Boundary Layer Meteorology

Chapter 06

Turbulence closure techniques

- The large number of equations we have developed would suggest that we have a fairly complete description of turbulent flow.
- A closer examination shows that there is a large number of unknowns. An unknown is a quantity for which we have no prognostic equation.
- In fact the number of unknowns in the set of equations for turbulent flow always exceeds the number of equations so that the problem is not **closed**.
- To make further progress we have to parameterize these unknowns.
- This is called the **turbulence closure problem**.

The nature of the closure problem

- Consider the prognostic equation for the mean potential temperature:

$$\frac{\partial\bar{\theta}}{\partial t}+\bar{u}_j\frac{\partial\bar{\theta}}{\partial x_j}=-\frac{1}{\bar{\rho}c_p}\frac{\partial\bar{Q}_j^*}{\partial x_i}+\nu\frac{\partial^2\bar{\theta}}{\partial x_j^2}-\frac{LE}{\bar{\rho}c_p}-\frac{\partial(\overline{u_j'\theta'})}{\partial x_j}$$

Called a **double correlation**, or a **second statistical moment**.

- To eliminate this as an unknown we derive a forecast equation for it. Unfortunately this equation contains **triple correlation** (or **third moment**) terms such as $\overline{\theta' u_i' u_j'}$.

- The prognostic equation for this third moment contains **fourth moment terms** and so on …

- The matter is actually worse, because $\overline{\theta' u_i' u_j'}$ really represents 9 terms, one for each value of i and j. Of these, 6 remain because of symmetries in the tensor matrix, e.g.

$$\overline{\theta' u_1' u_2'}=\overline{\theta' u_2' u_1'}$$

- Similar problems occur for the turbulence equations for momentum.

Next slide

- A tally of equations and unknowns for various **statistical moments of momentum** demonstrating the closure problem for turbulent flow.

Prognostic eqn. for	Moment	Equation	No of eqns	No of unknowns
$\overline{u}_i$	First	$\frac{\partial \overline{u}_i}{\partial t} = \ldots - \frac{\partial}{\partial x_j} \overline{u_i' u_j'}$	3	6
$\overline{u_i' u_j'}$	Second	$\frac{\partial}{\partial t} \overline{u_i' u_j'} = \ldots - \frac{\partial}{\partial x_k} \overline{u_i' u_j' u_k'}$	6	10
$\overline{u_i' u_j' u_k'}$	Third	$\frac{\partial}{\partial t} \overline{u_i' u_j' u_k'} = \ldots - \frac{\partial}{\partial x_m} \overline{u_i' u_j' u_k' u_m'}$	10	15

- The full set of equations includes even more unknowns.

- There is an easy way to anticipate which unknowns remain at any level of closure after symmetries are considered, as is shown in the following table for momentum correlations.
- In the full equations of motion there are additional unknowns such as pressure correlations and terms involving viscosity.

- Correlation triangles indicating the unknowns for various levels of turbulence closure, for the momentum equations only. Notice the pattern in these triangles, with the u, v, and w statistics at their respective vertices, and the cross correlations in between.

Order of closure	Correlation triangle of unknowns
Zero	$\overline{u}$ $\overline{v}$ $\overline{w}$
First	$\overline{u'^2}$ $\overline{u'v'}$ $\overline{u'w'}$ $\overline{v'^2}$ $\overline{v'w'}$ $\overline{w'^2}$

Order of closure	Correlation triangle of unknowns
Second	$\overline{u'^3}$ $\overline{u'^2v'}$ $\overline{u'^2w'}$ $\overline{u'v'^2}$ $\overline{u'v'w'}$ $\overline{u'w'^2}$ $\overline{v'^3}$ $\overline{v'^2w'}$ $\overline{v'w'^2}$ $\overline{w'^3}$

- To make make mathematical/statistical description of turbulence tractable, one approach is to use only a finite number of equations, and then approximate the remaining unknowns in terms of known quantities.
- Such **closure approximations** or closure assumptions are named by the highest order prognostic equations that are retained.

Next slide

- Using the equations in the table as an example, for **first-order closure** the first equation is retained and the second moments are approximated.
- Similarly, **second-order closure** retains the first two equations, and approximates involving third moments.

Prognostic eqn. for	Moment	Equation	No of eqns	No of unknowns
$\overline{u}_i$	First	$\frac{\partial \overline{u}_i}{\partial t} = \ldots - \frac{\partial}{\partial x_j}\overline{u'_i u'_j}$	3	6
$\overline{u'_i u'_j}$	Second	$\frac{\partial}{\partial t}\overline{u'_i u'_j} = \ldots - \frac{\partial}{\partial x_k}\overline{u'_i u'_j u'_k}$	6	10
$\overline{u'_i u'_j u'_k}$	Third	$\frac{\partial}{\partial t}\overline{u'_i u'_j u'_k} = \ldots - \frac{\partial}{\partial x_m}\overline{u'_i u'_j u'_k u'_m}$	10	15

- Some closure assumptions utilize only a portion of the equations available within a particular moment category.
- For example, if equations for the turbulence kinetic energy and temperature and moisture variance are used along with the first-moment equations of Table 6-1, the result can be classified as **one-and-a-half order closure**.
- It clearly would not be full second-order closure because not all of the prognostic equations for the second moments (i.e. for the fluxes) are retained, yet it is higher order than first-order closure.
- One can similarly define **zero-order closure** and **half-order closure** methods.

Local and nonlocal closure

- Two major schools of thought of turbulence have appeared in the literature: **local** and **nonlocal** closure.
- Neither local nor nonlocal methods are exact, but both appear to work well for the physical situations for which the parameterizations are designed.
- For **local closure**, an an unknown quantity at any point in space is parameterized by values and/or gradients of known quantities at the **same** point.
- Local closure thus assumes that turbulence is analogous to molecular diffusion.
- The Donaldson example in the next section demonstrates a local second-order closure. In the literature, local closure has been used at all orders up through third order.

Local and nonlocal closure

- For **nonlocal closure**, the unknown quantity at one point is parameterized by values of known quantities at **many points** in space.
- This assumes that turbulence is a superposition of eddies, each of which transports fluid like an advection process.
- Nonlocal methods have been used mostly with first-order closure.
- The **next table** summarizes the myriad of closure methods which have often appeared in the meteorological literature.
- Generally, the higher-order local closures and the nonlocal closures yield more accurate solutions than lower order, but they do so at added expense and complexity.

- Classification of closure techniques that have been frequently reported in the literature. Bulk and similarly methods are discussed later.

Order of closure	Local	Nonlocal	Other (bulk and similarity methods)
Zero			X
Half	X	X	X
First	X	X	
One-and-a-half	X		
Second	X		
Third	X		

Parameterization rules

- **Regardless of which order closure is used, there are unknown turbulence terms which must be parameterized as a function of known quantities and parameters.**
- **A known quantity is any quantity for which a prognostic or diagnostic equation is retained.**

Next slide

- **For example, if we decide to use second-order closure, the unknown quantity $\overline{u_i'u_j'u_k'}$ can be parameterized as a function of $\overline{u}_i$ and $\overline{u_i'u_j'}$ because we have prognostic equations for these quantities.**

Prognostic eqn. for	Moment	Equation	No of eqns	No of unknowns
$\overline{u}_i$	First	$\frac{\partial \overline{u}_i}{\partial t} = \ldots - \frac{\partial}{\partial x_j}\overline{u_i'u_j'}$	3	6
$\overline{u_i'u_j'}$	Second	$\frac{\partial}{\partial t}\overline{u_i'u_j'} = \ldots - \frac{\partial}{\partial x_k}\overline{u_i'u_j'u_k'}$	6	10
$\overline{u_i'u_j'u_k'}$	Third	$\frac{\partial}{\partial t}\overline{u_i'u_j'u_k'} = \ldots - \frac{\partial}{\partial x_m}\overline{u_i'u_j'u_k'u_m'}$	10	15

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- A **parameter** is usually a constant, the value of which is determined empirically. For example, the parameter can be a separate term, a multiplicative constant, or the value of a power or exponent.
- By definition, a **parameterization** is an approximation to nature. In other words, we are replacing the true (natural) equation describing a value with some artificially constructed approximation.
- Sometimes parameterizations are employed because the true physics has yet to be discovered.
- Sometimes the known physics are too complicated to use for particular application, given cost or computer limitations.
- Parameterization will rarely be perfect - the hope is that it will be adequate.

