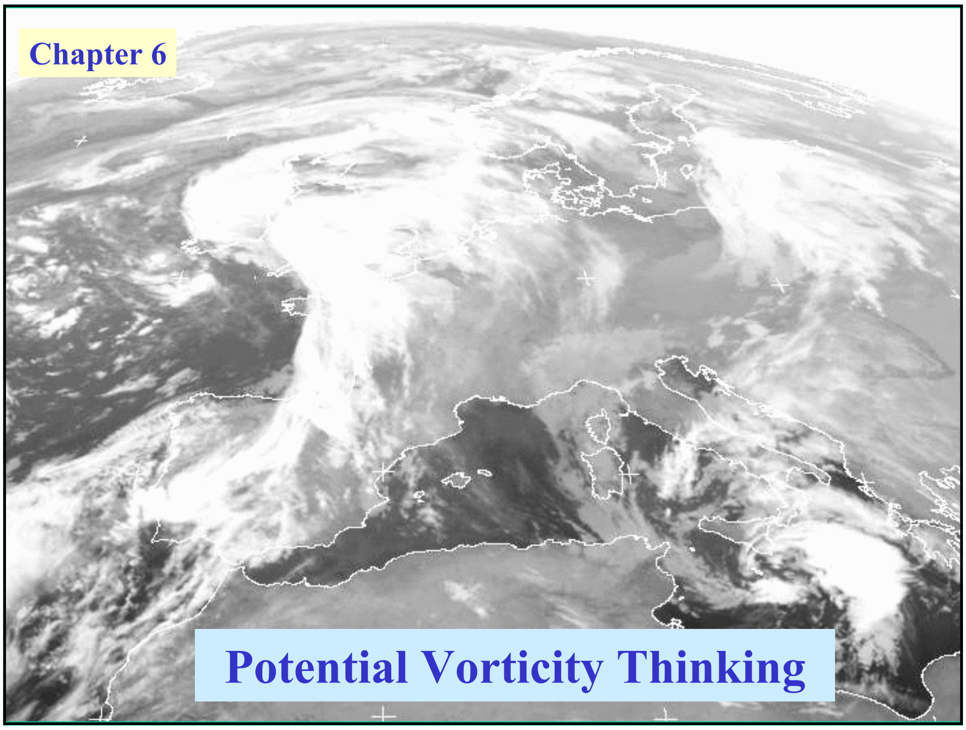
Chapter 6



Potential Vorticity Thinking

QUARTERLY JOURNAL

OF THE

ROYAL METEOROLOGICAL SOCIETY

Vol. 111 OCTOBER 1985 No. 470

Quart. J. R. Met. Soc. (1985), **111**, pp. 877–946 551.509.3:551.511.2:551.511.32

On the use and significance of isentropic potential vorticity maps

By B. J. HOSKINS[1], M. E. McINTYRE[2] and A. W. ROBERTSON[3]

[1] *Department of Meteorology, University of Reading*

[2] *Department of Applied Mathematics and Theoretical Physics, University of Cambridge*

[3] *Laboratoire de Physique et Chimie Marines, Université Pierre et Marie Curie*, 75230 *Paris Cédex* 05

(Received 12 February 1985; revised 2 July 1985)

Ertel potential vorticity

Define $P = \frac{1}{\rho}\zeta_a \cdot \nabla\theta$ — The Ertel potential vorticity

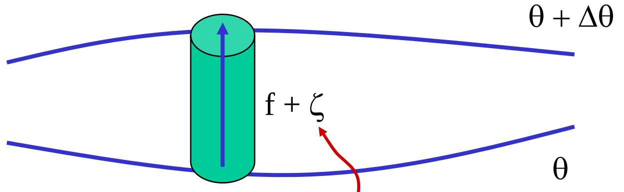
ζ_a: The 3D absolute vorticity $\mathbf{f} + \omega$

