Boundary Layer Meteorology

Chapter 2

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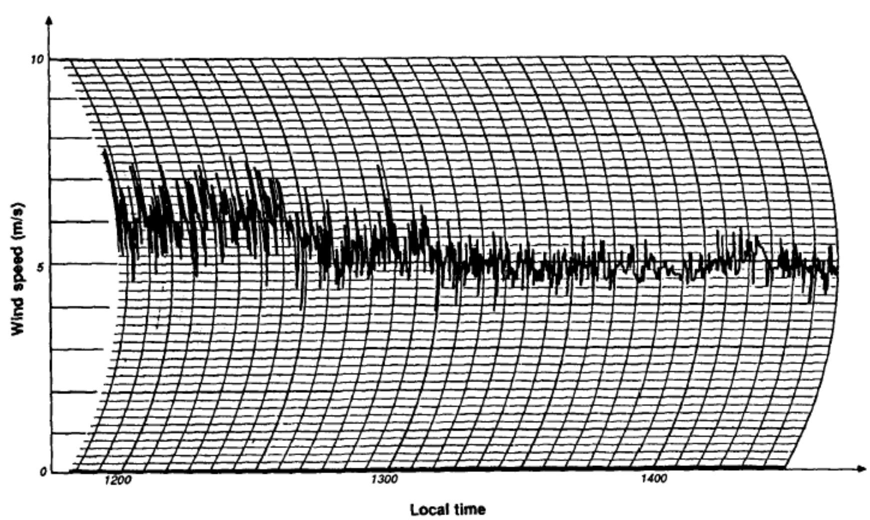
Some mathematical tools: Statistics

- Turbulence is an intrinsic feature of the atmospheric BL that must be quantified in order to study it.
- The randomness of turbulence precludes a deterministic approach and we are forced to use statistics, where we are limited to average or expected measures of turbulence.
- The procedure involves separating the turbulent from the nonturbulent part of the flow as described in this section.

Trace of wind speed observed in the early afternoon



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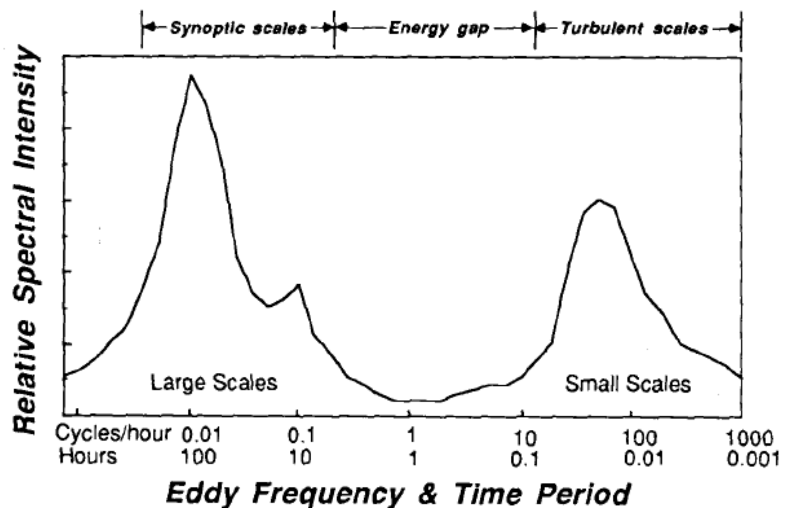
Eddy sizes

- There appears to be a wide variety of time-scales of wind variation superimposed on top of each other in the wind trace.
- If we look closely we see that the time period between each little peak in wind speed is about a minute.
- The larger peaks seem to happen about every 5 min and there are other variations that indicate a 10 min time period.
- The smallest detectable variations are about 10 sec long.
- If each time-scale is associated with a different size turbulent eddy, we can conclude (using Taylor's hypothesis) that we are seeing eddies ranging in size from about 50 m to about 3000 m, evidence of the spectrum of turbulence.

The turbulence spectrum

- The turbulence spectrum is analogous to the spectrum of colours in a rainbow.
- White light consists of many colours (i.e. many wavelengths or frequencies) superimposed on one another.
- Raindrops act like a prism that separates the colours.
- We could measure the intensity of each colour to learn the magnitude of its contribution to the original light beam.
- We can perform a similar analysis on a turbulence signal using mathematical rather than physical devices (i.e. a prism) to learn about the contribution of each different eddy size to the total turbulence kinetic energy.

Spectrum of near surface wind speed



Schematic spectrum of wind speed near the ground.

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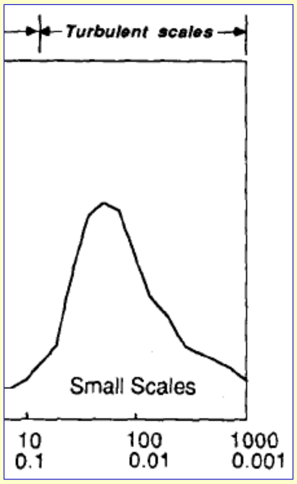
Energy cascade



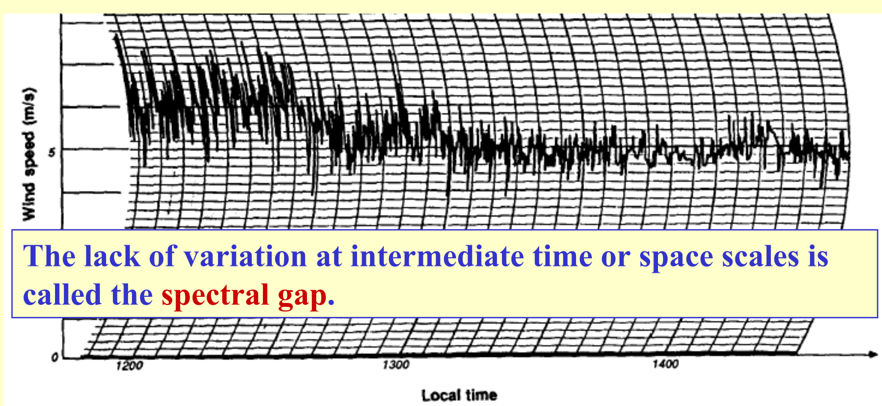
- The largest eddies in this range are usually the most intense.
- The smaller, high frequency, eddies are very weak.
- Large-eddy motions can create eddy-size wind-shear regions, which can generate smaller eddies.
- Such a net transfer of turbulence energy from the larger to the smaller scales is known as the energy cascade.
- At the smallest eddy sizes, the cascade of energy is dissipated into heat by molecular viscosity.

