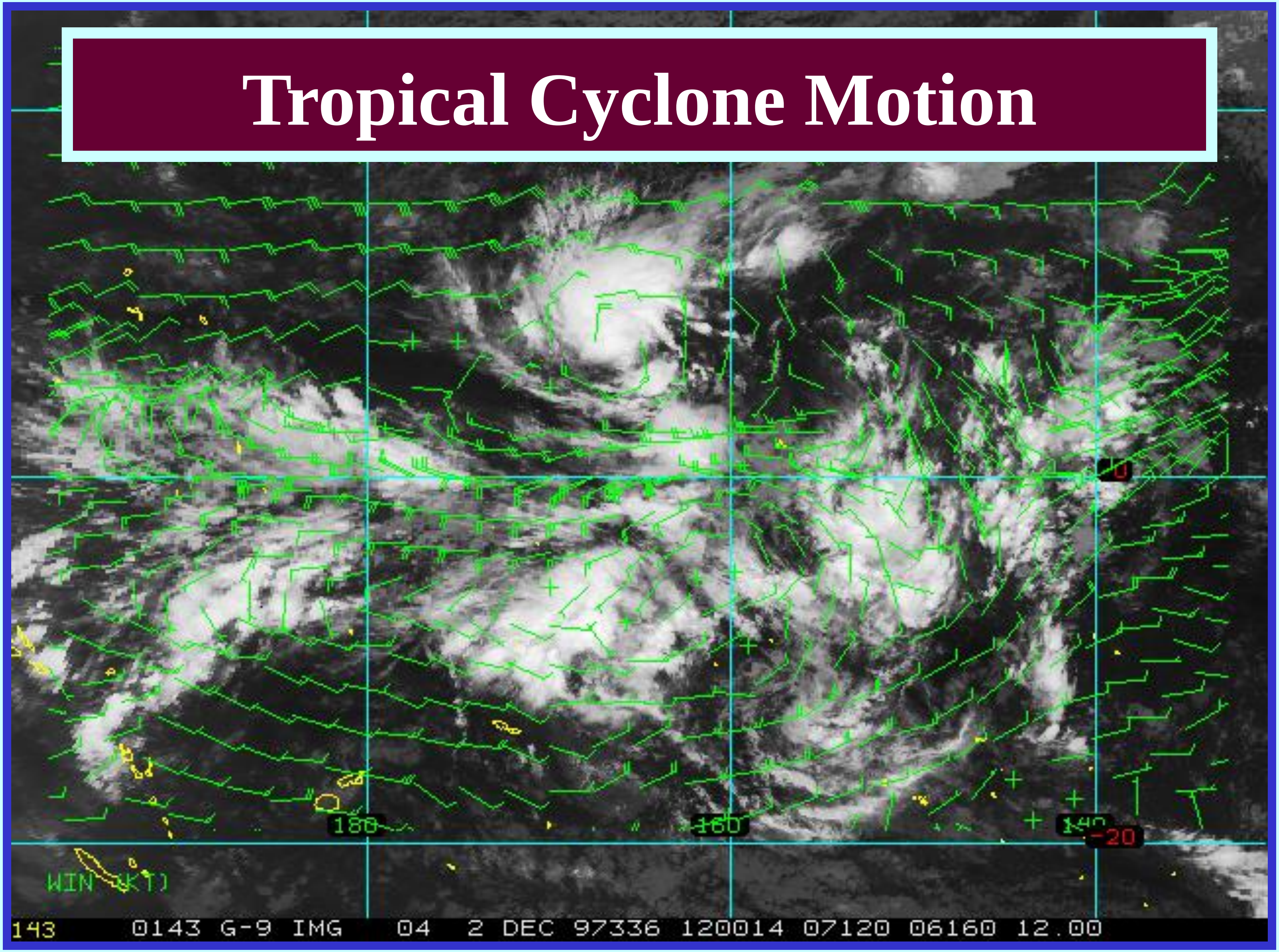
Tropical Cyclone Motion



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A numerical study of tropical cyclone motion using a barotropic model. I: The role of vortex asymmetries

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An Analytical Theory of Tropical Cyclone Motion Using a Barotropic Model

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Vortex motion in relation to the absolute vorticity gradient of the vortex environment