Ertel's theorem: for frictionless adiabatic motion

$$\frac{DP}{Dt} = 0$$

⇒ EPV is conserved following fluid parcels

EPV in isentropic coordinates



$$P = -g(\underbrace{f + \mathbf{k} \cdot \nabla_\theta \wedge \mathbf{u}}_{f+\zeta})\frac{\partial\theta}{\partial p}$$

$$\frac{\partial P}{\partial t} + \mathbf{u} \cdot \nabla_\theta P = 0$$

Standard PV distribution

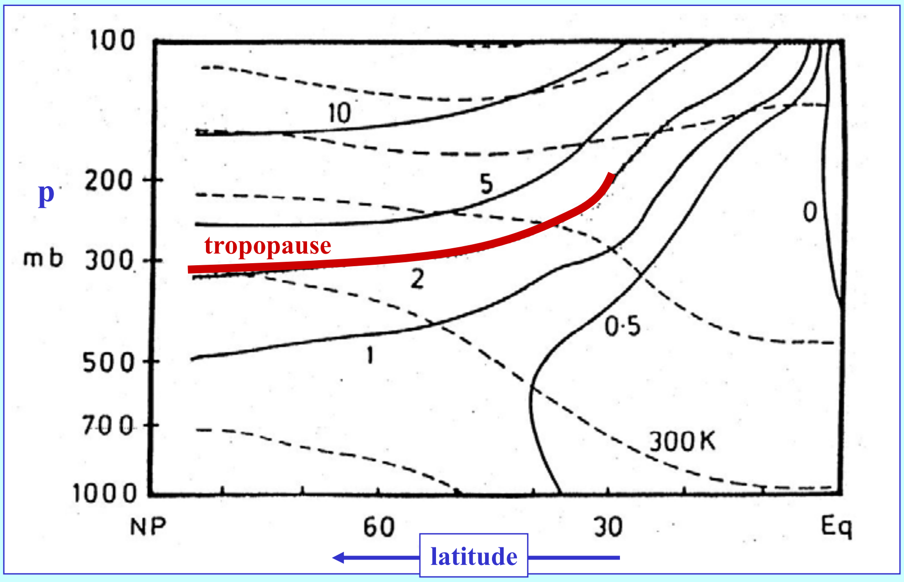
$$P = -g(f + \mathbf{k} \cdot \nabla_\theta \wedge \mathbf{u}) \frac{\partial \theta}{\partial p}$$

Standard distribution

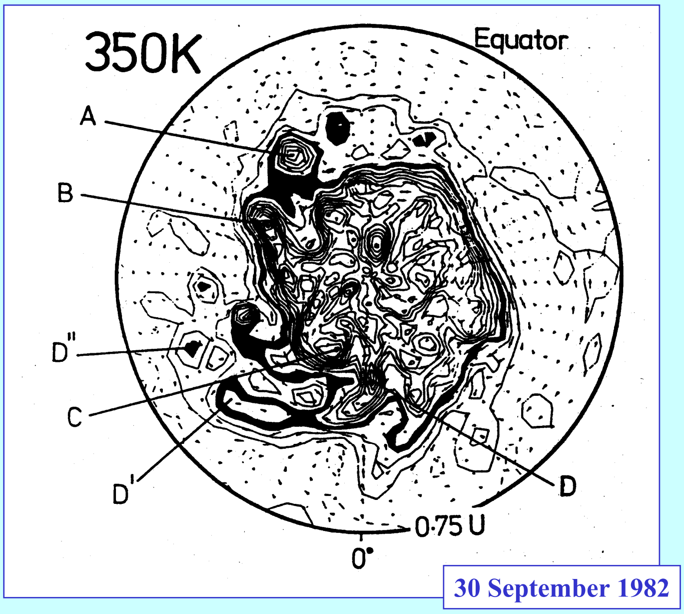
$$P = -fg \frac{\partial \theta}{\partial p} = \frac{f}{\rho} \frac{\partial \theta}{\partial z}$$

1 **PV unit =** $10^{-6}\ m^2 s^{-1}\ K\ kg^{-1} \approx 10$ K **per** 100 **mb at** 45^o **lat.**