➢ The flavour of the energy cascade is captured by Lewis Richardson's poem of 1922:

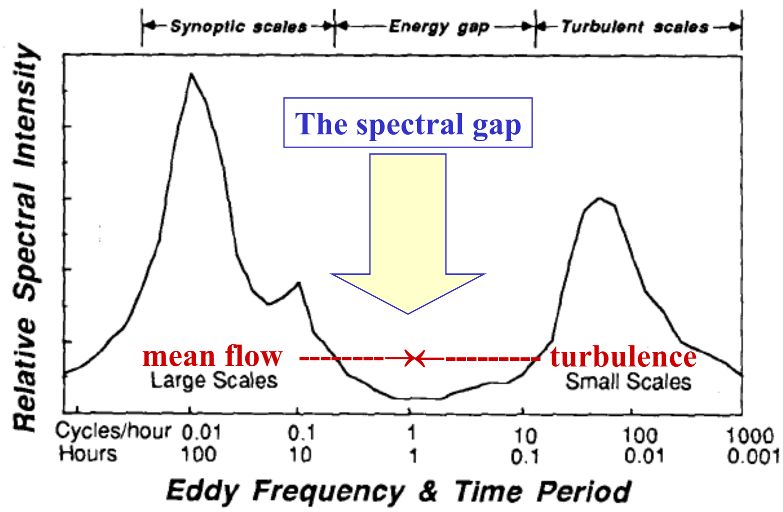
Big whirls have little whirls,
Which feed on their velocity;
And little whirls have lesser whirls,
And so on to viscosity.

The spectral gap

➢ There appears to be a distinct lack of wind-speed variation in the wind trace having time periods of about 30 min to 1 h. We already noted the slow variation of the mean wind speed from 6 to 5 m s^{-1}.



The lack of variation at intermediate time or space scales is called the **spectral gap**.



The **spectral gap** appears as the large valley separating the microscale from the synoptic scale peaks.

- For some flows there may not be a spectral gap.
- For example, larger cumulus clouds act like large eddies with timescales on the order of an hour. Thus a spectrum of wind speed made in the cloud layer might not exhibit a vivid separation of scales.
- Most analyses of turbulence rely on the separation of scales to simplify the problem; hence cloud-filled flow regimes might be difficult to properly describe.
- Many NWP models use grid spacings or wavelength cutoffs that fall within the spectral gap. This means that the larger-scale motions can be explicitly resolved and deterministically forecast.
- The smaller-scale motions, namely turbulence, are not modelled directly: they have to be **parameterized**.

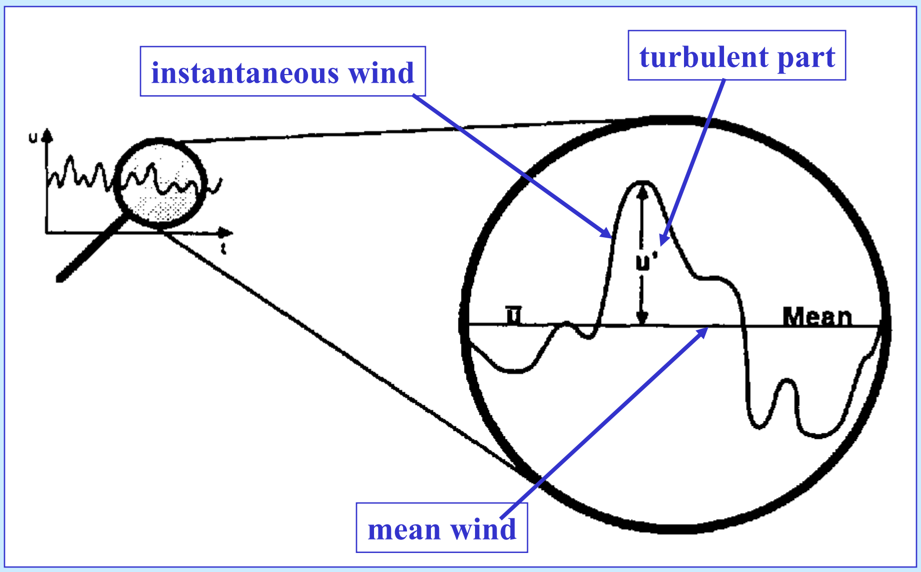
Mean and turbulent parts of the flow

- **There is a very easy way to isolate the large-scale variations from the turbulent ones: by averaging the wind speed measurements over a period of 30 min to one hour, we can eliminate or average out the positive and negative deviations of the turbulent velocities about the mean.**
- **Let**

$$u = \bar{U} + u'$$

instantaneous wind (u) — **mean wind** ($\bar{U}$) — **turbulent part** (u')

- **The existence of a spectral gap allows us to partition the flow field in this manner.**



Detailed view of the wind speed record shown earlier. u' is the gust or deviation of the actual instantaneous wind, u, from the local mean, ū.

Some basic statistical methods

Three types of mean: time average, space average, ensemble average.

The time average, applies at one specific point in space and consists of a sum or integral over a time period T.

Let A = A(t,s), t **time,** s **space. Then:**

$$\overline{{}^{t}A(s)} = \frac{1}{N}\sum_{i=0}^{N-1} A(t,s) \quad \text{or} \quad \overline{{}^{t}A(s)} = \frac{1}{T}\int_0^T A(t,s)dt$$

where t = iΔt, **for the discrete case.**

Δt = T/N, **where** N **is the number of data points.**

The space average, which applies at some instant of time is given by a sum or integral over a spatial domain S.

$$\overline{{}^{s}A(t)} = \frac{1}{N}\sum_{j=0}^{N-1} A(t,j) \quad \text{or} \quad \overline{{}^{s}A(t)} = \frac{1}{S}\int_0^S A(t,s)ds$$

