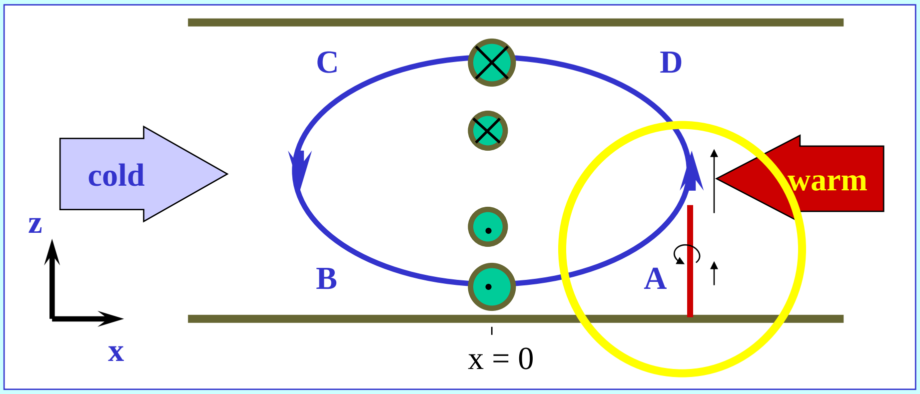
- **The quasi-geostrophic theory of frontogenesis in a field of pure geostrophic deformation was developed by Stone (1966), Williams and Plotkin (1968), and Williams (1968).**
- **The solutions obtained demonstrate the formation of large horizontal gradients near boundaries, but away from boundaries, the induced ageostrophic circulation prevents the contraction of the horizontal length scale of the temperature field below the Rossby length,** $L_R = N_o H/f$**; where** H **is the depth of the fluid.**
- **Because the ageostrophic circulation does not contribute to advection in quasi-geostrophic theory, the largest horizontal temperature gradient at each height remains coincident with the line of horizontal convergence (**$x = 0$**).**

Limitations of quasi-geostrophic theory

- Many unrealistic features of the quasi-geostrophic theory result from the omission of certain feedback mechanisms.
- The qualitative effect of some of these feedbacks can be deduced from the quasi-geostrophic results.

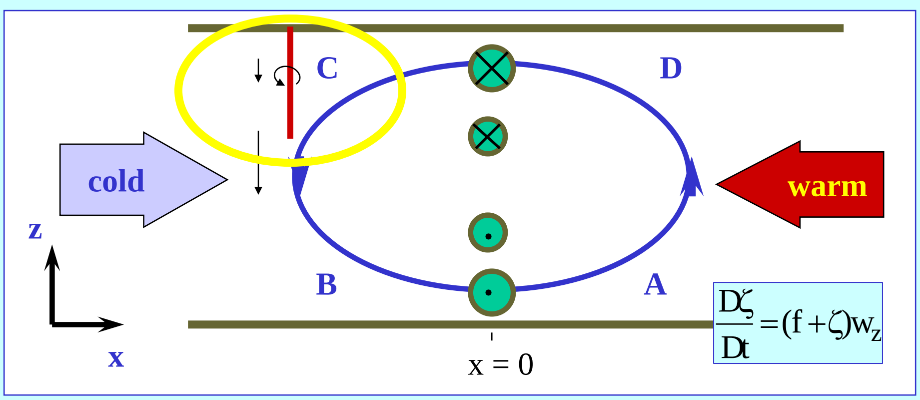


- **The ageostrophic velocity u_a is clearly convergent ($u_{ax} < 0$) in the vicinity of A on the warm side of the maximum T_x (b_x).**
- **If included in the advection of b it would lead to a larger gradient b_x.**



- At A, the generation of cyclonic relative vorticity ζ is underestimated because of the exclusion of the stretching term ζ_{wz} in the vertical vorticity equation,

$$\frac{D\zeta}{Dt} = (f + \zeta)w_z$$



- Similar arguments apply to the neighbourhood of **C** on the cold side of the maximum temperature gradient at upper levels.
- In the vicinity of **B** and **D**, the ageostrophic divergence would imply weaker gradients in ζ and the neglect of ζ_{wz} would imply smaller negative vorticity.

- In summary, QG-theory points to the formation of **sharp surface fronts** with cyclonic vorticity on the warm side of the temperature contrast, and with the maximum horizontal temperature gradient sloping in the vertical from **A** to **C**, **even though** these effects are excluded in the QG-solutions.
- The theory highlights the role of horizontal boundaries in frontogenesis and shows that the ageostrophic circulation acts to inhibit the formation of large gradients in the free atmosphere.
- **Hoskins** (1982) pointed out that unless the ageostrophic convergence at **A** and **C** increase as the local gradients increase, the vorticity and the gradients in b can only increase exponentially with time.
- Quasi-geostrophic theory does not even suggest the formation of frontal discontinuities in a finite time.

Semi-geostrophic frontogenesis

Next file