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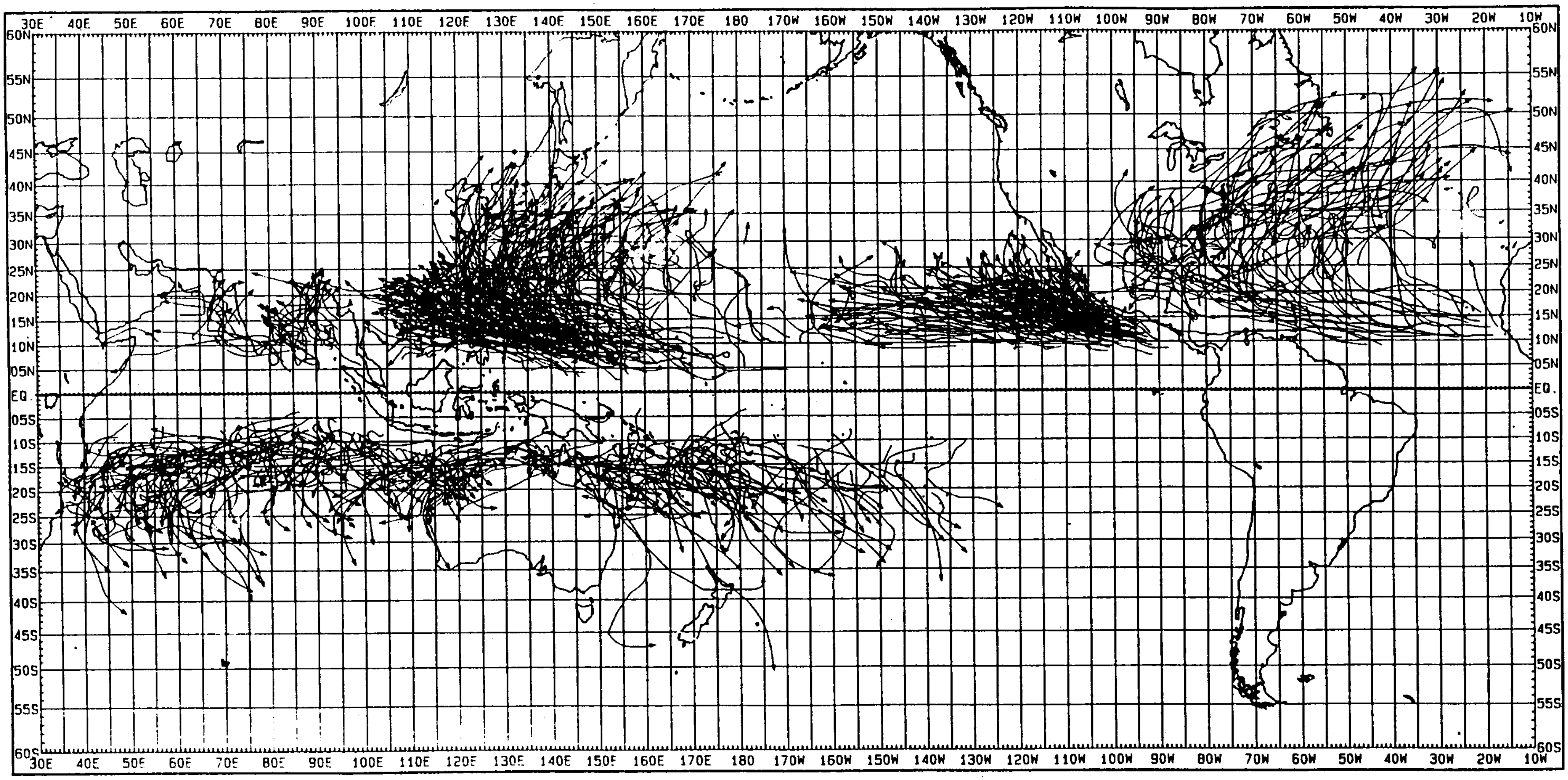
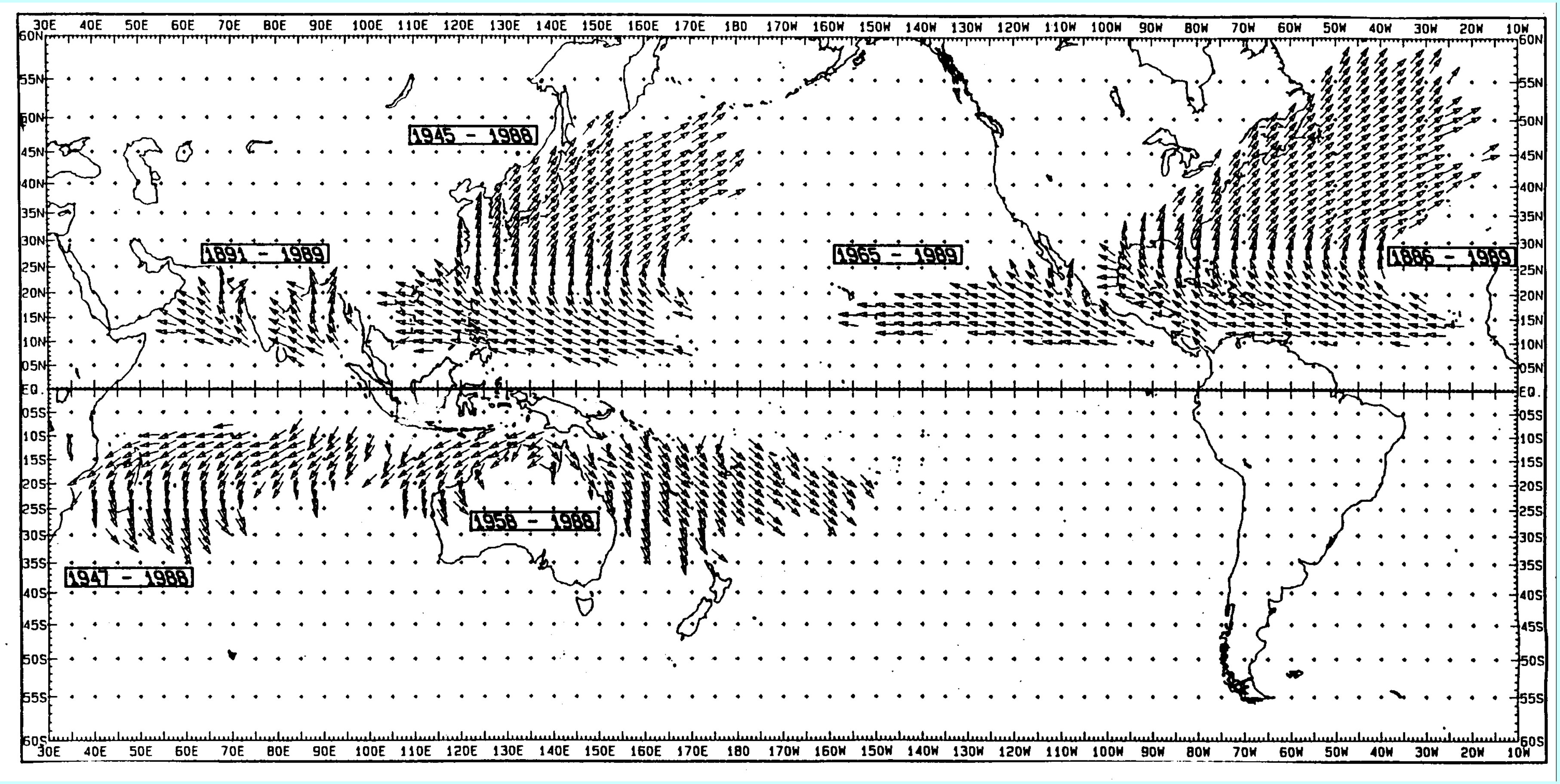
Tropical cyclone tracks (1979-1988)



Mean direction of TC motion



Two-dimensional barotropic flow

$$\frac{\partial}{\partial t}(\zeta + f) + u\frac{\partial}{\partial x}(\zeta + f) + v\frac{\partial}{\partial y}(\zeta + f) = 0, \tag{1.1}$$

where u and v are the velocity components in the x and y directions, respectively. For an incompressible fluid, the continuity equation is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} = 0, \tag{1.2}$$

and accordingly there exists a streamfunction ψ such that

$$u = -\frac{\partial \psi}{\partial y}, \quad v = \frac{\partial \psi}{\partial x}, \tag{1.3}$$

and

$$\zeta = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}, \tag{1.4}$$

The partitioning problem

The partitioning method can be illustrated mathematically as follows. Let the total wind be expressed as $\mathbf{u} = \mathbf{u_s} + \mathbf{U}$, where $\mathbf{u_s}$ denotes the symmetric velocity field and $\mathbf{U}$ is the vortex environment vorticity, and define $\zeta_s = \mathbf{k} \cdot \nabla \wedge \mathbf{u_s}$ and $\Gamma = \mathbf{k} \cdot \nabla \wedge \mathbf{U}$, where $\mathbf{k}$ is the unit vector in the vertical. Then Eq. (1.1) can be partitioned into two equations:

$$\frac{\partial \zeta_s}{\partial t} + \mathbf{c}(t) \cdot \nabla \zeta_s = 0, \tag{1.5}$$

and

$$\frac{\partial \Gamma}{\partial t} = -\mathbf{u_s} \cdot \nabla(\Gamma + f) - (\mathbf{U} - \mathbf{c}) \cdot \nabla \zeta_s - \mathbf{U} \cdot \nabla(\Gamma + f), \tag{1.6}$$

noting that $\mathbf{u_s} \cdot \nabla \zeta_s = 0$, because for a symmetric vortex $\mathbf{u_s}$ is normal to $\nabla \zeta_s$. Equation (1.5) states that the symmetric vortex translates with speed $\mathbf{c}$ and Eq. (1.6) is an equation for the evolution of the asymmetric vorticity. Having solved the latter equation for $\Gamma(\mathbf{x}, t)$, we can obtain the corresponding asymmetric streamfunction by solving Eq. (1.4) in the form $\nabla^2 \psi_a = \Gamma$. The vortex translation velocity $\mathbf{c}$ may be obtained by calculating the speed $\mathbf{U_c} = \mathbf{k} \wedge \nabla \psi_a$ at the vortex centre. In some

The partitioning problem

be obtained by calculating the speed $\mathbf{U_c} = \mathbf{k} \wedge \nabla \psi_a$ at the vortex centre. In some situations it is advantageous to transform the equations of motion into a frame of reference moving with the vortex[2]. Then Eq. (1.5) becomes $\partial \zeta_s / \partial t \equiv 0$ and the vorticity equation (1.6) becomes

$$\frac{\partial \Gamma}{\partial t} = -\mathbf{u_s} \cdot \nabla(\Gamma + f) - (\mathbf{U} - \mathbf{c}) \cdot \nabla \zeta_s - (\mathbf{U} - \mathbf{c}) \cdot \nabla(\Gamma + f). \qquad (1.7)$$