- Parameterization involves human interpretation and creativity, which means that different investigators can propose different parameterizations for the same unknown.
- In fact, **Donaldson** (1973) noted that "there are more models for closure of the equations of motion at the second-order correlation level than there are principal investigators working on the problem".
- Although there is likely to be an infinite set of possible parameterizations for any quantity, all acceptable parameterizations must follow certain common-sense rules.
- Most importantly, the parameterization for an unknown quantity should be physically reasonable.

- In addition, the parameterization must:
  - have the same dimensions as the unknown,
  - have the same tensor properties,
  - have the same symmetries,
  - be invariant under an arbitrary transformation of coordinate systems,
  - be invariant under a Galilean (i.e. inertial or Newtonian transformation,
  - satisfy the same budget equations and constraints.
- These rules apply to all orders of closure.

- As an example, Donaldson (1973) has proposed that the unknown $\overline{u_i' u_j' u_k'}$ be parameterized by:

$$-\Lambda \bar{e}^{1/2} \left[\frac{\partial}{\partial x_i} \overline{u_j' u_k'} + \frac{\partial}{\partial x_j} \overline{u_i' u_k'} + \frac{\partial}{\partial x_k} \overline{u_i' u_j'} \right]$$

where Λ is a parameter having the dimension of length (m), and the knowns are $\bar{e}$ (turbulent kinetic energy per unit mass, $m^2\ s^{-2}$) and $\overline{u_i' u_j'}$ (momentum flux, $m^2\ s^{-2}$).

See Stull, p202 for discussion

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- I review now some of the parameterizations that have been presented in the literature.
- The review is by no means comprehensive - it is meant only to demonstrate the various types of closure and their features.
- Regardless of the type of parameterization used, the result closes the equations of motion for turbulent flow and allows them to be solved for various forecasting, diagnostic, and other practical applications.

Local closure

- Local Closure: Zero and Half Order
- Zero-order closure implies that no prognostic equations are retained, not even the equations for the mean quantities.
- In other words, the mean wind, temperature, humidity, and other mean quantities are parameterized directly as a function of space and time.
- Obviously, this is neither local or nonlocal closure because it avoids the parameterization of turbulence altogether.
- For this reason, I will not dwell on zero-order closure here, but will return to it later under the topic of similarity theory.

Half-order closure

- **Half-order closure uses a subset of the first moment equations.**
- **A variation of this approach is called the bulk method: in this a profile shape for wind or temperature is assumed, but the resulting wind or temperature curve can be shifted depending on the bulk-average background wind or temperature within the whole layer.**
- **For example, a BL (bulk) average** $<\overline{\theta}(t)>$ **is forecast using equations like:**

$$\frac{\partial \overline{\theta}}{\partial t} + \overline{u}_j \frac{\partial \overline{\theta}}{\partial x_j} = -\frac{1}{\overline{\rho} c_p} \frac{\partial \overline{Q}_j^*}{\partial x_i} + \nu \frac{\partial^2 \overline{\theta}}{\partial x_j^2} - \frac{LE}{\overline{\rho} c_p} - \frac{\partial (\overline{u_j' \theta'})}{\partial x_j}$$

- **Then, a profile shape** $\Delta\theta(z)$ **is assumed, and the final values of** $\overline{\theta}(z,t)$ **are found from:** $\overline{\theta}(z,t) = \langle \overline{\theta}(t) \rangle + \Delta\overline{\theta}(z)$ **.**

Half-order closure

- **Such schemes are used for:**
  1) **bulk or slab mixed layer models with** $\Delta\overline{\theta}(z) = 0$ **at all heights;**
  2) **For cloud models with** $\Delta\overline{\theta}(z)$ **modelled as linear functions of height within separate cloud and subcloud layers; and**
  3) **For stable boundary layers wi~~t~~h** $\Delta\overline{\theta}(z)$ **approximated with either linear, polynomial, or exponential profile shapes.**

First-order closure

- **First-order closure** retains the prognostic equations for only the zero-order mean variables such as wind, temperature, and humidity.
- Consider the idealized scenario of a dry environment, horizontally homogeneous, with no subsidence.
- The geostrophic wind is assumed to be known.

First-order closure

- The governing prognostic equations for the zero-order variables then reduce to

$$\frac{\partial \bar{u}}{\partial t} = f(\bar{v} - \bar{v}_g) - \frac{\partial}{\partial z}\overline{u'w'}$$

$$\frac{\partial \bar{v}}{\partial t} = -f(\bar{u} - \bar{u}_g) - \frac{\partial}{\partial z}\overline{v'w'}$$

$$\frac{\partial \bar{\theta}}{\partial t} = \ldots - \frac{\partial}{\partial z}\overline{w'\theta'}$$

- The unknowns in this set of equations are the second moments: $\overline{u'w'}$ $\overline{v'w'}$ $\overline{w'\theta'}$

- If we let ξ be any variable, then one possible first-order closure approximation for flux $\overline{u_j'\xi'}$ is :

$$\overline{u_j'\xi'} = -K_\xi \frac{\partial \overline{\xi}}{\partial z}$$

- where the parameter K_ξ is a scalar with $m^2 s^{-1}$.
- For positive K_ξ, the above expression implies that the flux $\overline{u_j'\xi'}$ flows down the local gradient of $\overline{\xi}$.
- This closure approximation is often called **gradient transport theory** or **K-theory**.
- Although it is one of the simplest parameterizations, it frequently fails when large-size eddies are present in the flow.
- Hence, we can classify it as a **small-eddy closure** technique.

Local closure

- K is known by a variety of names:
  - eddy viscosity
  - eddy diffusivity
  - eddy-transfer coefficient
  - turbulent-transfer coefficient
  - gradient-transfer coefficient
- the latter because it relates the turbulent flux to the gradient of the associated mean variable.
- Sometimes, different K values are associated with different variables. A subscript "M" is used for momentum, resulting in K_M as the eddy viscosity.

Local closure

- **For heat and moisture, we will use K_H and K_E for the respective eddy diffusivities.**
- **There is some experimental evidence to suggest that for statically neutral conditions:**

$$K_H = K_E = 1.35K_M$$

- **It is not clear why K_M should be smaller than other K values.**
- **Perhaps pressure-correlation effects contaminated the measurements upon which the expression is based.**

Example 1

- **Given $K_H = 5\ \mathbf{m^2\,s^{-1}}$ for turbulence within a background stable environment, with lapse rate $\partial\bar{\theta}/\partial z = 0.01$ K/m. Find $\overline{w'\theta'}$.**
- **Solution**

Use $\overline{u_j'\xi'} = -K_\xi \dfrac{\partial\bar{\xi}}{\partial x_j}$ **Put** $\bar{\xi} = \bar{\theta}$ $j = 3$

$$\overline{w'\theta'} = -K_H \frac{\partial\bar{\theta}}{\partial z} = -5\ \mathrm{m^2\,s^{-1}} \times 0.01\ \mathrm{K\,m^{-1}} = -0.05\ \mathrm{K\,m\,s^{-1}}$$

Discussion

- **Normally a negative heat flux would be expected in a stably-stratified environment, assuming only small eddies were present: i.e. in an environment with warm air above colder air, turbulence moves warm air down the gradient to cooler air, which in this case is a downward (or negative) heat flux.**

Example 2

- Suggest a parameterization set to close the Ekman equations.
- **Solution**

$$\overline{u'w'} = -K_M \frac{\partial \overline{u}}{\partial z}$$

$$\overline{v'w'} = -K_M \frac{\partial \overline{v}}{\partial z}$$

$$\overline{w'\theta'} = -K_\theta \frac{\partial \overline{\theta}}{\partial z}$$

- **Discussion**: If these equations are inserted into the Ekman equations, there are three equations for three unknowns $\overline{\theta}$, $\overline{u}$, and $\overline{v}$. This is a closed set which can be solved numerically if K values are known.