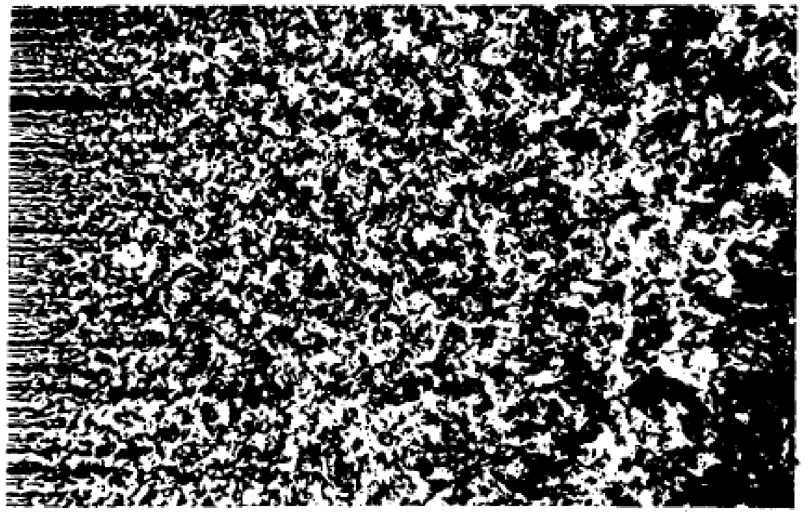
where, for the discrete case, s = jΔs, **and** Δs = S/N.

An ensemble average, consists of the sum over N **identical experiments, or realizations.**

$$\overline{{}^{e}A(t,s)} = \frac{1}{N}\sum_{j=0}^{N-1} A_i(t,s)$$

- For laboratory experiments, the ensemble average is the most desirable. It allows us to reduce random experimental errors by repeating the basic experiment.
- Unlike laboratory experiments, we have little control over the atmosphere so we are rarely able to observe reproducible weather events. Therefore we are unable to use the ensemble average.
- Spatial averages are possible by deploying an array of meteorological sensors covering a line, area, or volume.
- If the turbulence is homogenous (statistically the same at every point in space) then each of the sensors in the array will be measuring the same phenomenon, making a spatial average meaningful.

- The real atmosphere is only horizontally homogeneous in limited locations, meaning that most spatial means are averaged over a variety of different phenomena.
- By proper choice of sensor-array domain size as well as intra-array spacing, one can sometimes isolate scales of phenomena for study, while averaging out in the other scales.



Laboratory generation of homogeneous turbulence behind a grid. Using a finer resolution than in the figure, the merging unstable wakes quickly form a homogeneous field. As it decays downstream, it provides a useful approximation to homogeneous turbulence.

Types of averaging

- **Volume averaging** is virtually impossible using direct sensors such as thermometers because of the difficulty of deploying these sensors at all locations and altitudes throughout the BL.
- Remote sensors such as **radars**, **lidars** and **sodars** can scan volumes of the atmosphere, making volume averages of selected variables possible.
- **Area averaging** in the surface layer is frequently performed within small domains by deploying an array of small instrumented masts or instrument shelters on the ground.
- **Line averages** are similarly performed by erecting sensors along a road, for example.

Types of averaging

- Sensors mounted on a moving platform, such as a truck or an aircraft, can provide **quasi-line averages**.
- These are not true line (spatial) averages because the turbulence state of the flow may change during the time it takes the platform to move along the desired path.
- Most measurement paths are designed as a compromise between **long length** (to increase the statistical significance by observing a larger number of data points), and **short time** (because of the diurnal change that occur in the mean and turbulent state over most land surfaces.)

Time averages

- **Time averages** are frequently used, and are computed from sensors mounted on a single, fixed-location platform such as mast or tower.
- The relative ease of making observations at a fixed point has meant that time averaging has been the most popular in the lower BL.
- Some vertically-looking remote sensors also use this method to observe the middle and top of the BL.
- For turbulence that is both homogeneous and stationary (statistically not changing over time), the time, space and ensemble averages should all be equal. This is called the **ergodic condition**, which is often assumed to make the turbulence problem more tractable.

The rules of averaging

1. $\overline{c} = c$
2. $\overline{cA} = c\overline{A}$
3. $\overline{\overline{A}} = \overline{A}$
4. $\overline{\overline{A}B} = \overline{A}\,\overline{B}$
5. $\overline{A + B} = \overline{A} + \overline{B}$
6. $\overline{\dfrac{dA}{dt}} = \dfrac{d\overline{A}}{dt}$

Average of an average

3. $\overline{\overline{A}} = \overline{A}$

An average value acts like a constant when averaged a second time over the same time period ⇒

$$\overline{A}(T,s) = \frac{1}{T}\int_0^T A(t,s)dt$$

$$\Rightarrow \frac{1}{T}\int_0^T A(t,s)dt = \overline{A}(T,s)\frac{1}{T}\int_0^T dt = \overline{A}(T,s)$$

$$\Rightarrow \overline{\overline{A}} = \overline{A}$$

Also

$$\overline{\overline{A}B} = \overline{A}\,\overline{B}$$

Differentiation

$$\frac{d}{dt}\int_S A\,ds = \int_S \frac{\partial A}{\partial t}\,ds$$

Multiply both sides by 1/S**, where** $S = S_2 - S_1$ **gives:**

$$\frac{d(^S\overline{A})}{dt} = {}^S\overline{\left(\frac{\partial A}{\partial t}\right)}$$

This special case is not always valid for variable depth BLs.

Differentiation

Leibnitz' theorem

$$\frac{d}{dt}\int_{s_2(t)}^{s_1(t)} A(t,s)\,ds = \int_{s_2(t)}^{s_1(t)} \frac{\partial A(t,s)}{\partial t}\,ds + A(t,s_2)\frac{ds_2}{dt} - A(t,s_1)\frac{ds_1}{dt}$$

If s_1 **and** s_2 **are independent of time**

$$\frac{d}{dt}\int_{s_2}^{s_1} A(t,s)\,ds = \int_{s_2}^{s_1} \frac{\partial A(t,s)}{\partial t}\,ds$$

Example

Suppose we wish to find the time rate-of-change of a BL-averaged mixing ratio, $\overline{r}$, where the BL average is defined by integrating over the depth of the BL; i.e. from $z = 0$ to $z = z_i$.

Since z_i varies with time, we can use the full Leibnitz' theorem to give:

$$\frac{d}{dt}\left[z_i \, {}^{S}\overline{r}\right] = z_i \, {}^{S}\overline{\left[\frac{\partial r}{\partial t}\right]} + r(t, z_i^{+})\frac{dz_i}{dt}$$

where z_i^{+} represents a location just above the top of the BL.

The spectral gap

Let us re-examine the spectral gap.