Symmetric vortex in a uniform flow

Consider a barotropic vortex with an axisymmetric vorticity distribution embedded in a uniform zonal air stream on an f-plane. The streamfunction for the flow has the form:

$$\psi(x,y) = -Uy + \psi'(r), \tag{1.8}$$

where $r^2 = (x - Ut)^2 + y^2$. The corresponding velocity field is

$$\mathbf{u} = (U,0) + \left(-\frac{\partial \psi'}{\partial y}, \frac{\partial \psi'}{\partial x}\right), \tag{1.9}$$

The relative vorticity distribution, $\zeta = \nabla^2\psi$, is symmetric about the point $(x - Ut, 0)$, which translates with speed U in the x-direction. However, neither the streamfunction distribution $\psi(x,y,t)$, nor the pressure distribution $p(x,y,t)$, are symmetric and, in general, the locations of the minimum central pressure, maximum relative vorticity, and minimum streamfunction (where $\mathbf{u} = \mathbf{0}$) do not coincide. In particular, there are three important deductions from (1.9):

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- The total velocity field of the translating vortex is not symmetric, and
- The maximum wind speed is simply the arithmetic sum of U and the maximum tangential wind speed of the symmetric vortex, $V_m = (\partial \psi' / \partial r)_{max}$.
- The maximum wind speed occurs on the right-hand-side of the vortex in the direction of motion in the northern hemisphere and on the left-hand-side in the southern hemisphere.

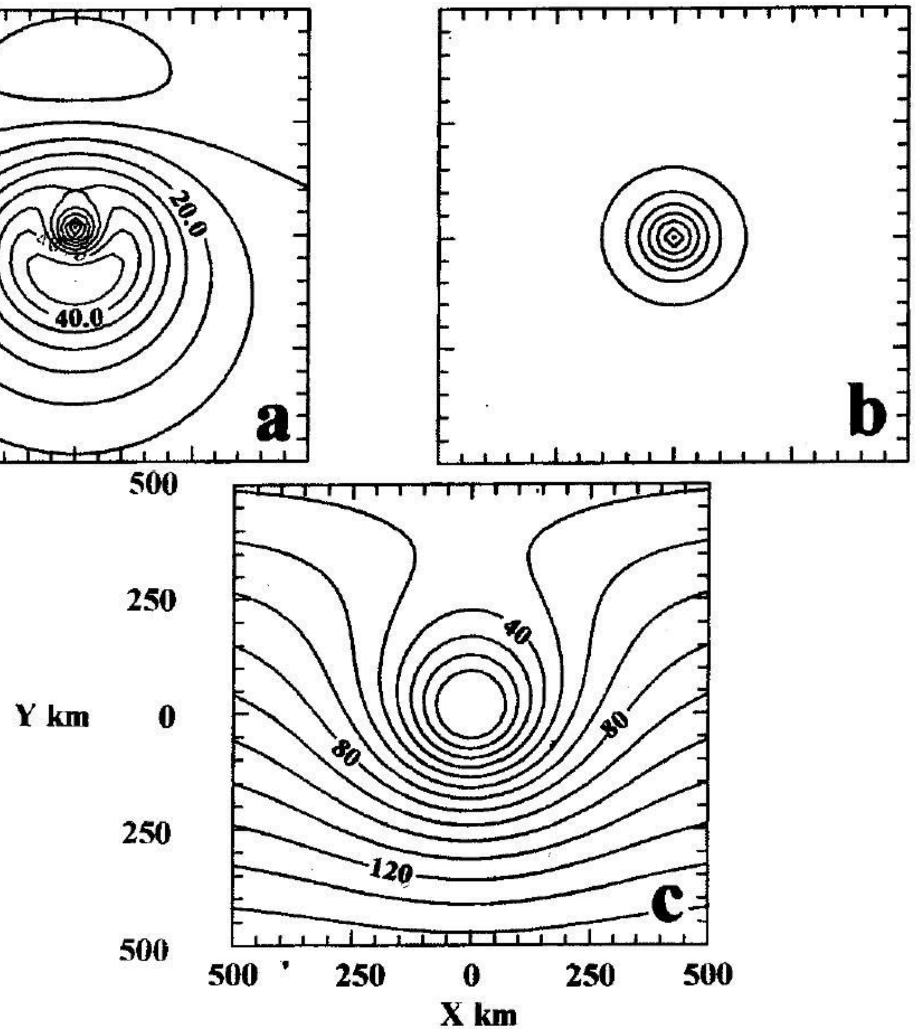


Figure 1.1: Contour plots of (a) total wind speed, (b) relative vorticity, and (c) streamlines, for a vortex with a symmetric relative vorticity distribution and maximum tangential wind speed of 40 m s^{-1} in a uniform zonal flow with speed 10 m s^{-1} on an f-plane. The maximum tangential wind speed occurs at a radius of 100 km (for the purpose of illustration). The contour intervals are: 5 m s^{-1} for wind speed, 2×10^{-4} s^{-1} for relative vorticity and 1×10^{4} m^2 s^{-1} for streamfunction.

Because the vorticity field is Galilean invariant while the pressure field and streamfunction fields are not, it is advantageous to define the vortex centre as the location of maximum relative vorticity and to transform the equations of motion to a coordinate system $(X, Y) = (x - Ut, y)$, whose origin is at this centre[3]. In this frame of reference, the streamfunction centre is at the point $(0, Y_s)$, where

$$U - \Phi(Y_s)Y_s = 0, \tag{1.10}$$

and $\Phi = \psi'(r)/r$. This point is to the left of the vorticity centre in the direction of motion in the northern hemisphere. In the moving coordinate system, the momentum equations may be written in the form

$$\nabla p = \rho\Phi(\Phi + f)(X, Y) + \rho f(0, U). \tag{1.11}$$

The minimum surface pressure occurs where $\nabla p = 0$, which from (1.11) is at the point $(0, Y_p)$ where

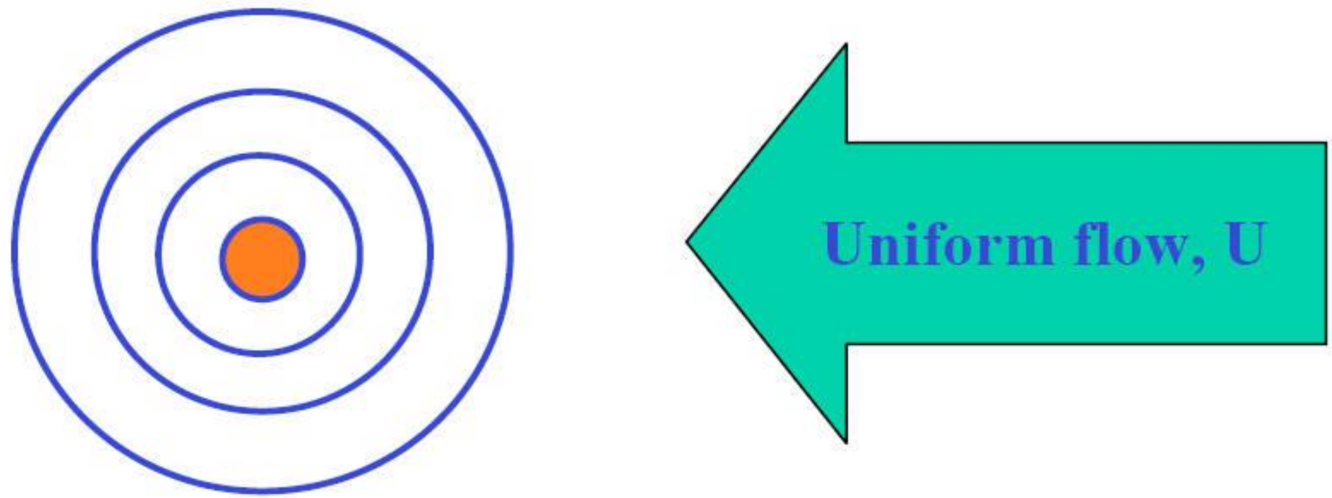
$$Y_p\Phi(Y_p)(\Phi(Y_p) + f) = fU. \tag{1.12}$$

We show that, although Y_p and Y_s are not zero and not equal, they are for practical purposes relatively small.