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Example 3

- Given $K_H = 5\ m^2\ s^{-1}$ for turbulence within a background horizontally-homogeneous environment, find $\overline{u'\theta'}$.
- **Solution**

$$\overline{u'\theta'} = -K_H \frac{\partial \overline{\theta}}{\partial x}$$

Horizontally homogeneous $\Rightarrow \quad \frac{\partial \overline{\theta}}{\partial x} = 0 \quad \Rightarrow \quad \overline{u'\theta'} = 0$

- **Discussion**: it makes no difference whether K_H is positive, negative or exceptionally large - K-theory will always yield zero flux in a uniform environment, regardless of the true flux.

Analogy with viscosity

- For a Newtonian fluid, the molecular stress τ_{mol} can be approximated by:

$$\tau_{mol} = \rho\nu \frac{\partial \overline{u}}{\partial z}$$

- By analogy, one might expect that the turbulent Reynolds stress can be expressed in terms of the mean shear, with ν replaced with an eddy viscosity K_M, i.e.

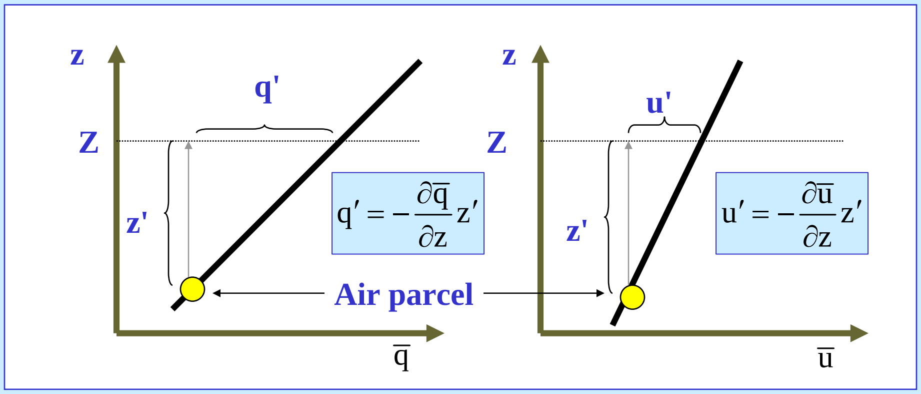
$$\tau_{Reynolds} = \rho K_M \frac{\partial \overline{u}}{\partial z}$$

- Dividing by ρ gives the usual kinematic form.
- ρK_M is sometimes called the **Austausch coefficient**.

- Since turbulence is much more effective than viscosity at causing mixing, one would expect $K_m > \nu$.
- Values of K_m in the literature vary from 0.1 $m^2 s^{-1}$ to 2000 $m^2 s^{-1}$, with typical values $\approx$ 1 to 10 $m^2 s^{-1}$.
- Values of ν are much smaller, $\approx 1.5 \times 10^{-5}$ $m^2 s^{-1}$.
- Magnitude is not the only difference between the molecular and eddy viscosities: a significant difference is that ν **is a function of the fluid**, while K_m **is a function of the flow.**
- Thus, while ν is uniquely determined by the chemical composition of the fluid and its state (temperature and pressure, etc.), K_m varies as the turbulence varies.
- Thus, on must parameterize K_m as a function of other variables such as z/L, Richardson number or the stability $\partial\theta_v / \partial z$.

Mixing length theory

- Assume that there is turbulence in a statically neutral environment, with a linear mean humidity gradient in the vertical.



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- In order for the parcel to move upward a distance z', it must have some vertical velocity w'.
- If the nature of turbulence is such that w' is proportional to u', then we might expect w' = –cu' for the linear wind shear sketched in the previous slide (i.e. for $\partial u/\partial z > 0$), and w' = cu' for $\partial u/\partial z < 0$, where c is some constant of proportionality.

$$u' = -\frac{\partial \bar{u}}{\partial z} z' \quad \Rightarrow \quad w' = c\left|\frac{\partial \bar{u}}{\partial z}\right| z' \qquad \textbf{6.4.4c}$$

It follows that the magnitude of the shear is important.

- **The kinematic eddy flux of moisture is R**
- **We know that** $q' = -\frac{\partial \bar{q}}{\partial z} z'$ **and** $w' = c\left|\frac{\partial \bar{u}}{\partial z}\right| z'$**.**
- **Multiply these expressions and average over the spectrum of different eddy sizes z' to obtain the average flux R:**

$$R = -c\,\overline{(z')^2}\left|\frac{\partial \bar{u}}{\partial z}\right|\frac{\partial \bar{q}}{\partial z}$$

$\overline{(z')^2}$ **is the variance of the parcel displacement distance.**

$\sqrt{\overline{(z')^2}}$ **is a measure of the average distance a parcel moves in the mixing process that generated the flux R.**

- **In this way we can define a mixing length, *l*, by** $l^2 = c\overline{(z')^2}$

$$\Rightarrow \quad R = -l^2\left|\frac{\partial \bar{u}}{\partial z}\right|\frac{\partial \bar{q}}{\partial z}$$

- **This is directly analogous to K-theory if**

$$K_E = l^2\left|\frac{\partial \bar{u}}{\partial z}\right| \qquad \textbf{Then} \qquad R = -K_E\frac{\partial \bar{q}}{\partial z}$$

Suggests that $|K_M|$ should increase as the shear increases (i.e. as a measure of the turbulence intensity) and as the mixing length increases (i.e. as a measure of the ability of turbulence to cause mixing).

- **In the surface layer, the size of the turbulent eddies is limited by the presence of the earth's surface. Thus it is sometimes assumed that $l^2 = k^2z^2$, k = von Kármán's constant ⇒ the eddy viscosity in the surface layer:**

$$K_E = k^2z^2\left|\frac{\partial \bar{u}}{\partial z}\right|$$

- For stable boundary layers, Delage (1974) proposed the following parameterization for mixing length that has been used since as a starting point for other parameterizations:

$$\frac{1}{l} = \frac{1}{kz} + \frac{1}{0.0004U_g / f} + \frac{\beta}{kL_L}$$

where L_L is a local Obukhov length based on local values of stress and heat flux above the surface, U_g is the geostrophic wind speed, and β is an empirical constant.

Limitations of mixing-length theory

- The relationship $w' = c\left|\frac{\partial \bar{u}}{\partial z}\right| z'$ is only valid when turbulence is generated mechanically.
- Hence, mixing-length derivation is valid only for statically neutral conditions, even though K-theory has been applied to statically stable conditions.
- Also, linear gradients of wind and moisture were assumed in deriving $q' = -\frac{\partial \bar{q}}{\partial z} z'$.
- In the real atmosphere, gradients are approximately linear only over small distances (i.e., the first-order term of a Taylor series expansion) ⇒ mixing-length theory is a small-eddy theory.

Sample parameterizations of K

- The eddy viscosity is best not kept constant, but should be parameterized as a function of the flow.
- The parameterizations for K should satisfy the following constraints:
  - K = 0 where there is no turbulence
  - K = 0 at the ground (z = 0).
  - K increases as TKE increases.
  - K varies with static stability (in fact, one might expect that a different value of K should be used in each of the coordinate directions for anisotropic turbulence).
  - K is non-negative (if one uses the analogy with viscosity).
- This latter constraint has occasionally been ignored.

Some remarks about eddy viscosity

- The normal concept of an eddy viscosity or a small-eddy theory is that a turbulent flux is down the gradient.
- Such a down-gradient transport means heat flows from hot to cold, moisture flows from moist to dry, and so forth.
- Down-gradient transport is associated with positive values for K, and is consistent with the molecular viscosity analogy.
- In the real atmosphere, however, there are occasions where transport appears to flow up the gradient (i. e. counter-gradient).
- This is explained physically by the fact that there are large eddies associated with rise of warm air parcels that transport heat from hot to cold, regardless of the local gradient of the background environment.