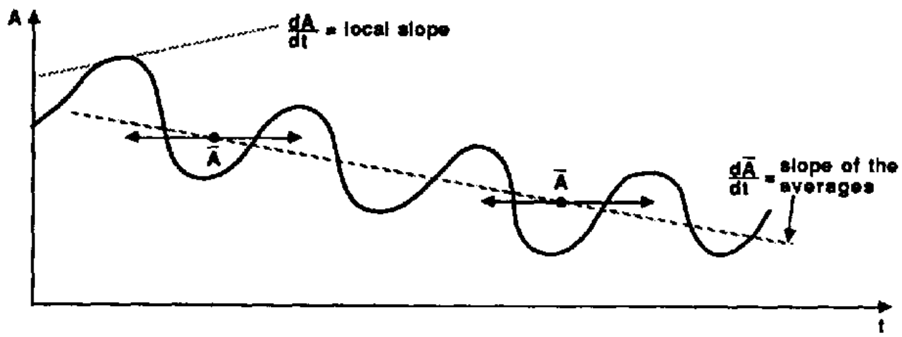
With an averaging time of 30 min to 1 hour, turbulent fluctuations will be eliminated, leaving the longer-period time variations.

We saw that the 30 min mean wind speed changes over a period of a few hours. Thus we can take the 30 min average of the time derivative of variable A to find out how $\overline{A}$ varies over longer periods:

$$ {}^{t}\overline{\left(\frac{\partial A}{\partial t}\right)} = \frac{d\,{}^{t}\overline{A}}{dt}$$

In other words, the average of the local slopes (slope = rate of change with time) equals the slope of the averages.

See figure



Schematic comparison of the slope of average values versus the average of local slopes. Averaging period, T, is indicated by the width of the horizontal arrows.

This is a difficult concept that deserves some thought on the part of the reader. It is an important consequence of the spectral gap because it allows us to make a deterministic forecast of a mean variable such as $\overline{A}$ using simplified, stochastic, representations of the turbulence. Otherwise, operational forecasts of seemingly simple variables such as temperature or wind would be much more difficult.

Reynolds averaging

Apply rules to mean and fluctuation

$$A = \overline{A} + a, \quad B = \overline{B} + b$$

$$\overline{A} = \overline{\overline{A} + a} = \overline{\overline{A}} + \overline{a} = \overline{A} + \overline{a}$$

$$\Rightarrow \quad \overline{a} = 0$$

$$\overline{AB} = \overline{(\overline{A} + a)(\overline{B} + b)}$$

$$= \overline{\overline{A}\,\overline{B} + a\overline{B} + \overline{A}b + ab}$$

$$= \overline{\overline{A}\,\overline{B}} + 0 + 0 + \overline{ab} = \overline{A}\,\overline{B} + \overline{ab}$$

$\neq 0$

Variance

One statistical measure of the dispersion of data about the mean is the variance, σ^2, defined by

$$\sigma_A^2 = \frac{1}{N}\sum_{i=0}^{N-1}(A_i - \overline{A})^2$$

Called the **biased variance**. It is a good measure of the dispersion of a sample of BL observations, but not the best measure of the dispersion of the whole population of possible observations. A better estimate of the variance (an **unbiased variance**) of the population, given a sample of data, is

$$\sigma_A^2 = \frac{1}{N-1}\sum_{i=0}^{N-1}(A_i - \overline{A})^2$$

For $1 \ll N$, there is little difference in these two estimates.

Standard deviation

The turbulent part of a turbulence variable is: $a' = A - \overline{A}$.

$$\sigma_A^2 = \frac{1}{N}\sum_{i=0}^{N-1} a_i'^2 = \overline{a'^2}$$

Thus turbulence quantities such as

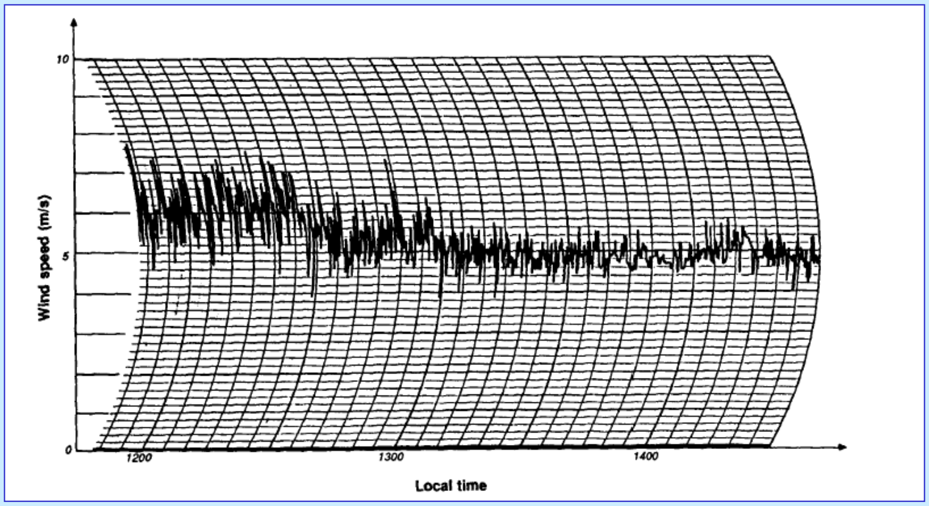
$$\overline{u'^2}, \overline{v'^2}, \overline{w'^2}, \overline{\theta'^2}, \overline{r'^2}, \overline{q'^2}$$

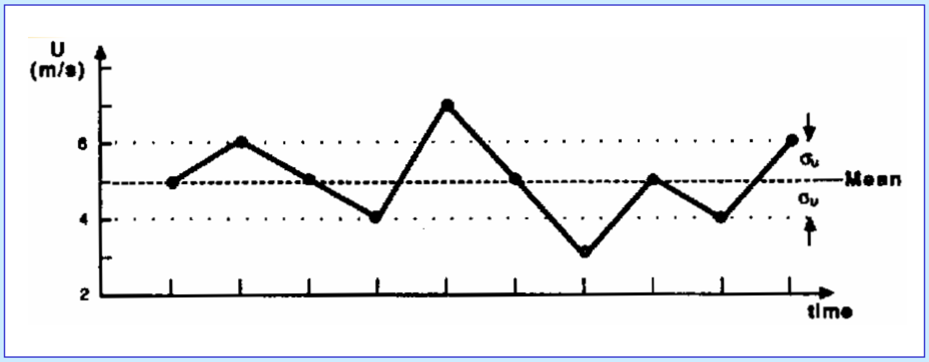
can be interpreted as variances.