Consider the case where the inner core is in solid body rotation out to the radius r_m, of maximum tangential wind speed v_m, with uniform angular velocity $\Omega = v_m/r_m$. Then $\psi'(r) = \Omega r$ and $\Phi = \Omega$. It follows readily that $Y_s/r_m = U/v_m$ and $Y_p/r_m = U/(v_m Ro_m)$, where $Ro_m = v_m/(r_m f)$ is the Rossby number of the vortex core which is large compared with unity in a tropical cyclone. Taking typical values: $f = 5 \times 10^{-5}\ s^{-1}$, $U = 10$ m s^{-1}, $v_m = 50$ m s^{-1}, $r_m = 50$ km, $Ro_m = 20$ and $Y_s = 10$ km, $Y_p = 0.5$ km, the latter being much smaller than r_m. Clearly, for weaker vortices (smaller v_m) and/or stronger basic flows (larger U), the values of Y_s/r_m and Y_p/r_m are comparatively larger and the difference between the various centres may be significant.

Tropical cyclone motion

f-plane



Symmetric vortex $\boldsymbol{\zeta} = \boldsymbol{\zeta}(\mathbf{r},\mathbf{t}),\ \mathbf{v} = \mathbf{v}(\mathbf{r},\mathbf{t})$

$$\frac{\partial \zeta}{\partial t} + \mathbf{u} \cdot \nabla \zeta = 0$$

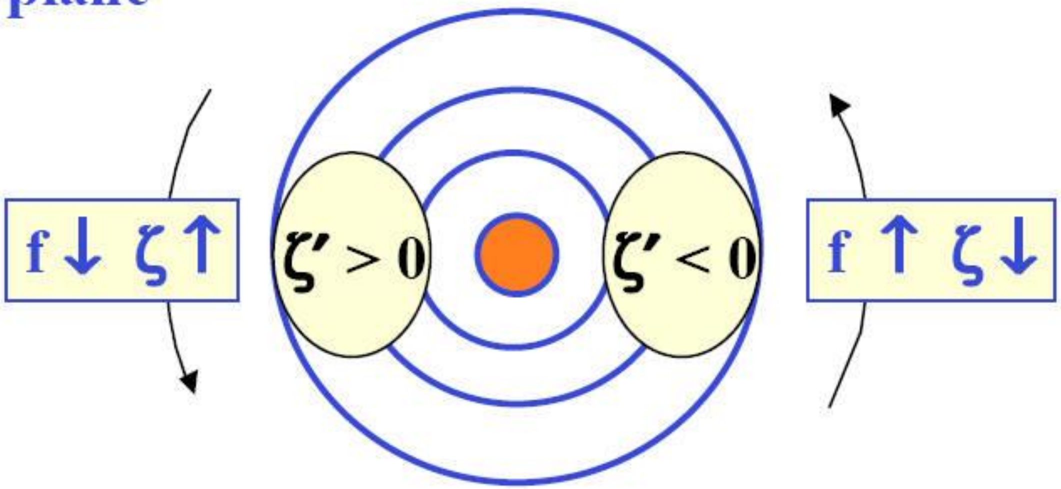
$$\mathbf{u} = U\mathbf{i} + \mathbf{v}$$

$$\mathbf{v} \cdot \nabla \zeta = 0 \quad \Rightarrow \quad \frac{\partial \zeta}{\partial t} + U \frac{\partial \zeta}{\partial x} = 0$$

Tropical cyclone motion

β-plane

$\mathbf{f = f_o + \beta y}$



Symmetric vortex $\boldsymbol{\zeta = \zeta(r,t), v = v(r,t)}$

$$\frac{\partial \zeta}{\partial t} + \mathbf{u} \cdot \nabla \zeta + \beta y = 0 \qquad \mathbf{v} \cdot \nabla \zeta \approx 0$$

Vortex motion on a beta-plane

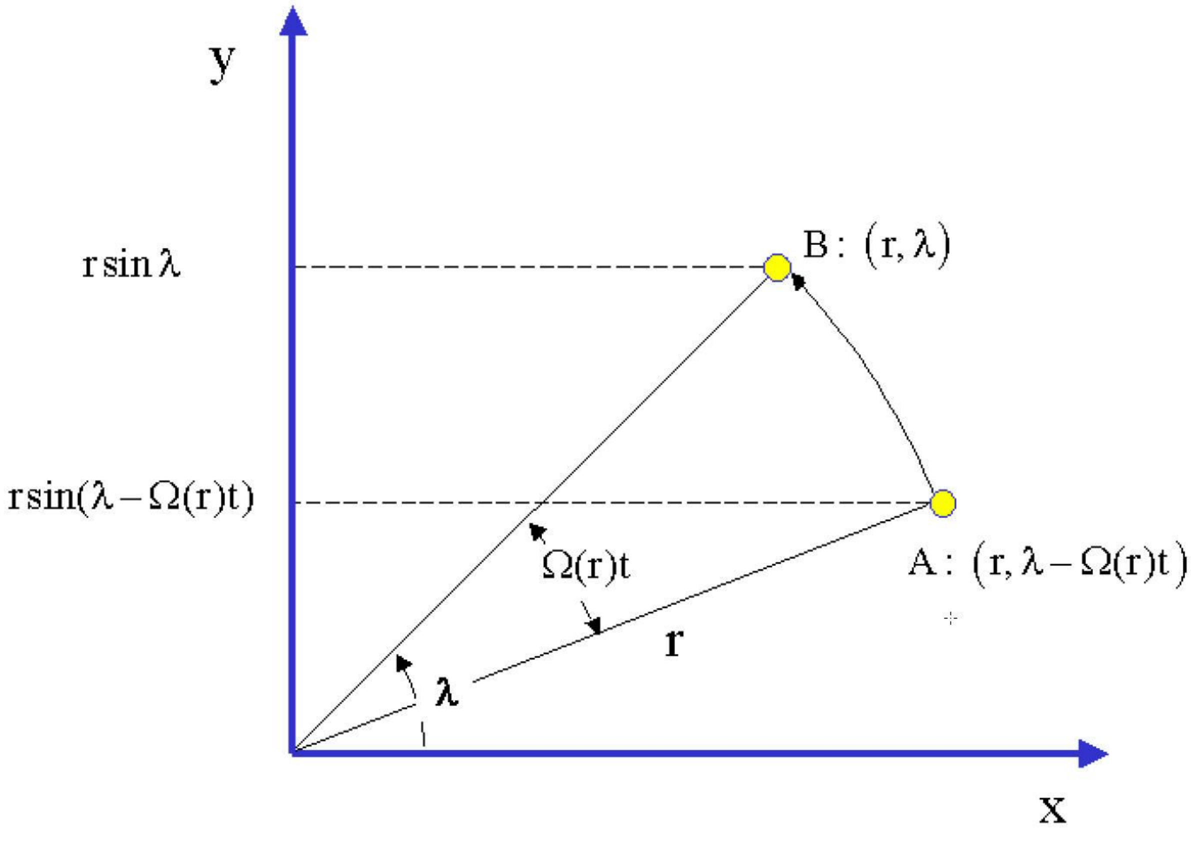


Figure 1.2: An air parcel moving in a circular orbit of radius r with angular velocity $\Omega(r)$ is located at the point B with polar coordinates (r, λ) at time t. At time $t = 0$ the parcel was located at point A with coordinates $(r, \lambda - \Omega(r)t)$. During this time it undergoes a meridional displacement $r[\sin\lambda - \sin(\lambda - \Omega(r)t)]$.