- Thus, in an attempt to make small-eddy K-theory work in large-eddy convective boundary layers, one must resort to negative values of K.
- Since this results in heat flowing from cold to hot, it is counter to our common-sense concept of diffusion.
- Thus, K-theory is not for use in convective mixed layers.
- There has been no lack of creativity by investigators in designing parameterizations for K.
- The following table lists some of the parameterizations for K that have appeared in the literature (Bhumralkar, 1975).
- Variations of K in the horizontal have also been suggested to explain phenomena such as mesoscale cellular convection (Ray, 1986).

Examples of parameterizations for K in the BL

Neutral surface layer

$K = \text{constant}$	not the best parameterization
$K = u_*^2 T_o$	where u_* is the friction velocity
$K = U^2 T_o$	where T_o is a timescale
$K = k z u_*$	where k is von Karman's constant
$K = k^2 z^2 [(\partial\bar{U}/\partial z)^2 + (\partial\bar{V}/\partial z)^2]^{1/2}$	from mixing-length theory
$K = l^2 (\partial\bar{U}/\partial z)^2$	where $l = k(z+z_o)/\{1+[k(z+z_o)/\Lambda]\}$, Λ=length scale

Neutral surface layer

$K_{\text{statically unstable}} > K_{\text{neutral}} > K_{\text{statically stable}}$)

$K = k z u_* / \phi_M (z/L)$ where ϕ_M a dimensionless shear (see appendix A), and L is the Obukhov length (appendix A)

$K = k^2 z^2 [(\partial\bar{U}/\partial z) + \{(g/\overline{\theta_v})\cdot|\partial\overline{\theta_v}/\partial z|\}^{1/2}]$ for statically unstable conditions

$K = k^2 z^2 [(\partial\bar{U}/\partial z) - (L_*/z)^{1/6}\{(15g/\overline{\theta_v})\cdot|\partial\overline{\theta_v}/\partial z|\}^{1/2}]$ for statically stable conditions, where

$L_* = -\theta u_*^2/(15\, k\, g\, \theta_*)$

Examples of parameterizations for K in the BL

Neutral or stable boundary layer

$K = \text{constant}$ — see Ekman Spiral derivation in next subsection

$$K = K(h) + [(h-z)/(h-z_{SL})]^2 \{K(z_{SL}) - K(h) + (z-z_{SL})[\partial K/\partial z|_{z_{SL}} + 2(K(z_{SL})-K(h))/(h-z_{SL})]\}$$

this is known as the O'Brien cubic polynomial approximation (O'Brien, 1970), see Fig 6-2, where z_{SL} represents the surface layer depth.

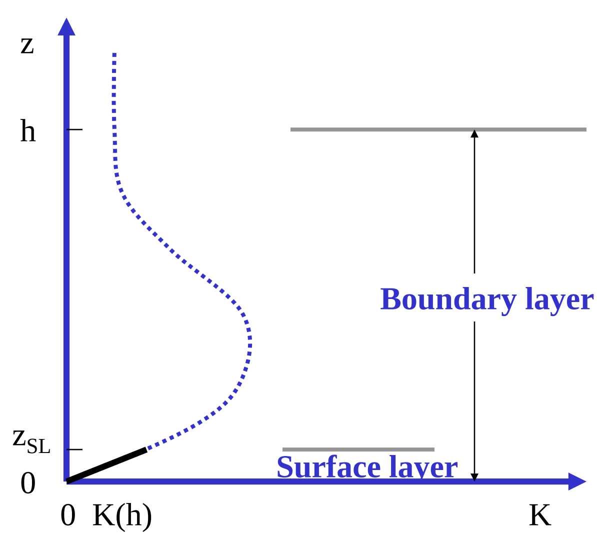
Unstable (convective) boundary layer

$K = 1.1\,[(R_c - Ri)\,l^2 / Ri\,]\,|\partial\bar{U}/\partial z|$ for $\partial\overline{\theta_v}/\partial z > 0$ where $l = kz$ for $z < 200$ m and

$K = (1 - 18\,Ri)^{-1/2}\,l^2\,|\partial\bar{U}/\partial z|$ for $\partial\overline{\theta_v}/\partial z < 0$ $l = 70$ m for $z > 200$ m.

Numerical model approximation for anelastic 3-D flow

$K = (0.25\,\Delta)^2 \cdot |0.5\,\Sigma_i\,\Sigma_j\,[\partial\bar{U}_i/\partial x_j + \partial\bar{U}_j/\partial x_i - (2/3)\delta_{ij}\Sigma_k(\partial\bar{U}_k/\partial x_k)]^2\,|^{1/2}$ where Δ=grid size



Typical variation of eddy viscosity, K, with height in the boundary layer (After O'Brien, 1970).

The Ekman spiral

- Even with first-order closure, the Ekman equations are often too difficult to solve analytically.
- The exception is the case of a steady ($\partial/\partial t = 0$), horizontally homogeneous ($\partial/\partial x = 0$, $\partial/\partial y = 0$), statically neutral ($\partial\theta_v/\partial t = 0$), barotropic atmosphere (u_g,v_g constant with height) with no subsidence ($w = 0$).

$$-f(\overline{v}-\overline{v}_g) = -\frac{\partial}{\partial z}\overline{u'w'}$$

$$f(\overline{u}-\overline{u}_g) = -\frac{\partial}{\partial z}\overline{v'w'}$$

- An analytic solution of these equations for the ocean was obtained by Ekman in 1905 and was soon modified for the atmosphere.

- Align the x-axis with the geostrophic wind (i.e. put $v_g = 0$).
- Use first-order local closure K-theory, with constant K_M.

$$\overline{u'w'} = -K_M\frac{\partial\overline{u}}{\partial z} \quad , \quad \overline{v'w'} = -K_M\frac{\partial\overline{v}}{\partial z}$$

$$f\overline{v} = -K_M\frac{\partial^2\overline{u}}{\partial z^2}$$

$$f(\overline{u}-\overline{u}_g) = K_M\frac{\partial^2\overline{v}}{\partial z^2}$$

- The boundary conditions are:

$$(\overline{u},\overline{v}) = 0 \ \text{ at } \ z = 0 \ \text{ and } \ (\overline{u},\overline{v}) \to (\overline{u}_g, 0) \ \text{ as } \ z \to \infty$$

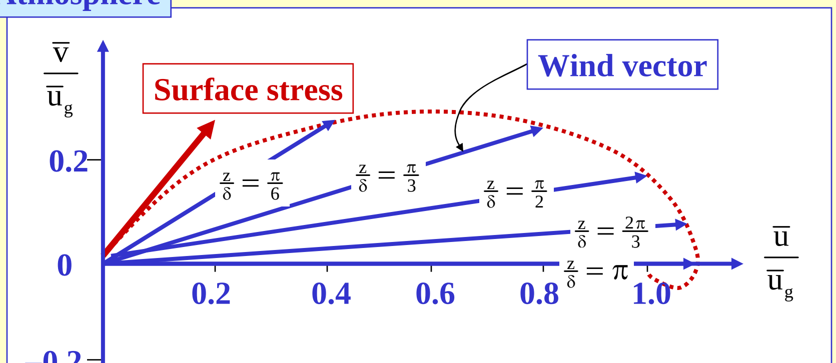
- The solution is (see DM, Ch. 5): ⇨

Ekman layer solution

$$\bar{u} = \bar{u}_g[1 - e^{-z/\delta}\cos(z/\delta)] \qquad \bar{v} = \bar{u}_g[e^{-z/\delta}\sin(z/\delta)]$$

$$\delta = (f/2K_M)^{1/2}$$

Atmosphere



Define the Ekman layer depth, $\delta_E = \pi\delta$.