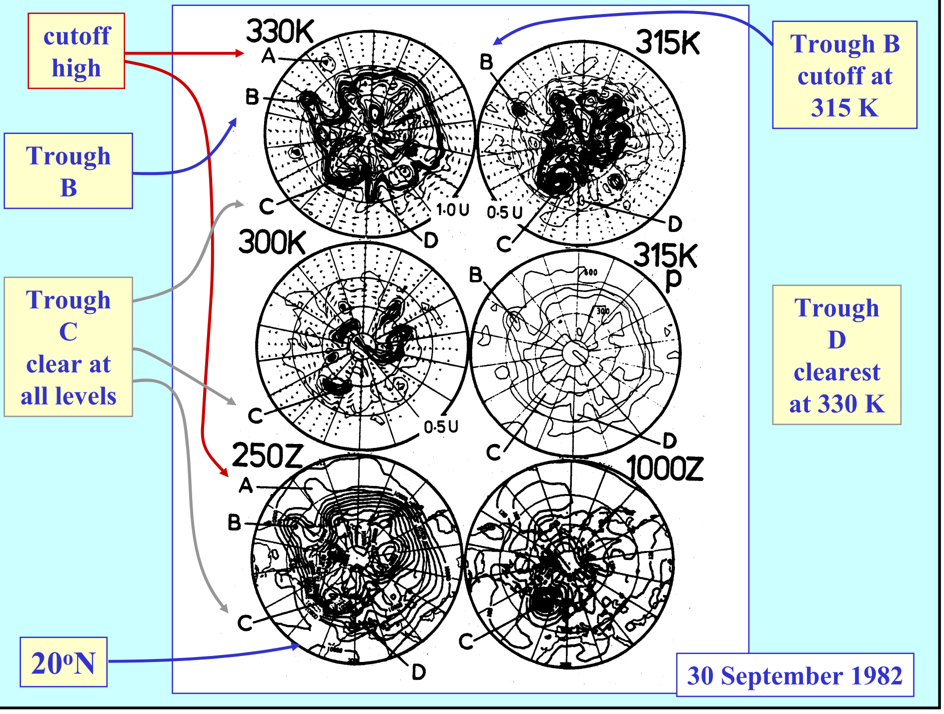
Mean meridional distribution of PV



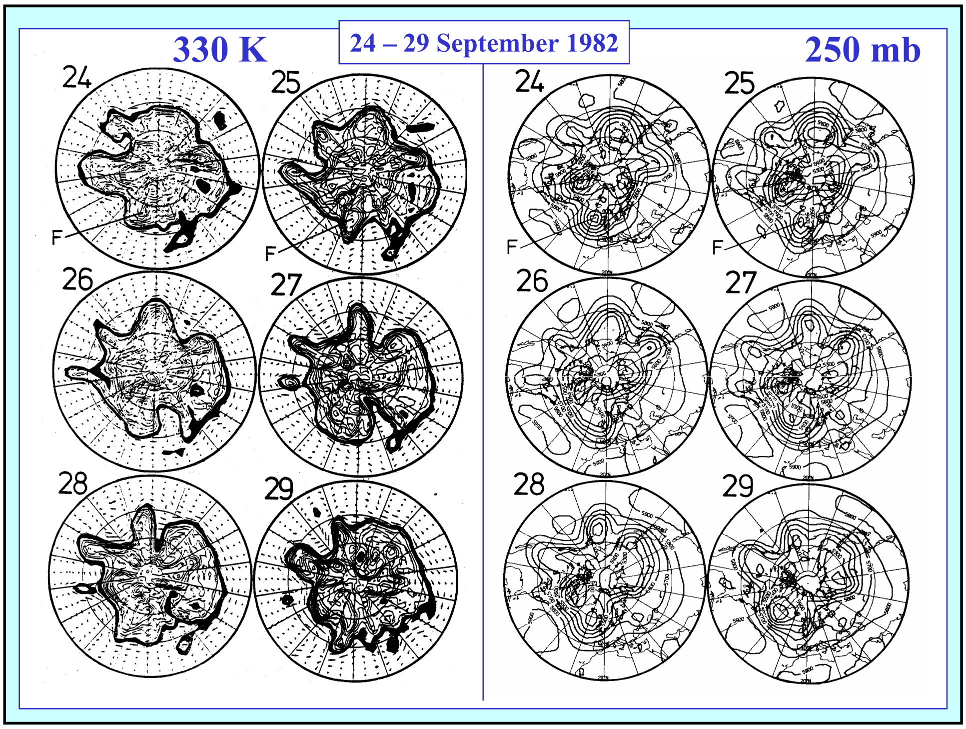
A PV chart



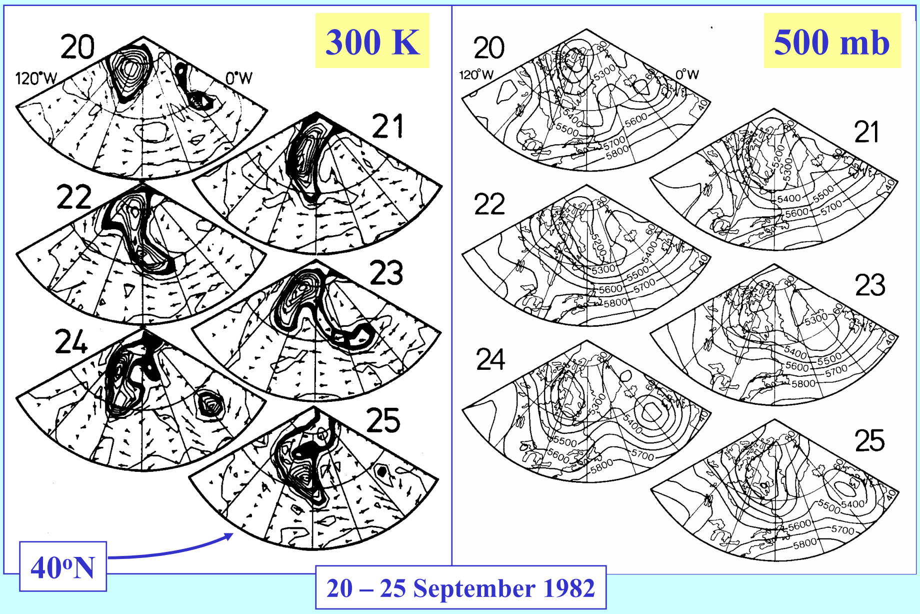
30 September 1982



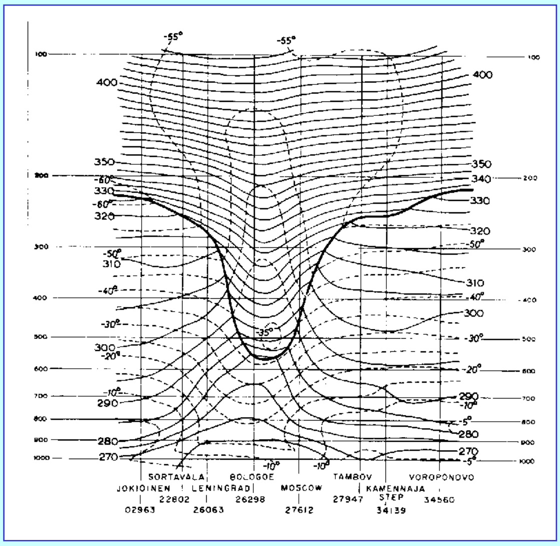