Vortex motion on a beta-plane

the moving vortex (we discuss the reason for the vortex movement below). Consider an air parcel that at time t is at the point with polar coordinate (r, λ) located at the (moving) vortex centre (Fig. 1.2). This parcel would have been at the position $(r, \lambda - \Omega(r)t)$ at the initial instant, where $\Omega(r) = V(r)/r$ is the angular velocity at radius r and $V(r)$ is the tangential wind speed at that radius. The initial vorticity of the parcel is $\zeta_s(r) + f_0 + \beta r \sin(\lambda - \Omega(r)t)$ while the vorticity of a parcel at its current location is $\zeta(r) + f_0 + \beta r \sin \lambda$. Therefore the vorticity perturbation $\zeta_a(r, \lambda)$ at the point (r, λ) at time t is $\zeta(r) - \zeta_s(r)$, or

$$\zeta_a(r, \lambda) = \beta r[\sin \lambda - \sin(\lambda - \Omega(r)t)]$$

or

$$\zeta_a(r, \lambda) = \zeta_1(r, t) \cos \lambda + \zeta_2(r, t) \sin \lambda, \qquad (1.13)$$

where

$$\zeta_1(r, t) = -\beta r \sin(\Omega(r)t), \quad \zeta_2(r, t) = -\beta r[1 - \cos(\Omega(r)t)]. \qquad (1.14)$$

Vortex motion on a beta-plane

$$\zeta_1(r,t) = -\beta r \sin(\Omega(r)t), \quad \zeta_2(r,t) = -\beta r[1-\cos(\Omega(r)t)]. \qquad (1.14)$$

We can now calculate the asymmetric streamfunction $\psi_a(r,\lambda,t)$ corresponding to this asymmetry using Eq. (1.4). The solution should satisfy the boundary condition that $\psi \to 0$ as $r \to \infty$. It is reasonable to expect that ψ_a will have the form:

$$\psi_a(r,\lambda) = \Psi_1(r,t)\cos\lambda + \Psi_2(r,t)\sin\lambda, \qquad (1.15)$$

and it is shown in Appendix 3.4.1 that

$$\Psi_n(r,t) = -\frac{r}{2}\int_r^\infty \zeta_n(p,t)\,dp - \frac{1}{2r}\int_0^r p^2\zeta_n(p,t)\,dp \quad (n=1,2), \qquad (1.16)$$

The Cartesian velocity components $(U_a, V_a) = (-\partial\Psi_a/\partial y, \partial\Psi_a/\partial x)$ are given by

$$U_a = \cos\lambda\sin\lambda\left[\frac{\Psi_1}{r} - \frac{\partial\Psi_1}{\partial r}\right] - \sin^2\lambda\frac{\partial\Psi_2}{\partial r} - \cos^2\lambda\frac{\Psi_2}{r}, \qquad (1.17)$$

$$V_a = \cos^2\lambda\frac{\partial\Psi_1}{\partial r} + \sin^2\lambda\frac{\Psi_1}{r} - \cos\lambda\sin\lambda\left[\frac{\Psi_2}{r} - \frac{\partial\Psi_2}{\partial r}\right]. \qquad (1.18)$$

Vortex motion on a beta-plane

In order that these expressions give a unique velocity at the origin, they must be independent of λ as $r \to 0$, in which case

$$\left.\frac{\partial \Psi_n}{\partial r}\right|_{r=0} = \lim_{r \to 0} \frac{\Psi_n}{r}, \quad (n = 1, 2).$$

Then

$$(U_a, V_a)_{r=0} = \left[-\left.\frac{\partial \Psi_2}{\partial r}\right|_{r=0}, \left.\frac{\partial \Psi_1}{\partial r}\right|_{r=0} \right], \tag{1.19}$$

and using (1.16) it follows that

$$\left.\frac{\partial \Psi_n}{\partial r}\right|_{r=0} = -\frac{1}{2} \int_0^\infty \zeta_n(p, t)\, dp. \tag{1.20}$$

If we make the reasonable assumption that the symmetric vortex moves with the velocity of the asymmetric flow across its centre, the vortex speed is simply

$$\mathbf{c}(t) = \left[-\left.\frac{\partial \Psi_2}{\partial r}\right|_{r=0}, \left.\frac{\partial \Psi_1}{\partial r}\right|_{r=0} \right], \tag{1.21}$$

which can be evaluated using (1.14) and (1.20).

Vortex motion on a beta-plane

If we make the reasonable assumption that the symmetric vortex moves with the velocity of the asymmetric flow across its centre, the vortex speed is simply

$$\mathbf{c}(t) = \left[-\frac{\partial \Psi_2}{\partial r}\bigg|_{r=0}, \frac{\partial \Psi_1}{\partial r}\bigg|_{r=0} \right], \tag{1.21}$$

which can be evaluated using (1.14) and (1.20).

The assumption is reasonable because *at the vortex centre* $\zeta \gg f$ and the governing equation (1.1) expresses the fact that $\zeta + f$ is conserved following the motion. Since the symmetric circulation does not contribute to advection across the vortex centre (recall that the vortex centre is defined as the location of the maximum relative vorticity), advection must be by the asymmetric component. With the method of partitioning discussed in section 3.2, this component is simply the environmental flow by definition. The slight error committed in supposing that ζ is conserved rather than $\zeta + f$ is equivalent to neglecting the propagation of the vortex centre. The track error amounts to no more than a few kilometers per day which is negligible compared with the actual vortex displacements (e.g., see Fig. 1.6).