Surface stress

- **The surface stress is characterized by** u_*

$$u_*^2 = \sqrt{\left(\overline{u'w'}\right)_s^2 + \left(\overline{v'w'}\right)_s^2} = K_M\sqrt{\left(\frac{\partial\bar{u}}{\partial z}\right)_s^2 + \left(\frac{\partial\bar{v}}{\partial z}\right)_s^2}$$

- **Use** $\bar{u} = \bar{u}_g[1 - e^{-z/\delta}\cos(z/\delta)] \qquad \bar{v} = \bar{u}_g[e^{-z/\delta}\sin(z/\delta)]$

- **Put** $z = 0$ ⇒ $u_*^2 = \bar{u}_g\sqrt{K_M f}$

Ekman layer depth

- **The wind speed is supergeostrophic at** $z = \pi/\gamma_E$**, which is also the lowest height where the wind is parallel to geostrophic.**
- **Sometimes this height is used as an estimate of the depth of the neutral boundary layer.**
- **Hence the Ekman layer depth,** h_E**, is defined as** $h_E = \pi/\gamma_E$**.**
- **Assuming that** $K_M = cku_* h_E$**, where** c **is a constant of proportionality** ≈ 0.1**, and** k **is the von Kármán constant, then:**

$$h_E = 2ck\pi^2 u_* / f$$

Ekman pumping

- **The major conclusion from the Ekman solution is that friction reduces the boundary layer wind speed below geostrophic, and causes it to cross the isobars from high towards low pressure.**
- **In a synoptic situation where the isobars are curved, such as a low or high pressure system, the cross-isobaric component of flow near the surface causes convergence or divergence, respectively.**
- **Hence, mass continuity requires that there be rising air in low pressure systems, and descending air in highs.**
- **The process of inducing vertical motions by boundary layer friction is called Ekman pumping.**

The oceanic Ekman layer

- **The ocean drift current is driven by the surface wind stress, neglecting pressure gradients in the ocean ⇒**

$$-f\overline{v} = -K_M \frac{\partial^2 \overline{u}}{\partial z^2}, \qquad f\overline{u} = K_M \frac{\partial^2 \overline{v}}{\partial z^2}$$

- **Now choose a coordinate system with the x-axis aligned with the surface stress and z positive up.**
- **The boundary conditions are:**

$$K_M \frac{\partial \overline{u}}{\partial z} = u_*^2, \quad \frac{\partial \overline{v}}{\partial z} = 0 \ \text{ at } \ z = 0 \ \text{ and } \ (\overline{u}, \overline{v}) \to (0,0) \ \text{ as } \ z \to -\infty$$

K_M **and** u_* **refer to their ocean values**

$$\rho u_*^2\big|_{water} = \text{surface stress} = \rho u_*^2\big|_{air}$$

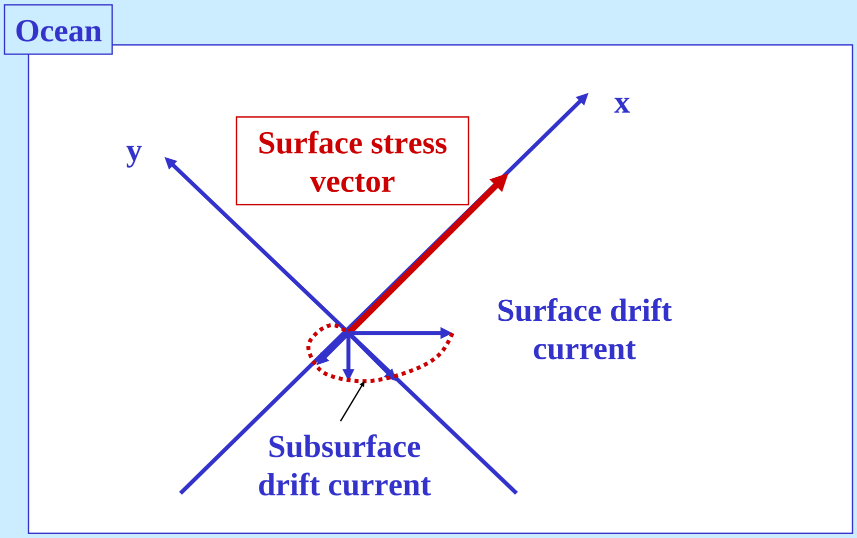
- **Solution:**

$$\overline{u} = \overline{u}_o e^{z/\delta_E} \cos(z/\delta_E - \pi/4)$$
$$\overline{v} = \overline{u}_o e^{z/\delta_E} \sin(z/\delta_E - \pi/4)$$

$$\overline{u}_o = \frac{u_*^2}{\sqrt{K_M f}}$$

- **Now** K_M **and** δ_E **apply to ocean values.**

Plot



The surface current is 45^o to the right of the surface stress, in the direction of $\mathbf{u_g}$. The surface drift current is ≈ | $\mathbf{u_g}$|/30.

Discussion

- Although the Ekman solution is analytic and has been around for a long time, the conditions under which it was derived rarely happen in nature in the atmosphere.
- At best, it gives an approximate quantitative solution for statically neutral boundary layers (i.e., mechanical turbulence production characteristic of strong winds, with no buoyancy effects).
- For convective mixed layers, the Ekman profile shape is not observed, although it agrees qualitatively with the observed winds, which are subgeostrophic and cross-isobaric.
- Observed stable boundary layers can have supergeostrophic winds at low altitudes, making the Ekman solution even qualitatively incorrect.

Local closure – one-and-a-half order

- One-and-a-half-order closure retains the prognostic equations for the zero-order statistics such as mean wind, temperature, and humidity, and also retains equations for the variances of those variables.
- The TKE equation is usually used in place of the velocity variance equations.
- Example (based on Yamada & Mellor, 1975), consider a horizontally homogeneous, dry environment, with no subsidence.
- Again, consider the idealized scenario of a horizontally homogeneous dry atmosphere, with no subsidence.
- The governing equations are $\Rightarrow$

Momentum

$$\frac{\partial \bar{u}}{\partial t} - f(\bar{v} - \bar{v}_g) = -\frac{\partial}{\partial z}\overline{u'w'}$$

$$\frac{\partial \bar{v}}{\partial t} + f(\bar{u} - \bar{u}_g) = -\frac{\partial}{\partial z}\overline{v'w'}$$

Heat

$$\frac{\partial \bar{\theta}}{\partial t} = -\frac{\partial}{\partial z}\overline{w'\theta'}$$

TKE

$$\frac{\partial \bar{e}}{\partial t} = -\overline{u'w'}\frac{\partial \bar{u}}{\partial z} - \overline{v'w'}\frac{\partial \bar{v}}{\partial z} + \frac{g}{\bar{\theta}}\overline{w'\theta'} - \frac{\partial}{\partial z}\left[\overline{w'\left(\frac{p'}{\bar{\rho}} + e\right)}\right] - \varepsilon$$

Temperature variance

$$\frac{\partial \overline{\theta'^2}}{\partial t} = -2\overline{w'\theta'}\frac{\partial \bar{\theta}}{\partial z} - \frac{\partial \overline{w'\theta'^2}}{\partial z} - 2\varepsilon_\theta - \varepsilon_R$$

Unknowns

$\overline{u'w'}$, $\overline{v'w'}$, $\overline{w'\theta'}$, $\overline{w'p'}/\bar{\rho}$ — Second moments (fluxes)

$\overline{w'e}$, $\overline{w'\theta'^2}$ — third moments

ε, ε_θ, ε_R — dissipation terms

Discussion

- At first glance, the addition of the variance equations seems to have hurt us rather than help us.
- With first-order closure we had 3 unknowns for:

$$\overline{u'w'},\ \overline{v'w'},\ \overline{w'\theta'}$$

- Now we have an additional 6 unknowns! So why do it?
- The reason is that knowledge of the TKE and temperature variance provide a measure of the turbulence intensity.
- We can use this information to formulate an improved parameterization for the eddy diffusivity $K_M(\overline{e}, \overline{\theta'^2})$.
- One suggested parameterization is ⇒

$$\overline{u'w'} = -K_M(\overline{e}, \overline{\theta'^2}) \frac{\partial \overline{u}}{\partial z}$$

$$\overline{v'w'} = -K_M(\overline{e}, \overline{\theta'^2}) \frac{\partial \overline{v}}{\partial z}$$

$$\overline{w'\theta'} = -K_H(\overline{e}, \overline{\theta'^2}) \frac{\partial \overline{\theta}}{\partial z} - \gamma_c(\overline{e}, \overline{\theta'^2})$$

$$\overline{w'\left(\frac{p'}{\overline{\rho}} + e\right)} = \frac{5}{3}\Lambda_4 \overline{e}^{-1/2} \frac{\partial \overline{e}}{\partial z}$$

$$\overline{w'\theta'^2} = \Lambda_3 \overline{e}^{-1/2} \frac{\partial \overline{\theta'^2}}{\partial z}$$

$$\varepsilon_R = 0 \qquad \varepsilon = \frac{\overline{e}^{3/2}}{\Lambda_1} \qquad \varepsilon_\theta = \frac{\overline{e}^{1/2}\,\overline{\theta'^2}}{\Lambda_2}$$

The Λ_n are empirical length-scale parameters. They are often chosen by trial and error to match model simulations with data.