30 September 1982

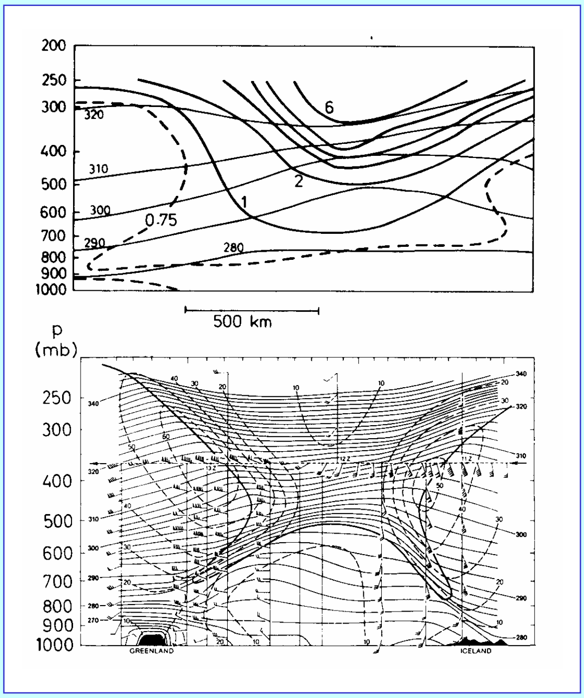


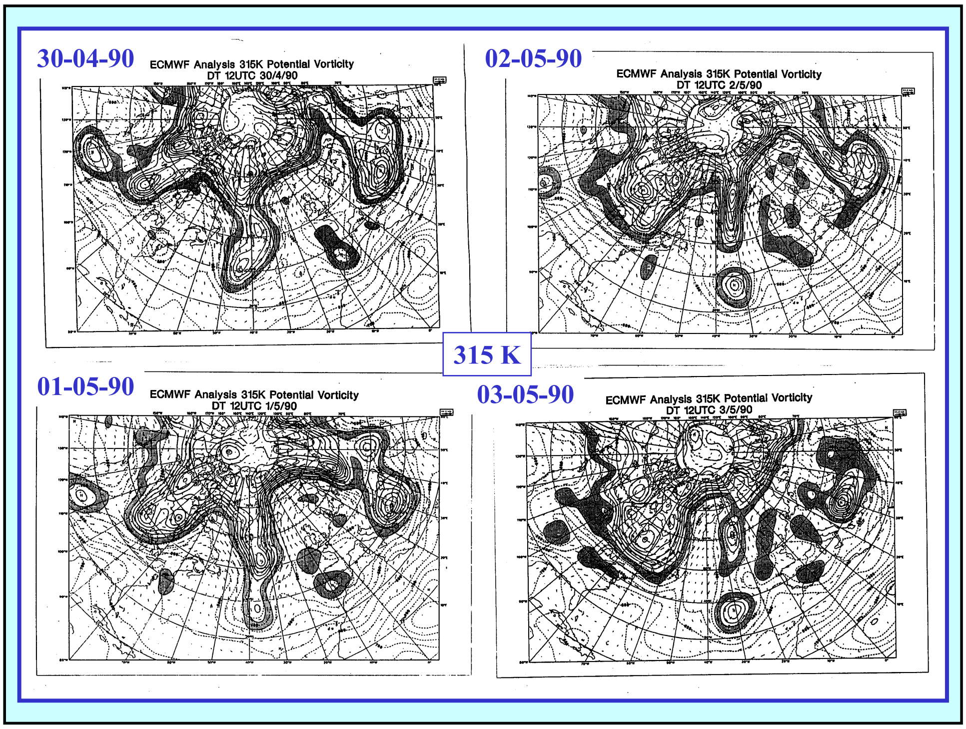
Development of an Atlantic cutoff low

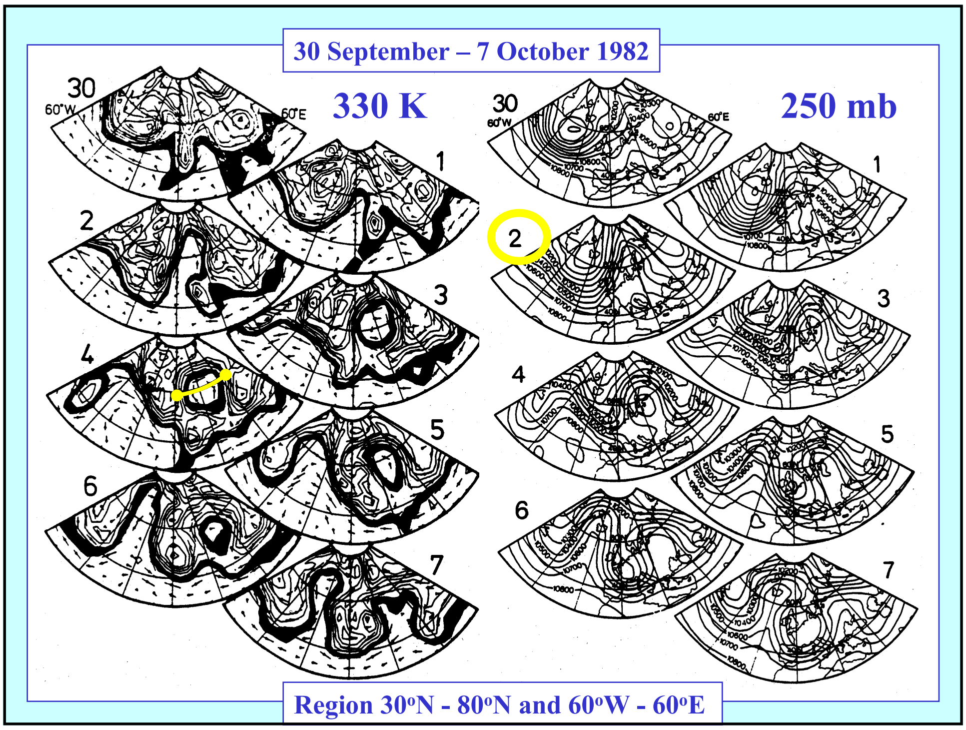


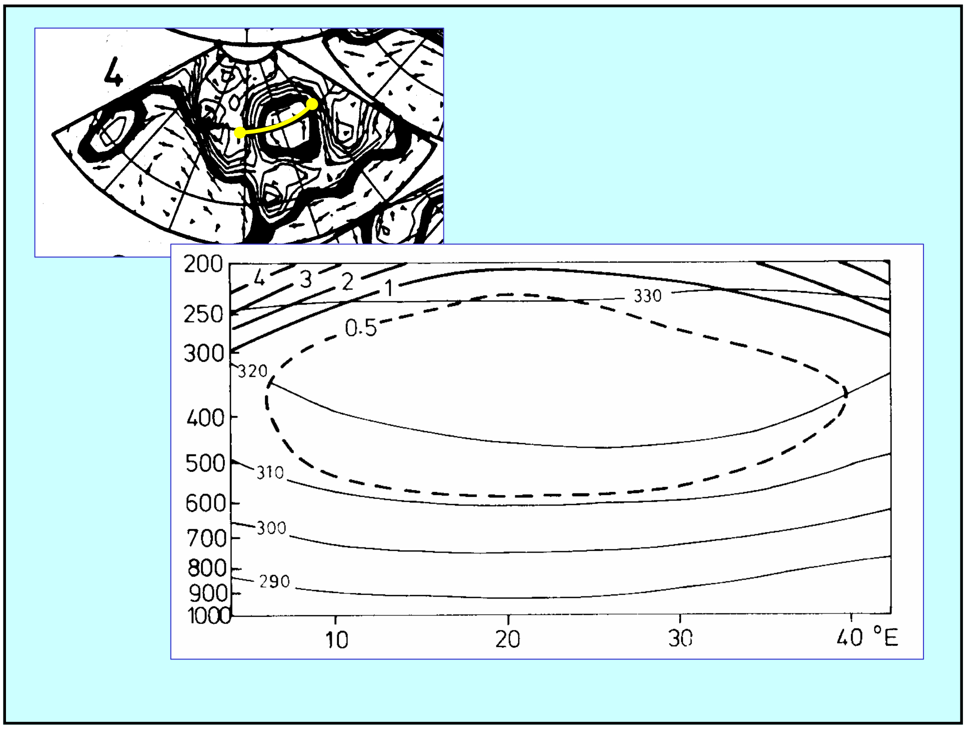