The **standard deviation** is defined as the square root of the variance:

$$\sigma_A = \sqrt{\overline{a'^2}}$$

The standard deviation always has the same dimensions as the original variable. In this figure, for example, we might guess the standard deviation to be about 0.5-0.6 m s^{-1} at noon, dropping to about 0.3 m s^{-1} by 1400 local time.





Relationship between the standard deviation and turbulence variations:

- **solid line connects the data points**
- **heavy dashed line is the average**
- **dotted lines are drawn one standard deviation above and below the mean**

Turbulence intensity

Near the ground the turbulence intensity might be expected to increase as the mean wind speed U increases.

A dimensionless measure of the turbulence intensity, I, is often defined as

$$I = \sigma_M/U$$

For mechanically generated turbulence, one might expect σ_M to be a simple function of U.

Recall that $I < 0.5$ is required for Taylor's hypothesis to be valid.

Covariance and correlation

Covariance between two variables

$$\mathrm{covar}(A,B) = \frac{1}{N}\sum_{i=0}^{N-1}(A_i - \overline{A})(B_i - \overline{B})$$

Using Reynolds' averaging methods:

$$\mathrm{covar}(A,B) = \frac{1}{N}\sum_{i=0}^{N-1}A_i B_i = \overline{ab}$$

Thus, the nonlinear turbulence products introduced earlier have the same meaning as covariances.

Interpretation of covariances

- The covariance indicates the degree of common relationship between the two variables, A and B.
- Example: A = air temperature T, B = vertical velocity w.
- On a hot summer day over land, we might expect the warmer air to rise (T' and w' both > 0), and the cooler than average air to sink (T' and w' both < 0).
- $\Rightarrow$ $w'\,T' > 0$ on average, indicating that w and T vary together.
- The covariance $\overline{w'T'}$ is indeed found to be positive throughout the bottom 80% of the convective mixed layer.

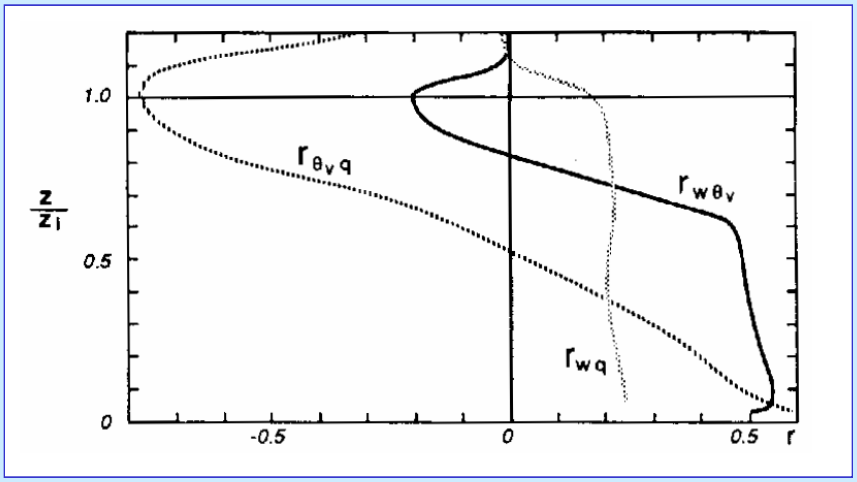
Normalized covariance

- Sometimes one is interested in a normalized covariance.
- Such a relationship is defined as the linear correlation coefficient:

$$r_{AB} = \frac{\overline{ab}}{\sigma_A \sigma_B}$$

- This variable ranges between –1 and +1 by definition.
- Two variables that are perfectly correlated (i.e. vary together) yield r = 1. Two variables that are perfectly negatively correlated (i.e. vary oppositely) yield r = –1.
- Variables with no net variation together yield r = 0.

Example



Typical correlation coefficient profiles in the convective mixed layer.

Turbulence kinetic energy

- The usual definition of **kinetic energy** (KE) is $0.5m\mathbf{U}^2$.
- For a fluid we often talk about **KE/unit mass** = $0.5\mathbf{U}^2$.
- We can partition into **mean KE** + **turbulence KE**:

$$MKE/m = \tfrac{1}{2}(\bar{u}^2 + \bar{v}^2 + \bar{w}^2)$$

$$TKE/m = \tfrac{1}{2}(u'^2 + v'^2 + w'^2)$$

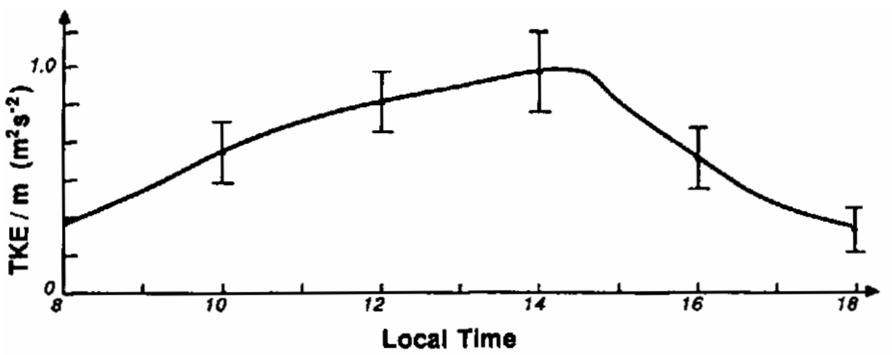
Instantaneous KE/unit mass

- We can expect rapid variations in TKE as we measure faster and slower gusts ⇒ define a **mean turbulent KE/unit mass**:

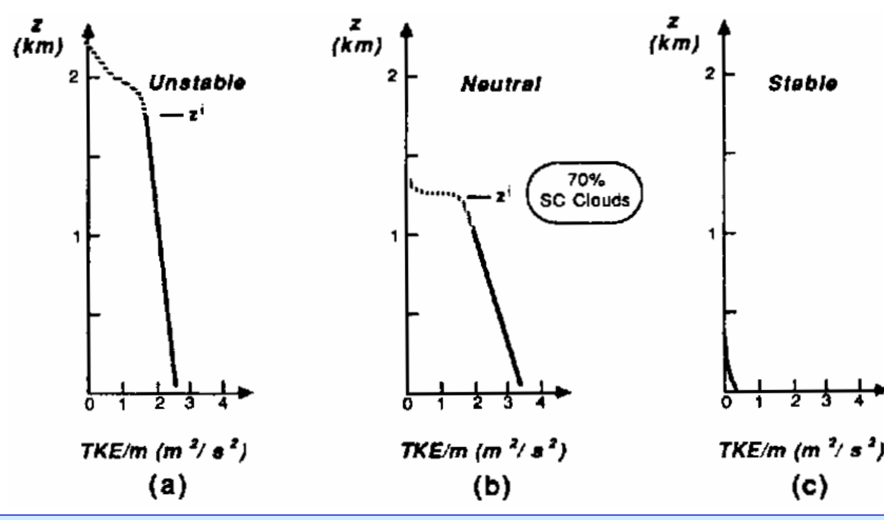
$$TKE/m = \tfrac{1}{2}(\overline{u'^2} + \overline{v'^2} + \overline{w'^2})$$

- Note the appearance of the variance!

A typical daytime variation of TKE in convective conditions



Diurnal cycle of $\mathrm{TKE/m} = \frac{1}{2}(\overline{u'^2} + \overline{v'^2} + \overline{w'^2})$ **measured by aircraft below 300 m AGL over Tennessee, August 1978**



Examples of TKE/m **for various BLs. (a) Daytime convective mixed layer with mostly clear skies and light winds. (b) Near neutral day with strong winds** (10 – 15 m s^{-1} **near the surface) and broken clouds. (c) nocturnal stable BL at** 1000 **local time.**

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Kinematic flux

Flux is the transfer of a quantity per unit area per unit time.

Examples

Flux	Symbol	Units
mass	$\tilde{M}$	$\left[\frac{kg_{air}}{m^2 \cdot s}\right]$
heat	$\tilde{Q}_H$	$\left[\frac{J}{m^2 \cdot s}\right]$
moisture	$\tilde{R}$	$\left[\frac{kg_{water}}{m^2 \cdot s}\right]$
momentum	$\tilde{F}$	$\left[\frac{kg \cdot (m \cdot s^{-1})}{m^2 \cdot s}\right]$
pollutant	$\tilde{\chi}$	$\left[\frac{kg_{pollutant}}{m^2 \cdot s}\right]$ or $\left[\frac{kg_{pollutant}}{m^3} \cdot \frac{m}{s}\right]$

- **Sometimes the moisture flux is rewritten as a latent heat flux:**

$$\tilde{Q}_E = L_v \tilde{R}$$

Latent heat of vaporization (→ L_v)

- **We rarely measure quantities like momentum directly. Instead we measure quantities like temperature or wind speed.**

- **⇒ we define kinematic fluxes by dividing by the density of moist air, ρ_{air}. In the case of sensible heat flux we divide also by the specific heat at constant pressure, c_p.**

$\rho c_p = 1.216 \times 10^3$ **W m^{-3} / (K m s^{-1}) allows us to convert easily between kinematic heat fluxes and normal heat fluxes.**

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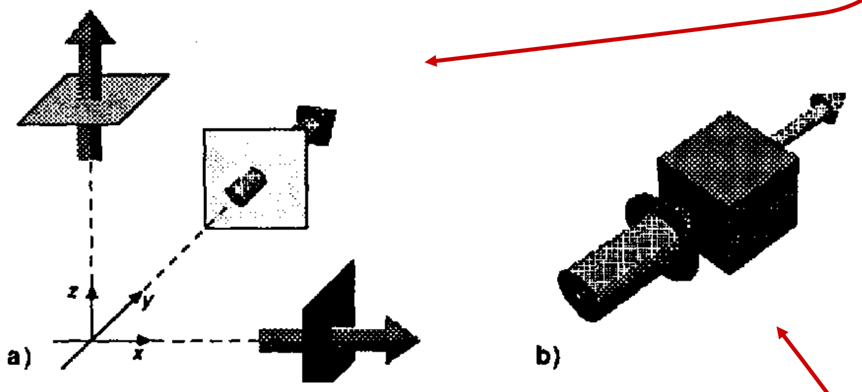
- Most of the fluxes can be split into three components: a vertical component and two horizontal components as in (a):

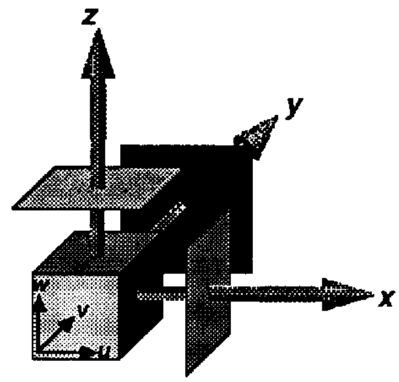


- If a greater flux enters a volume than leaves it, there must be an increase of that quantity within the volume as in (b):

We can picture these fluxes as vectors

- In the case of momentum, the flux in one direction might be the flux of u, v or w $\Rightarrow$ there are nine components of this flux. Each of the three momentum components can pass through a plane normal to any of the three Cartesian directions.

Momentum can be split into the three cartesion directions, based on the u, v, and w components of wind. Momentum flux can consist of the transfer of any of these three components in any of three directions: x, y, and z, yielding a total of nine momentum flux components.



The momentum flux is a (second-order) tensor

- We can split the fluxes into mean and turbulent parts.
- Some of the fluxes associated with the mean wind (i.e. advection) are:
  - vertical kinematic advective heat flux $\overline{w}\overline{\theta}$
  - vertical kinematic advective moisture flux $\overline{w}\overline{q}$
  - kinematic advective heat flux in the x-direction $\overline{u}\overline{\theta}$
  - vertical kinematic advective flux of u-momentum $\overline{w}\overline{u}$

  also the kinematic flux of w-momentum in the x-direction

- Fluxes in other directions can be constructed in a similar way.
- These fluxes have the proper dimensions for kinematic fluxes and make physical sense: for example, a larger vertical velocity or a larger potential temperature both create a larger vertical heat flux, as would be expected intuitively.