N

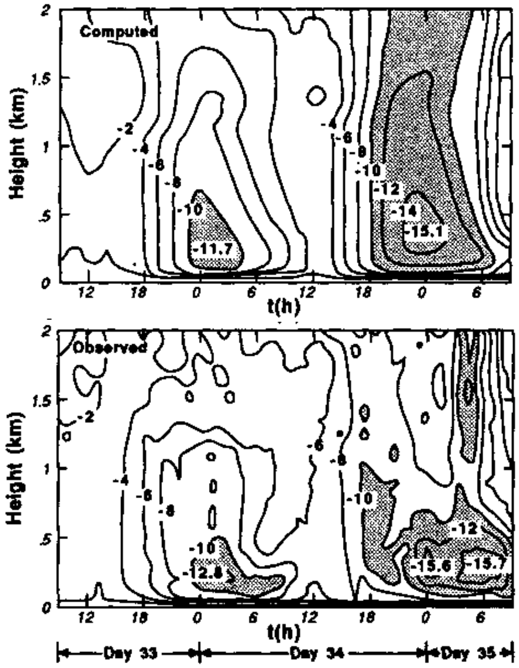
- One problem with the foregoing closure is that the length scales are rather arbitrary.
- The expressions for K are rather complex also, but can be represented approximately by:

$$K = \Lambda \overline{e}^{1/2}$$

Λ represents one of the length scales.

A calculation

- The set of equations is far too complex to solve analytically.
- Typically the equations are solved numerically using finite difference methods.
- The next figures show a numerical BL simulation of a two-day period from the Wangara field experiment using the Yamada-Mellor one-and-a-half order closure.



⇐ Simulated

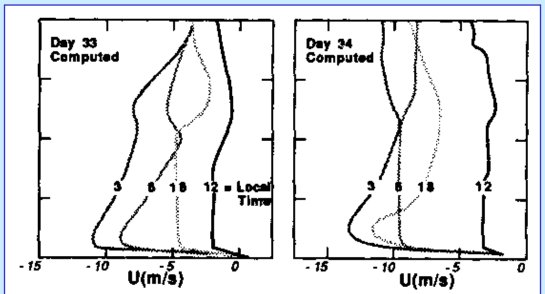
using 1-1/2 closure

Eastward velocity

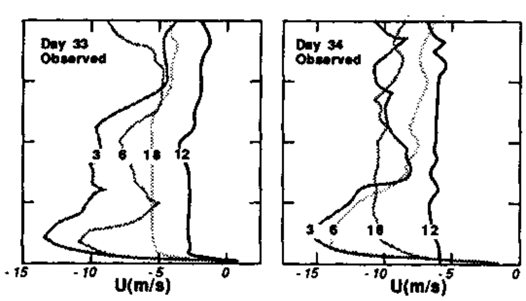
⇐ Observed

Wangara days 33-35

From Yamada & Mellor, 1975



Eastward velocity profiles



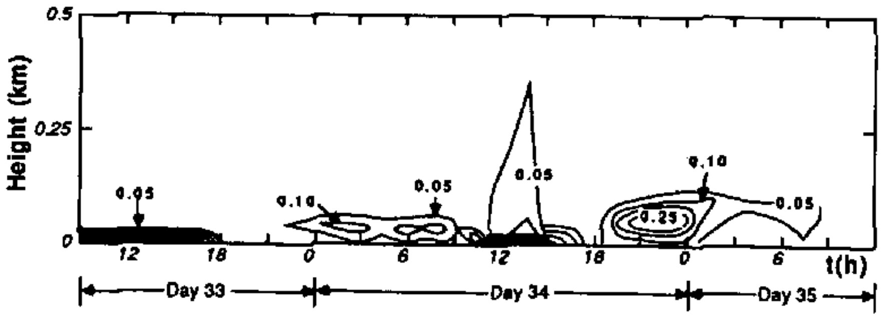
⇐ Simulated

using 1-1/2 closure

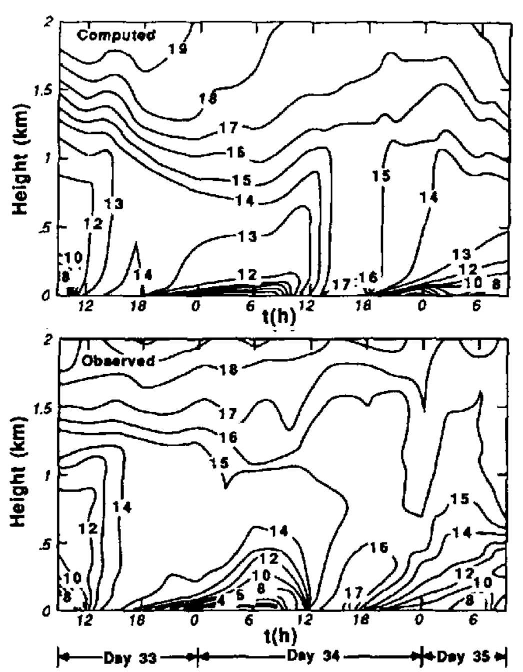
⇐ Observed

Wangara days 33-35

From Yamada & Mellor, 1975



Wangara simulated one-and-a-half order closure time and space variation of virtual potential temperature variance (units K^2).



⇐ Simulated

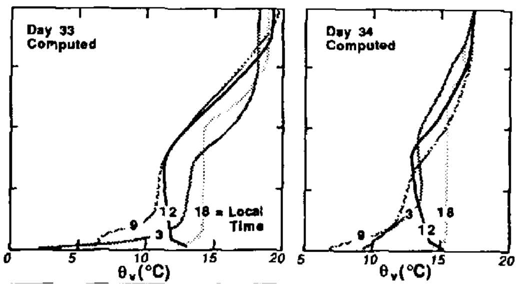
using 1-1/2 closure

Virtual potential temperature

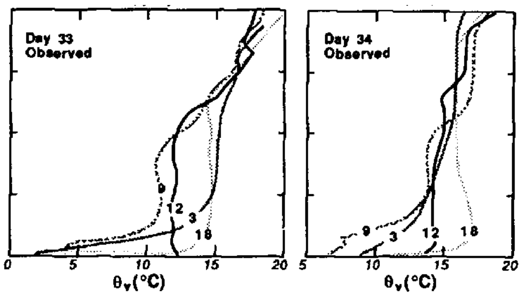
⇐ Observed

Wangara days 33-35

From Yamada & Mellor, 1975



Virtual potential temperature



⇐ Simulated

using 1-1/2 closure

⇐ Observed

Wangara days 33-35

From Yamada & Mellor, 1975

Discussion

- By studying the foregoing figures, we can learn some of the advantages of higher-order closure:
  1) The higher-order scheme creates nearly well-mixed layers during the daytime that increase in depth with time.
  2) At night, there is evidence of nocturnal jet formation along with the development of statically stable layer near the ground.
  3) Turbulence intensity increases to large values during the day, but maintains smaller values at night in the nocturnal boundary layer.

Discussion

- First-order closure, on the other hand, gives no information on turbulence intensity or temperature variance.
- Furthermore, it has difficulty with the well mixed layers that have zero gradients of mean variables.
- However, the benefits of higher-order closure do not come cheaply; they are gained at the expense of increased computer time and cost to first-order closure.

Local closure – second order – history

- The development of **higher-order-closure** (usually meaning anything higher than first-order-closure) was closely tied to the evolution of digital computer power.
- Although the use of higher-moment equations for turbulence forecasting was suggested in the early 1940's, the large number of unknown variables remained a stumbling block.
- Around 1950, Rotta and Chou and others suggested parameterizations for some of the unknowns.
- By the late 1960's, computer power improved to the point where second-order closure forecasts for clear air turbulence and shear flows were first made.