Vertical structure through a cutoff low

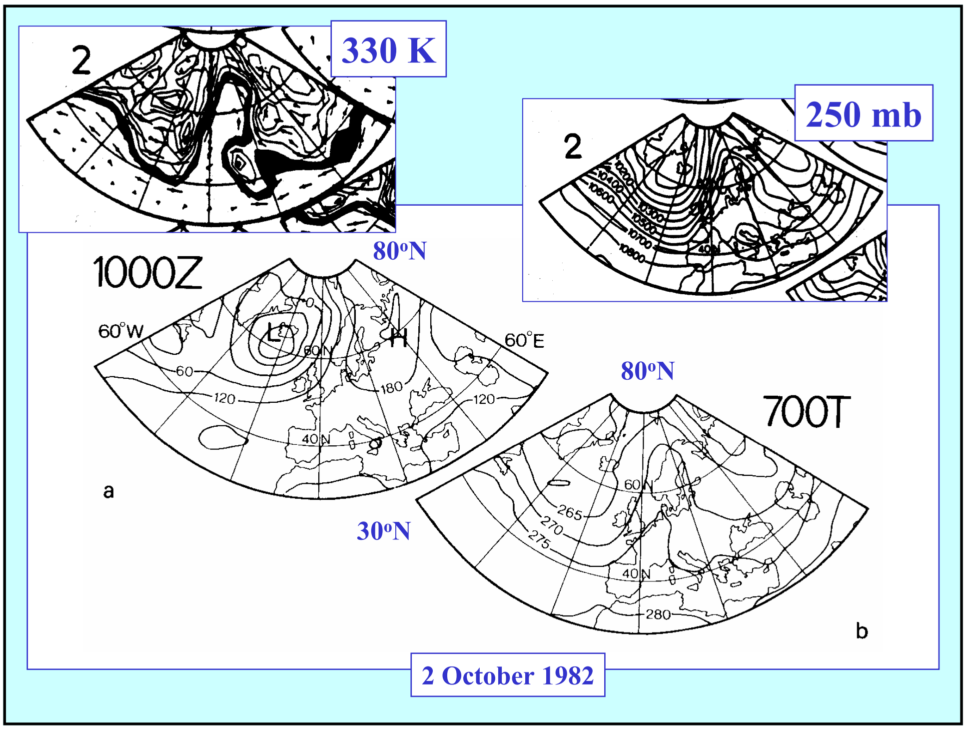





30-04-90
ECMWF Analysis 315K Potential Vorticity
DT 12UTC 30/4/90
02-05-90
ECMWF Analysis 315K Potential Vorticity
DT 12UTC 2/5/90
315 K
01-05-90