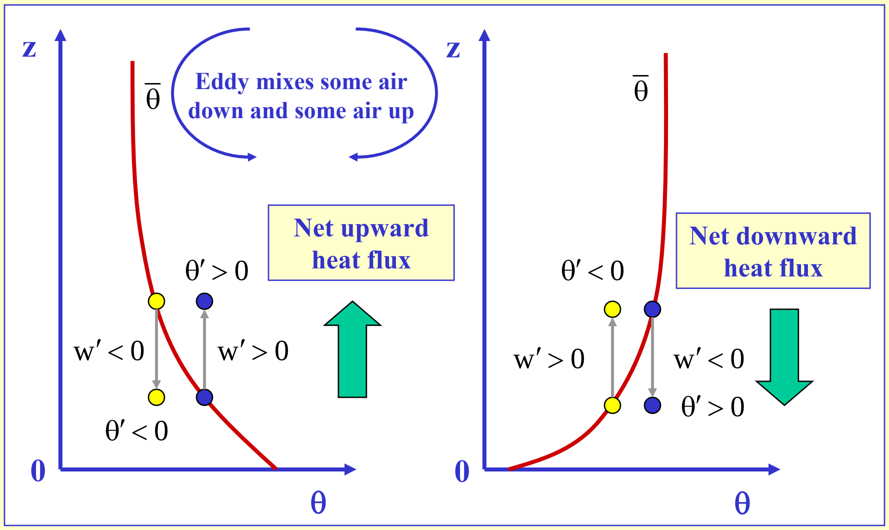
Turbulent transport

- Fluid motion can transport quantities, resulting in fluxes.
- Turbulence involves motion also.
- Thus we expect that turbulence transports quantities as well.

Concepts

- A term like $\overline{w'\theta'}$ looks similar to the kinematic terms, except that the perturbation values are used instead of mean values of w and θ.
- If turbulence is completely random, then a positive w'θ' at one instant might cancel a negative w'θ' at some later instant, resulting in a near zero value for the average turbulent heat flux.

- However, there are situations where the average turbulent flux $\overline{w'\theta'}$ might be significantly different from zero.



Turbulent transport

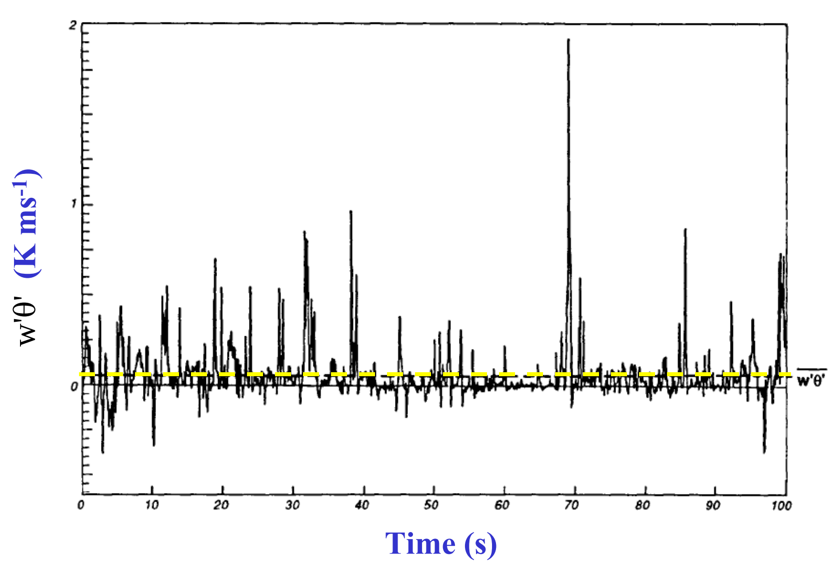
- Note that turbulence can transport heat $\overline{w'\theta'} \neq 0$ although there is no mass transport $\overline{w'} = 0$.
- These form of these fluxes highlights the statistical nature of turbulence: a flux such as $\overline{w'\theta'} \neq 0$ is just a **statistical covariance**.
  - vertical kinematic eddy heat flux $\overline{w'\theta'}$
  - vertical kinematic eddy moisture flux $\overline{w'q'}$
  - kinematic eddy heat flux in the x-direction $\overline{u'\theta'}$
  - vertical kinematic eddy flux of u-momentum $\overline{w'u'}$

also the kinematic eddy flux of w-momentum in the x-direction

Turbulent transport

- It is important to recognize that $\overline{w'} = 0$ throughout most of the BL.
- Thus the vertical advective fluxes are usually negligible compared with the vertical turbulent fluxes.
- No such statement can be made about the horizontal fluxes.
- Turbulence in the real atmosphere usually consists of many large positive and negative values of the instantaneous fluxes, such as the heat flux $w'\theta'$.
- Only after averaging does a smaller, but significant, net flux $\overline{w'\theta'}$ become apparent.

Example



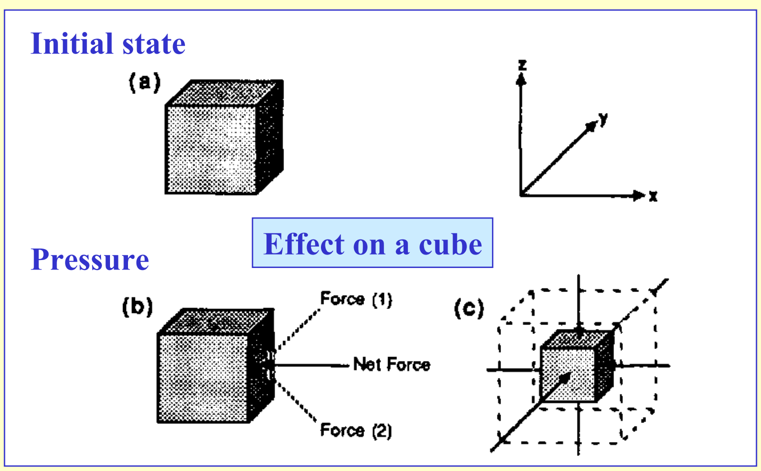
Instantaneous kinematic surface heat flux $w'\theta'$ trace measured by aircraft. Dashed line is the average heat flux $\overline{w'\theta'}$.

Stress

- **Stress** is the force tending to produce deformation in a body.
- It is measured as a force per unit area.
- There are three types of stress that we have to consider:
  - pressure
  - Reynolds stress, and
  - viscous shear stress.

Pressure

- **Pressure** is a familiar concept: it is isotropic (i.e. independent of direction) and therefore a **scalar** quantity.