Some calculations

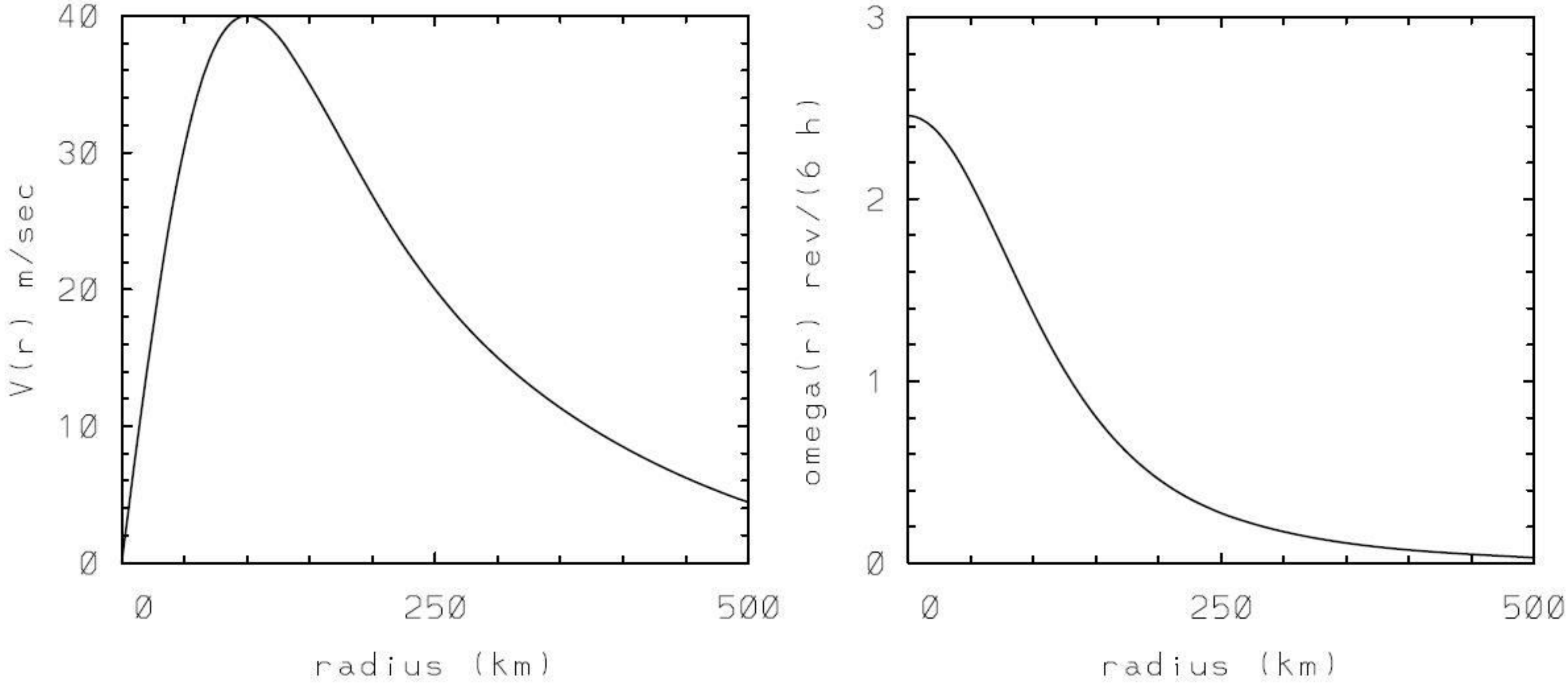
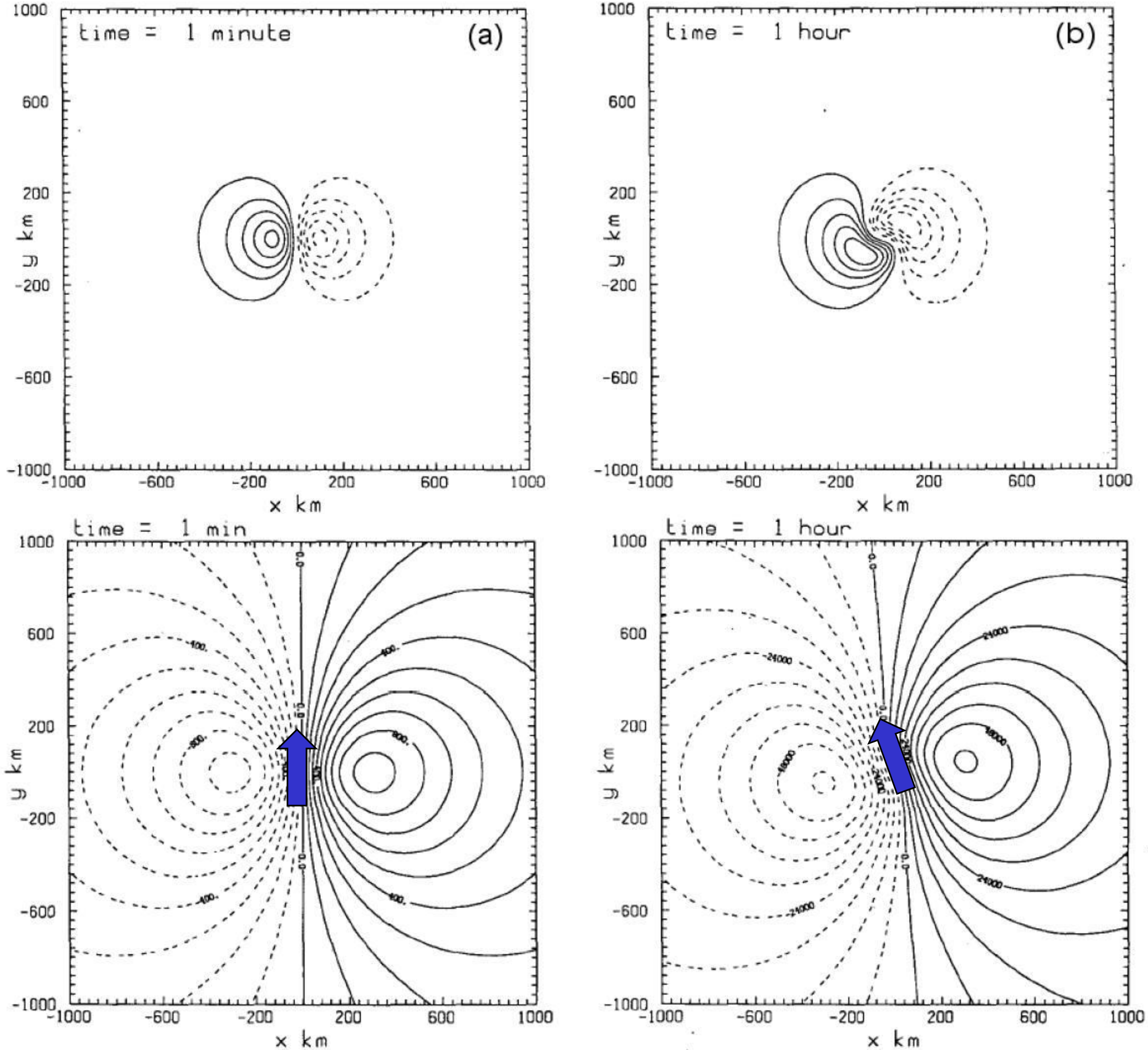
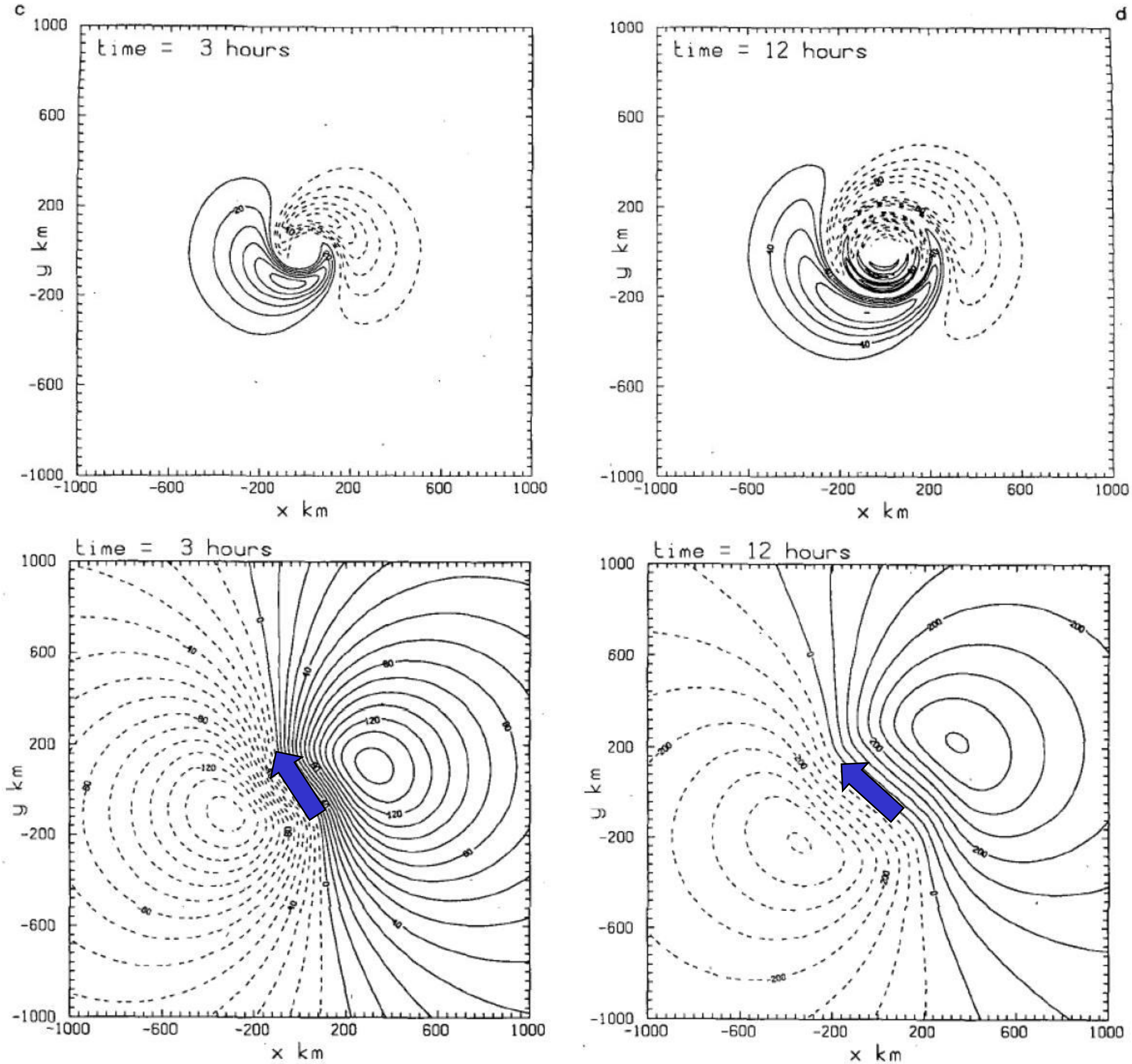