Local closure – second order – history

- In the early 1970’s, the United States Environmental Protection Agency began funding some second-order closure pollution dispersion models, and by the mid 1970’s a number of investigators were using such models.
- In fact, second-order closure appears to have started before one-and-a-half-order closure.
- In the late 1970’s, some third-order closure models also started to appear in the literature, with many more third-order simulations published in the 1980’s.

- The set of second-order turbulence equations includes not only those from one-and-a-half order, but it includes second moment terms as well.
- Using the same idealized example as above, consider a dry environment, horizontally homogeneous, with no subsidence.
- The additional governing prognostic equations are those for $\overline{u_i' u_j'}$ and $\overline{u_i' \theta'}$.
- The resulting set of coupled equations is:

$$\frac{\partial \overline{U}_i}{\partial t} = - f_c \varepsilon_{ij3} (\overline{U}_{gj} - \overline{U}_j) - \frac{\partial (\overline{u_i'w'})}{\partial z} \qquad \text{(for } i \neq 3\text{)} \qquad (6.6a)$$

$$\frac{\partial \overline{\theta}}{\partial t} = - \frac{\partial (\overline{w'\theta'})}{\partial z}$$

$$\frac{\partial \overline{e}}{\partial t} = -\overline{u'w'} \frac{\partial \overline{U}}{\partial z} - \overline{v'w'} \frac{\partial \overline{V}}{\partial z} + \left(\frac{g}{\overline{\theta}}\right) \overline{w'\theta'} - \frac{\partial \left[\overline{w' ((p'/\overline{\rho}) + e)}\right]}{\partial z} - \varepsilon$$

$$\frac{\partial \left(\overline{\theta'^2}\right)}{\partial t} = - 2\, \overline{w'\theta'} \frac{\partial \overline{\theta}}{\partial z} - \frac{\partial (\overline{w'\theta'^2})}{\partial z} - 2\varepsilon_\theta - \varepsilon_R$$

$$\frac{\partial (\overline{u_i'u_j'})}{\partial t} = - \overline{u_i'w'} \frac{\partial \overline{U}_j}{\partial z} - \overline{u_j'w'} \frac{\partial \overline{U}_i}{\partial z} - \frac{\partial (\overline{u_i'u_j'w'})}{\partial z} + \left(\frac{g}{\overline{\theta}}\right)\left[\delta_{i3}\overline{u_j'\theta'} + \delta_{j3}\overline{u_i'\theta'}\right]$$

$$+ \overline{\left(\frac{p'}{\rho}\right)\left[\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right]} - 2\varepsilon_{u_i u_j}$$

$$\frac{\partial (\overline{u_i'\theta'})}{\partial t} = - \overline{w'\theta'} \frac{\partial \overline{U}_i}{\partial z} - \overline{u_i'w'} \frac{\partial \overline{\theta}}{\partial z} - \frac{\partial (\overline{u_i'w'\theta'})}{\partial z} + \delta_{i3}\, g \frac{\overline{\theta'^2}}{\overline{\theta}} + \left(\frac{1}{\overline{\rho}}\right)\left[\overline{p' \frac{\partial \theta'}{\partial x_i}}\right] - \varepsilon_{u\theta}$$

Unknowns

pressure-correlation terms:

$$\frac{1}{\overline{\rho}} \overline{p' \frac{\partial \theta'}{\partial x_i}}, \quad \overline{\frac{p'}{\overline{\rho}}\left(\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right)}, \quad \frac{\overline{w'p'}}{\overline{\rho}}$$

third moments:

$$\overline{w'e}, \quad \overline{w'\theta'^2}, \quad \overline{u_j'w'\theta'}, \quad \overline{u_i'u_j'w'}$$

dissipation terms:

$$\varepsilon, \quad \varepsilon_R, \quad \varepsilon_\theta, \quad \varepsilon_{u\theta}, \quad \varepsilon_{u_i u_j}$$

Table 6-5. Sample second-order closure parameterizations suggested by (A) Donaldson, and (B) Deardorff. (Reference: Workshop on Micrometeorology, 1973). The Λ_i are length scales, which are either held constant or based on mixing-length arguments.

1

$$\overline{u_i'u_j'u_k'} = -\Lambda_2 \bar{e}^{1/2}\left[\frac{\partial \overline{u_i'u_j'}}{\partial x_k} + \frac{\partial \overline{u_i'u_k'}}{\partial x_j} + \frac{\partial \overline{u_k'u_j'}}{\partial x_i}\right] \quad \text{(A)}$$

$$= -\frac{3}{2}\left(\frac{\Lambda_2}{\bar{e}^{1/2}}\right)\left[\overline{u_k'u_m'}\,\frac{\partial \overline{u_i'u_j'}}{\partial x_m} + \overline{u_j'u_m'}\,\frac{\partial \overline{u_i'u_k'}}{\partial x_m} + \overline{u_i'u_m'}\,\frac{\partial \overline{u_k'u_j'}}{\partial x_m}\right] \quad \text{(B)}$$

2

$$\overline{u_i'u_j'\theta'} = -\Lambda_2 \bar{e}^{1/2}\left[\frac{\partial \overline{u_i'\theta'}}{\partial x_j} + \frac{\partial \overline{u_j'\theta'}}{\partial x_i}\right] \quad \text{(A)}$$

$$= -\frac{3}{2}\left(\frac{\Lambda_2}{\bar{e}^{1/2}}\right)\left[\overline{\theta'u_m'}\,\frac{\partial \overline{u_i'u_j'}}{\partial x_m} + \overline{u_j'u_m'}\,\frac{\partial \overline{u_i'\theta'}}{\partial x_m} + \overline{u_i'u_m'}\,\frac{\partial \overline{\theta'u_j'}}{\partial x_m}\right] \quad \text{(B)}$$

$$\overline{u_j'\theta'^2} = -\Lambda_2 \bar{e}^{1/2}\left[\frac{\partial \overline{\theta'^2}}{\partial x_j}\right] \quad \text{(A)}$$

$$= -\frac{3}{2}\left(\frac{\Lambda_2}{\bar{e}^{1/2}}\right)\left[2\,\overline{\theta'u_m'}\,\frac{\partial \overline{u_i'\theta'}}{\partial x_m} + \overline{u_i'u_m'}\,\frac{\partial \overline{\theta'^2}}{\partial x_m}\right] \quad \text{(B)}$$

4

$$\overline{\left(\frac{p'}{\rho}\right)\left[\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right]} = -\left(\frac{\bar{e}^{1/2}}{\Lambda_1}\right)\left[\overline{u_i'u_j'} - \frac{2}{3}\delta_{ij}\bar{e}\right] \quad \text{(Rotta, 1951) (A)}$$

$$= -\left(\frac{\bar{e}^{1/2}}{\Lambda_1}\right)\left[\overline{u_i'u_j'} - \frac{2}{3}\delta_{ij}\bar{e}\right] + \frac{2}{5}\bar{e}\left[\frac{\partial \bar{U}_i}{\partial x_j} + \frac{\partial \bar{U}_j}{\partial x_i}\right] \quad \text{(B)}$$

5

$$\left(\frac{1}{\bar{\rho}}\right)\left[\overline{p'\frac{\partial \theta'}{\partial x_i}}\right] = -\left(\frac{\bar{e}^{1/2}}{\Lambda_1}\right)\overline{u_i'\theta'} \quad \text{(A)}$$

$$= -\left(\frac{\bar{e}^{1/2}}{\Lambda_1}\right)\overline{u_i'\theta'} - \frac{1}{3}\delta_{13}\frac{g}{\bar{\theta}}\overline{\theta'^2} \quad \text{(B)}$$

6			
$\frac{1}{\rho}\left(\overline{p'u_i'}\right)$	=	$-\left(\frac{\overline{e}^{1/2}}{\Lambda_3}\right)\frac{\partial\overline{u_i'u_j'}}{\partial x_j}$	(A,B)
7			
ε	=	$\frac{\overline{e}^{3/2}}{\Lambda_4}$	(A,B)
8			
ε_θ	=	$\frac{\overline{e}^{1/2}\,\overline{\theta'^2}}{\Lambda_4}$	(A,B)

Note: the $\overline{w'e}$ parameterization is the same as that for $\overline{u_i'u_j'u_k'}$.