ECMWF Analysis 315K Potential Vorticity
DT 12UTC 1/5/90
03-05-90
ECMWF Analysis 315K Potential Vorticity
DT 12UTC 3/5/90

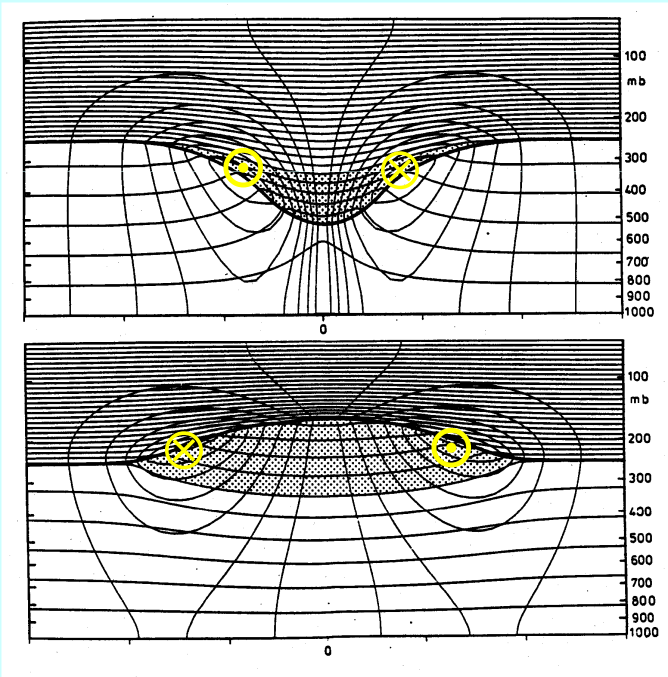

30 September – 7 October 1982
330 K
250 mb
30
60°W
60°E
1
2
3
4
5
6
7
Region 30°N - 80°N and 60°W - 60°E


4
200
250
300
400
500
600
700
800
900
1000
4
3
2
1
0.5
330
320
310
300
290
10
20
30
40 °E


2
330 K
250 mb
1000Z
80°N
60°W
60°E
L
H
60N
40N
a
30°N
80°N
700T
265
270
275
60N
40N
280
b
2 October 1982