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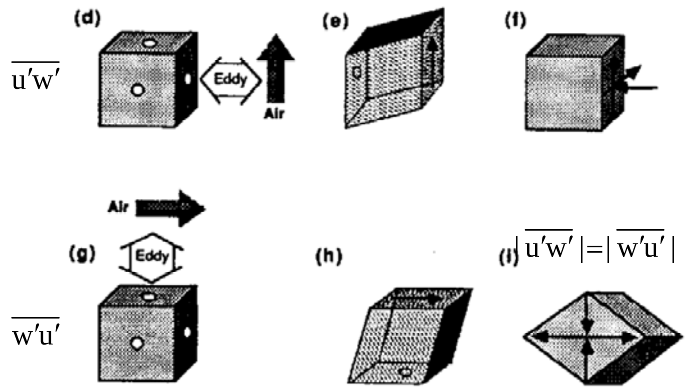
Reynolds stress

- **Reynolds stress** exists only when the fluid is in turbulent (or wavy) motion.

Effect on a cube

Reynolds stress



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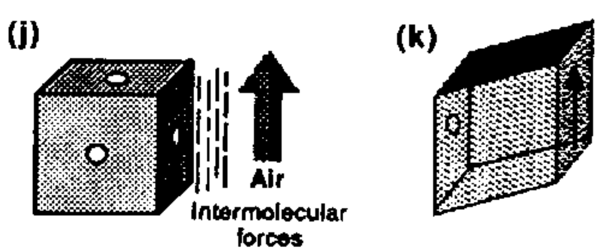
Viscous stress

- **Viscous shear stress** exists when there are shearing motions in the fluid. The motion can be laminar or turbulent.

Viscous shear stress

Effect on a cube



- In this case, molecular motions rather than turbulent eddies are responsible for the transport of momentum.

Viscous stress

- For a fluid for the viscous stress is linearly dependent on the shear (deformation) is called a **Newtonian fluid**. The stress, τ_{ij}, is given by:

$$\tau_{ij} = \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) + (\mu_B - \tfrac{2}{3}\mu)\delta_{ij}\frac{\partial u_k}{\partial x_k}$$

μ: Dynamic viscosity coefficient

μ_B: Bulk viscosity coefficient

μ_B is near zero for most gases

- Interpret τ_{ij} as the force per unit area in the x_i-direction acting on the face that is normal to the x_j-direction.

Viscous stress

- The viscous stress can be put into kinematic form: the **kinematic viscosity** is $\nu = \mu/\rho$.
- The standard atmosphere sea-level value for air is: $\nu = 1.4607 \times 10^{-5}$ m^2 s^{-1}.
- For a mean wind shear of $du/dz = 0.5$ s^{-1} (typical for atmospheric surface layers), the resulting viscous stress is $\tau/\rho = 7.304 \times 10^{-6}$ m^2 s^{-2}.
- This value is so much smaller than the Reynolds stresses in the BL that the viscous stress is usually neglected in mean wind forecasts. However, turbulent eddies can have much larger values of shear in localized eddy-size regions. Thus **we cannot ignore viscosity when forecasting turbulence**.

Friction velocity

- During situations where turbulence is generated or modulated by wind shear near the ground, the magnitude of the surface Reynolds stress proves to be an important scaling variable.
- The total vertical flux of horizontal momentum near the surface is given by:

$$\tau_{xz} = -\bar{\rho}_s \overline{u'w'} \quad \text{and} \quad \tau_{yz} = -\bar{\rho}_s \overline{v'w'}$$

$$|\boldsymbol{\tau}_s| = \sqrt{\tau_{xz}^2 + \tau_{yz}^2}$$

Define a friction velocity, u_*, by:

$$u_*^2 = |\boldsymbol{\tau}_s| / \bar{\rho}_s = \sqrt{(\overline{u'w'})^2 + (\overline{v'w'})^2}$$

Friction velocity and other surface scales

- For the special case where the coordinate system is aligned so that the x-axis points in the direction of the surface stress, we can write the friction velocity as

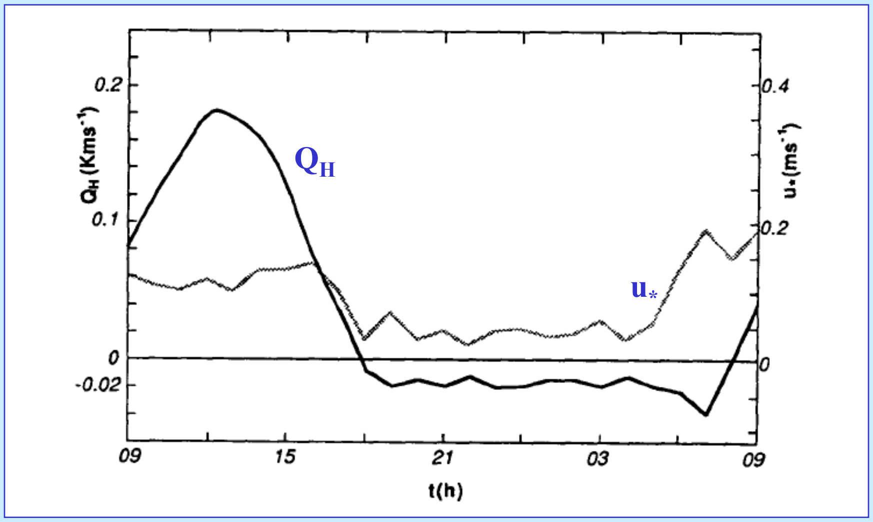
$$u_*^2 = |\overline{u'w'}|_s = |\boldsymbol{\tau}_s| / \bar{\rho}$$

- Similarly we can define a surface layer temperature (θ_*^{SL}) and specific humidity (q_*^{SL}) scales defined by:

$$\theta_*^{SL} = \frac{-\overline{w'\theta'}|_s}{u_*}$$

$$q_*^{SL} = \frac{-\overline{w'q'}|_s}{u_*}$$

Diurnal variation of friction velocity



Example of diurnal variation of kinematic heat flux and friction velocity.

Topics covered so far

- Some mathematical tools: Statistics
- The turbulence spectrum
- Energy cascade, The spectral gap
- Mean and turbulent parts of the flow
- Some basic statistical methods
- Types of averaging
- The rules of averaging
- Variance, covariance and correlation
- Turbulence intensity, turbulent transport
- Reynolds stress, viscous stress, friction velocity


End