There are three basic closure ideas contained in Table 6-5:

• **Down-gradient diffusion** (items 1-3 and 6 in the table), diffusion of the third-order statistics down the gradient of the second-order statistics;

• **Return to isotropy** (items 4 and 5), proportional to the amount of anisotropy;

• **Decay** (items 7 and 8), proportional to the magnitude of the turbulence.

I will show next a sample second-order closure model forecast, based on the moist convective boundary layer simulations of Sun and Ogura (1980).

Besides the equations listed above, they include prognostic equations for mixing ration, $\overline{r}$**, . moisture variance** $\overline{r'^2}$**, moisture flux** $\overline{w'r'}$**, and temperature-moisture covariance** $\overline{r'\theta'}$**.**

Using the full second-order set of equations, they could produce forecasts of mean variables, as can be produced (with poorer accuracy) by first-order closure.

They could forecast variances, as can be produced (with poorer accuracy) by on-and-a-half-order closure.

Most importantly, they can also produce forecasts of fluxes and other covariances that the lower-order schemes can not forecast.

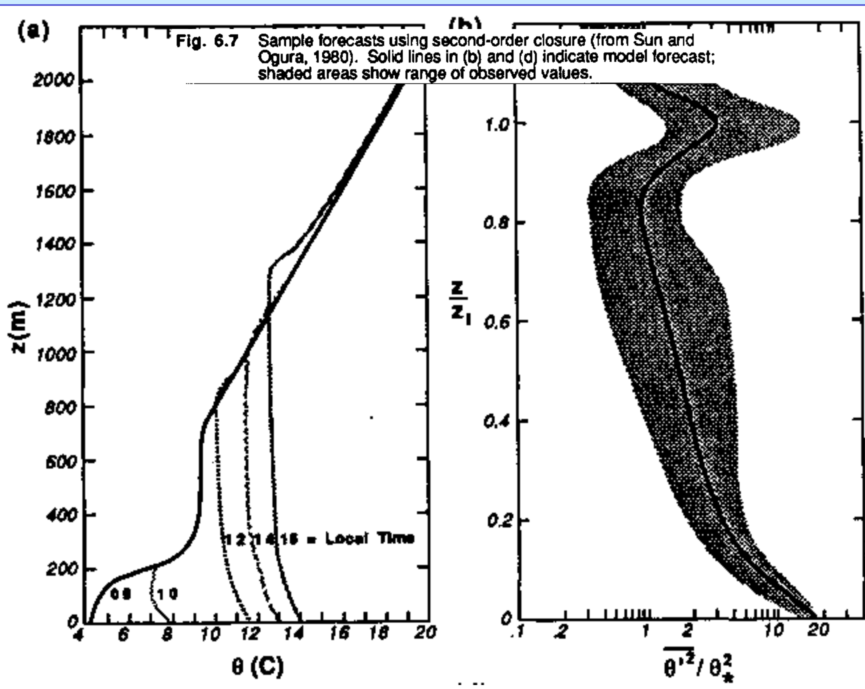


Fig. 6.7 Sample forecasts using second-order closure (from Sun and Ogura, 1980). Solid lines in (b) and (d) indicate model forecast; shaded areas show range of observed values.

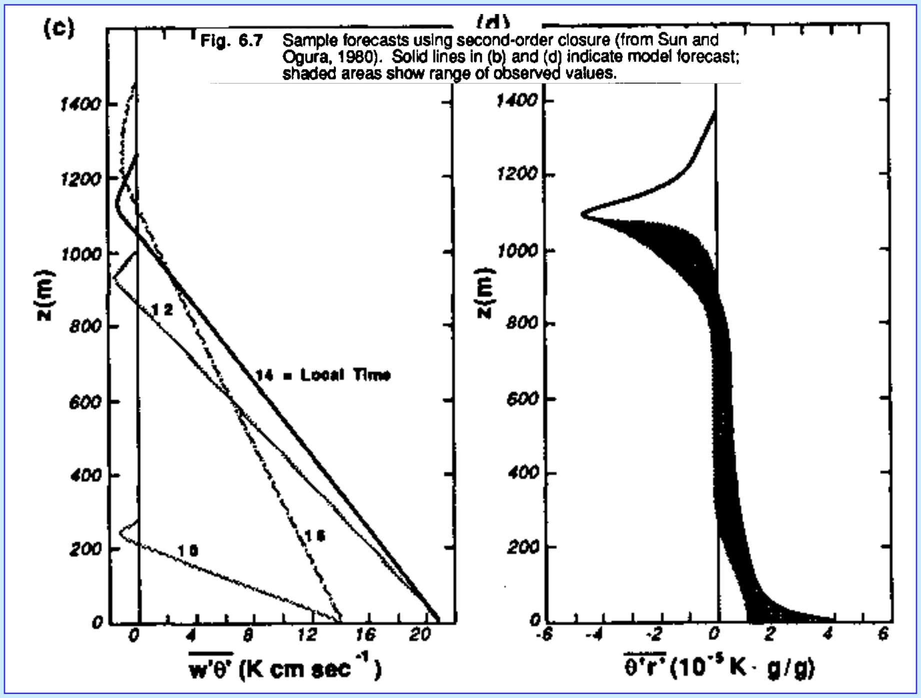


Fig. 6.7 Sample forecasts using second-order closure (from Sun and Ogura, 1980). Solid lines in (b) and (d) indicate model forecast; shaded areas show range of observed values.

Local closure – third order

- It is beyond the scope of this course to go into the details of third-order closure.
- In general, the prognostic equations for the triple-correlation terms are retained, while parameterizations are devised for the fourth-order correlations, for the pressure correlations, and for viscous dissipation.
- Some of the parameterizations presented in the literature assume that the fourth-order moments have a quasi-Gaussian probability distribution, and can be approximated as a function of second-moment terms.
- Any unrealistic values of some of the third moments are truncated or clipped to remain within physically realistic ranges, and various eddy damping schemes are used to prevent negative variances.

- It is generally assumed that equations for lower-order variables (such as mean wind fluxes) become more accurate, as the closure approximations are pushed to higher orders.
- In other words, parameterizations for the fourth-order terms might be very crude, but there are enough remaining physics (unparameterized terms) in the equations for the third moments that these third moments are less crude.
- The second moment equations bring in more physics, making them even more precise - and so on down to the equations for the mean wind and temperature, etc.
- Based on the successful simulations published in the literature, this philosophy indeed seems to work.
- Higher-order moments are extremely difficult to measure in the real atmosphere.

- Measurements of fluxes (second moments) typically have a large amount of scatter.
- Eddy correlation estimates of third moments are even worse, with noise or error levels larger than the signal level.
- Accurate forth-order moment measurements are virtually nonexistent.
- This means that we have very little knowledge of how these third and fourth moments behave; therefore, we have little guidance for suggesting good parameterizations for these moments.
- Now we see why such crude approximations are made in third-order closure models.

- Higher-order closure models have many parameters that can be adjusted advantageously to yield good forecasts.
- These parameters are fine-tuned using special limiting case studies and laboratory flows where simplifications cause some of the terms to disappear, allowing better determination of the few remaining terms.

Nonlocal closure – transilient turbulence theory

- Nonlocal closure recognizes that larger-size eddies can transport fluid across finite distances before the smaller eddies have a chance to cause mixing.
- This advective-like concept is supported by observations of thermals rising with undiluted cores, finite size swirls of leaves or snow, and the organized circulation patterns sometimes visible from cloud photographs.
- Stull presents two first-order nonlocal closure models:
  - Transilient turbulence theory, approaches the subject from a physical space perspective.
  - Spectral diffusivity theory, uses a spectral or phase-space approach.
- Both allow a range of eddy sizes to contribute to the turbulent mixing process.

See Stull, pp225-242