Figure 1.3: (left) Tangential velocity profile $V(r)$ and (right) angular velocity profile $\Omega(r)$ for the symmetric vortex.



time = 1 minute
(a)
time = 1 hour
(b)
time = 1 min
time = 1 hour
x km
y km



c
time = 3 hours
d
time = 12 hours
x km
y km
time = 3 hours
time = 12 hours
x km
y km

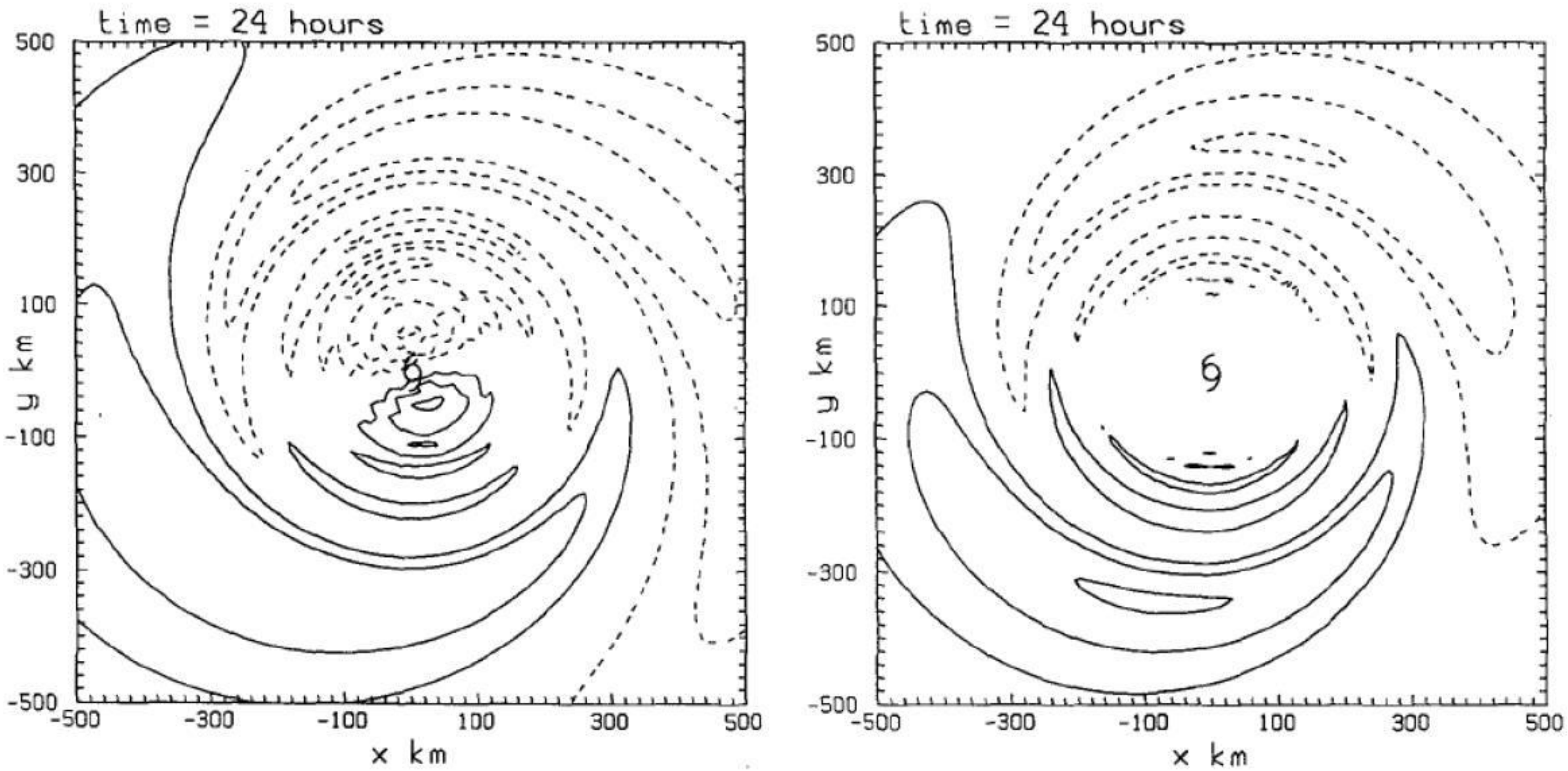
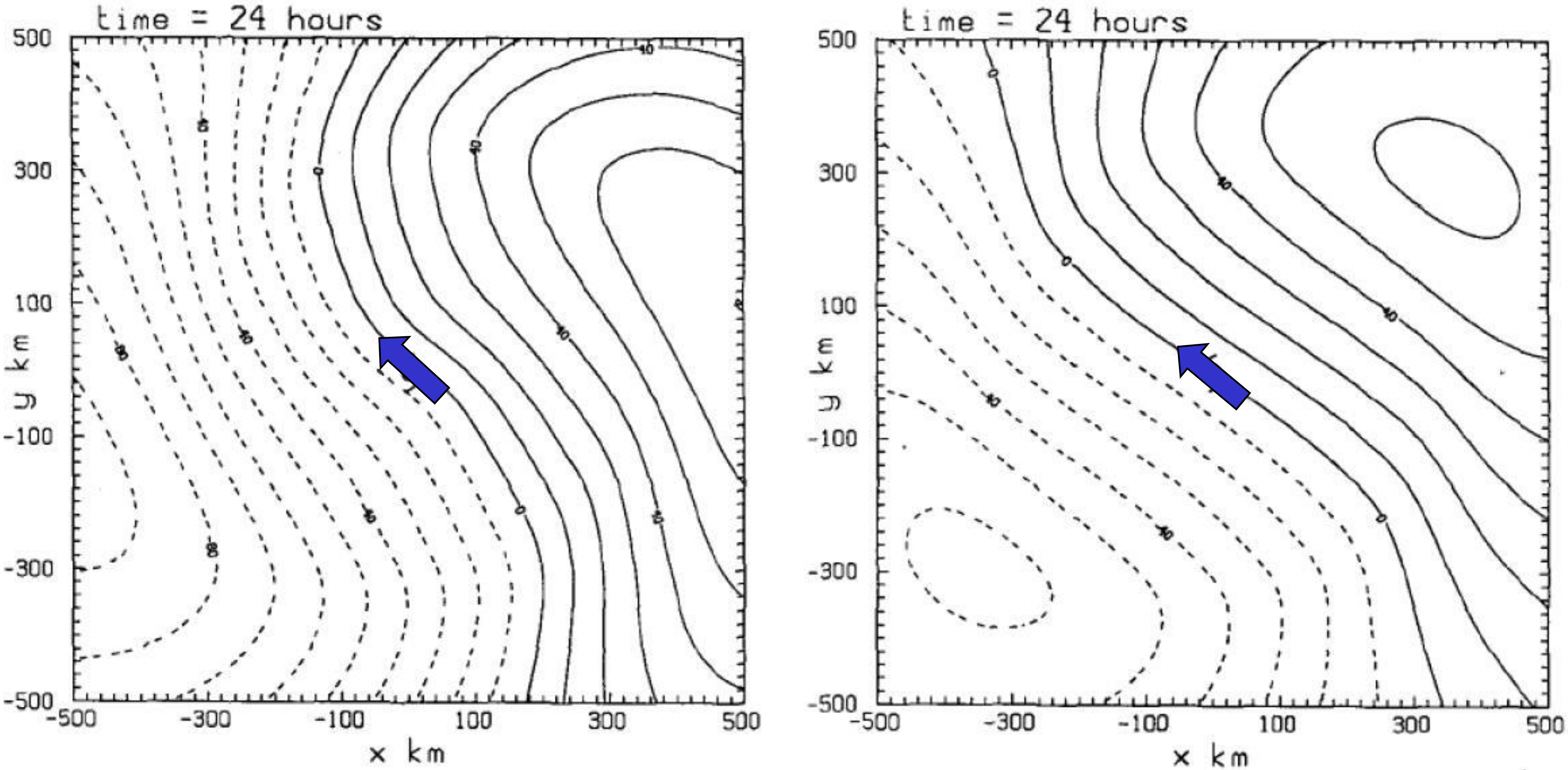