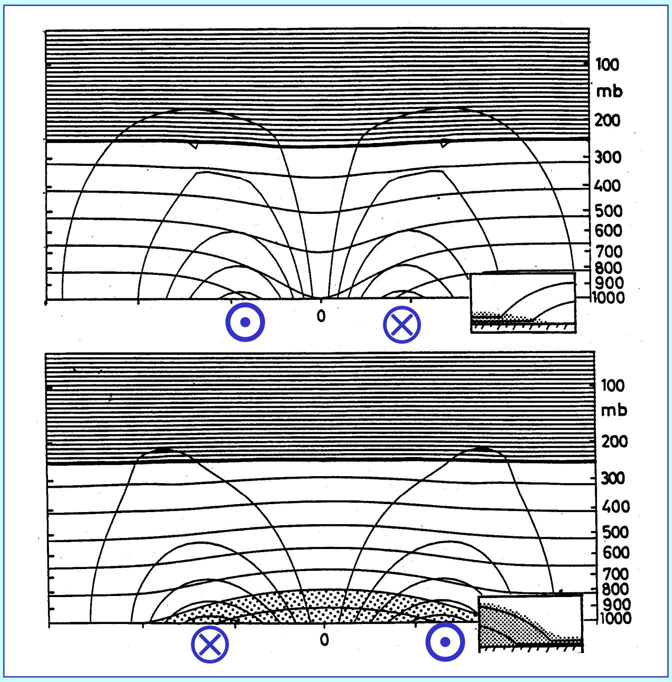
+ PV anomaly

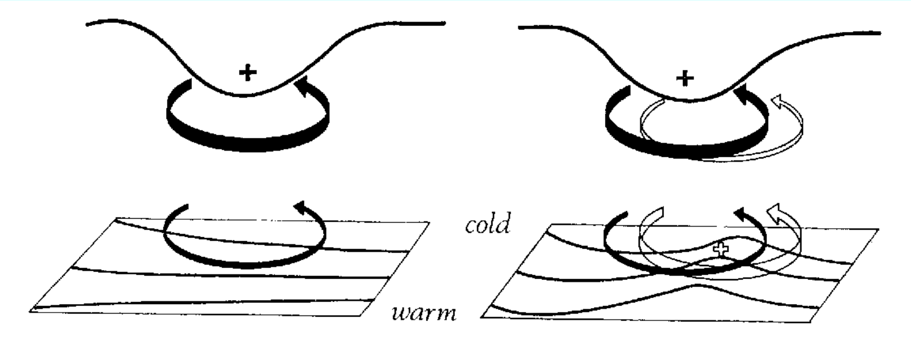
– PV anomaly



+ PV anomaly

– PV anomaly





Elements of PV thinking

- **PV anomaly:**
  - **defined as a deviation of PV contours from a background or reference state.**
  - **e.g. troughs may be defined as positive PV anomalies (NH) resulting from equatorward displacement of PV contours relative to reference state.**
- **Conservation:**
  - **emphasizes dynamical properties of flow features that depend on their material nature (e.g. propagation of Rossby waves arising from displacement of PV contours; motion of vortices due to advection of isolated regions of fluid).**

Elements of PV thinking

- **Invertibility:**
  - Given specification of a reference state, balance condition, and boundary conditions, the PV field uniquely determines (i.e. induces) the flow field.
  - Allows inference of action at a distance.
- **Attributability:**
  - PV field may be partitioned in a piecewise sense, allowing consideration of interactions among the respective constituents through their induced flow fields.

Elements of PV thinking

- **Scale effect:**
  - For a given magnitude of PV, small-scale features contribute weakly to the velocity field induced by a given PV anomaly, whereas large-scale features contribute strongly to the velocity field.
  - Scale effect depends also on the anisotropy of a PV anomaly (i.e. maximized for isotropic anomalies and reduced for increasing anisotropy).

Mechanisms for system evolution

- **Rossby-wave dispersion:**
  - **Referred to as downstream development; it is a consequence of the property of Rossby/PV waves and tropopause-based edge waves that $c_{group} > c_{phase}$, resulting in the sequential formation of troughs and ridges in the downstream direction and dissipation in the upstream direction.**

$$c_{phase} = U - \frac{\beta}{k^2}, \qquad c_{group} = U + \frac{\beta}{k^2}$$

Mechanisms for system evolution

- **Superposition:**
  - **Increase in the total perturbation energy arising from a reconfiguration of a given PV anomaly (e.g. through axisymmetrization in a deformation flow) or from a change in the relative position between separate PV anomalies.**
  - **Perturbation enstrophy is conserved**

Mechanisms for system evolution

- **Exponential (modal) growth:**
  - **Mutual intensification of counter-propagating wave trains on opposite-signed basic-state PV gradients in the presence of background vertical shear.**
  - **Characterized by fixed vertical structure resulting from phase locking of wave trains.**
  - **Total perturbation energy and enstrophy increase.**