Figure 1.5: Comparison of the analytically-computed asymmetric vorticity and streamfunction fields (upper right and lower right) with those for the corresponding numerical solutions at 24 h. Only the inner part of the numerical domain, centred on the vortex centre, is shown (the calculations were carried out on a 2000 km $\times$ 2000 km domain). Contour intervals are 5×10^{-6} s^{-1} for ζ_a and 10^5 m^2 s^{-1} for ψ_a. The tropical cyclone symbol represents the vortex centre.



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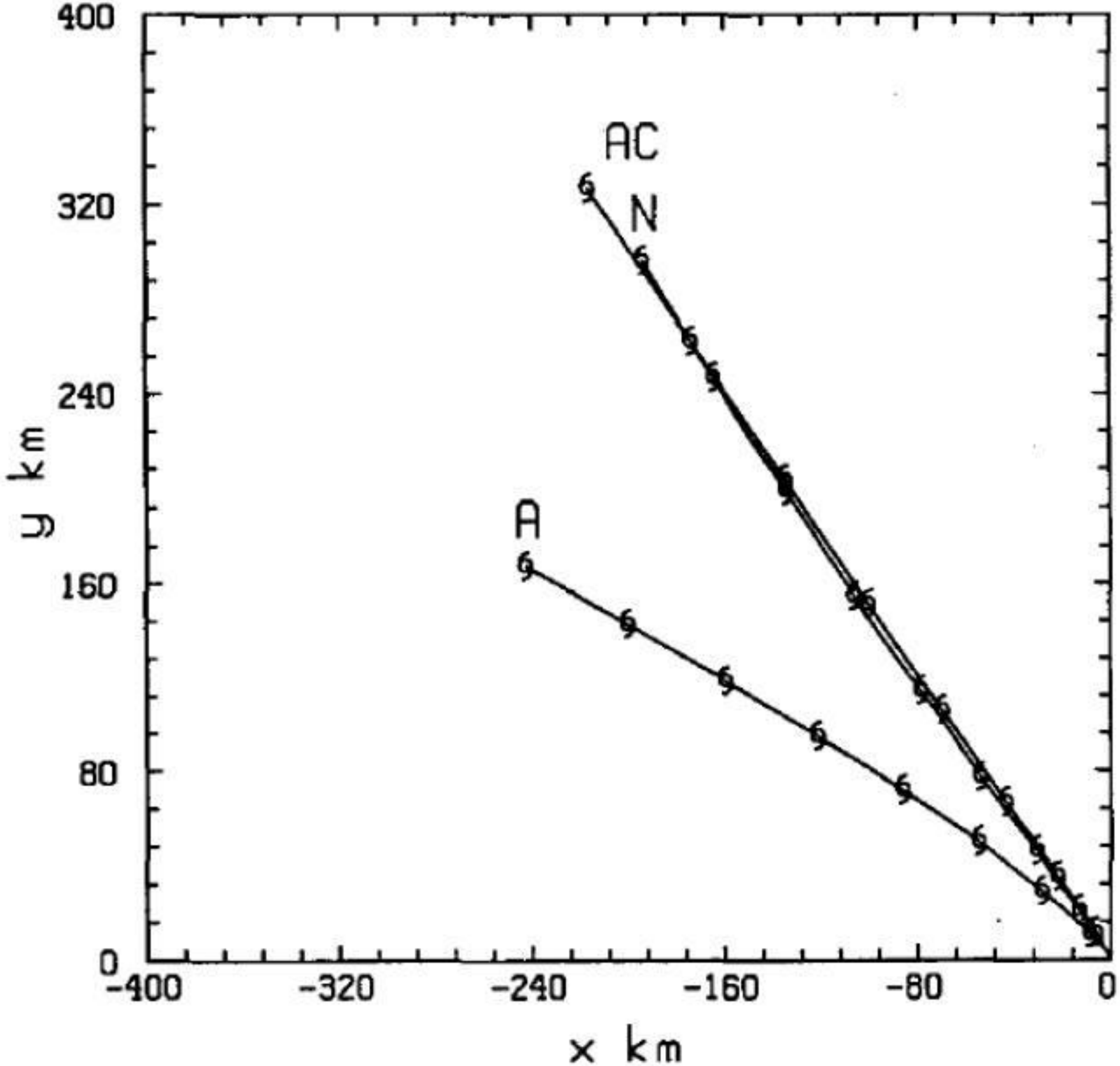
Vortex track

The vortex track, $\mathbf{X}(t) = [X(t), Y(t)]$ may be obtained by integrating the equation $d\mathbf{X}/dt = c(t)$, and using (1.20) and (1.21), we obtain

$$\begin{bmatrix} X(t) \\ Y(t) \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\int_0^\infty \left\{ \int_0^1 \zeta_2(p,t)dt \right\} dp \\ -\frac{1}{2}\int_0^\infty \left\{ \int_0^1 \zeta_1(p,t)dt \right\} dp \end{bmatrix}. \tag{1.22}$$

With the expressions for ζ_n in (1.14), this expression reduces to

$$\begin{bmatrix} X(t) \\ Y(t) \end{bmatrix} = \begin{bmatrix} -\frac{1}{2}\beta\int_0^\infty r\left[t - \frac{\sin(\Omega(r)t)}{\Omega(r)}\right] dr \\ \frac{1}{2}\beta\int_0^\infty r\left[\frac{1-\cos(\Omega(r)t)}{\Omega(r)}\right] dr \end{bmatrix}. \tag{1.23}$$



400
320
240
160
80
0
y km
-400
-320
-240
-160
-80
0
x km
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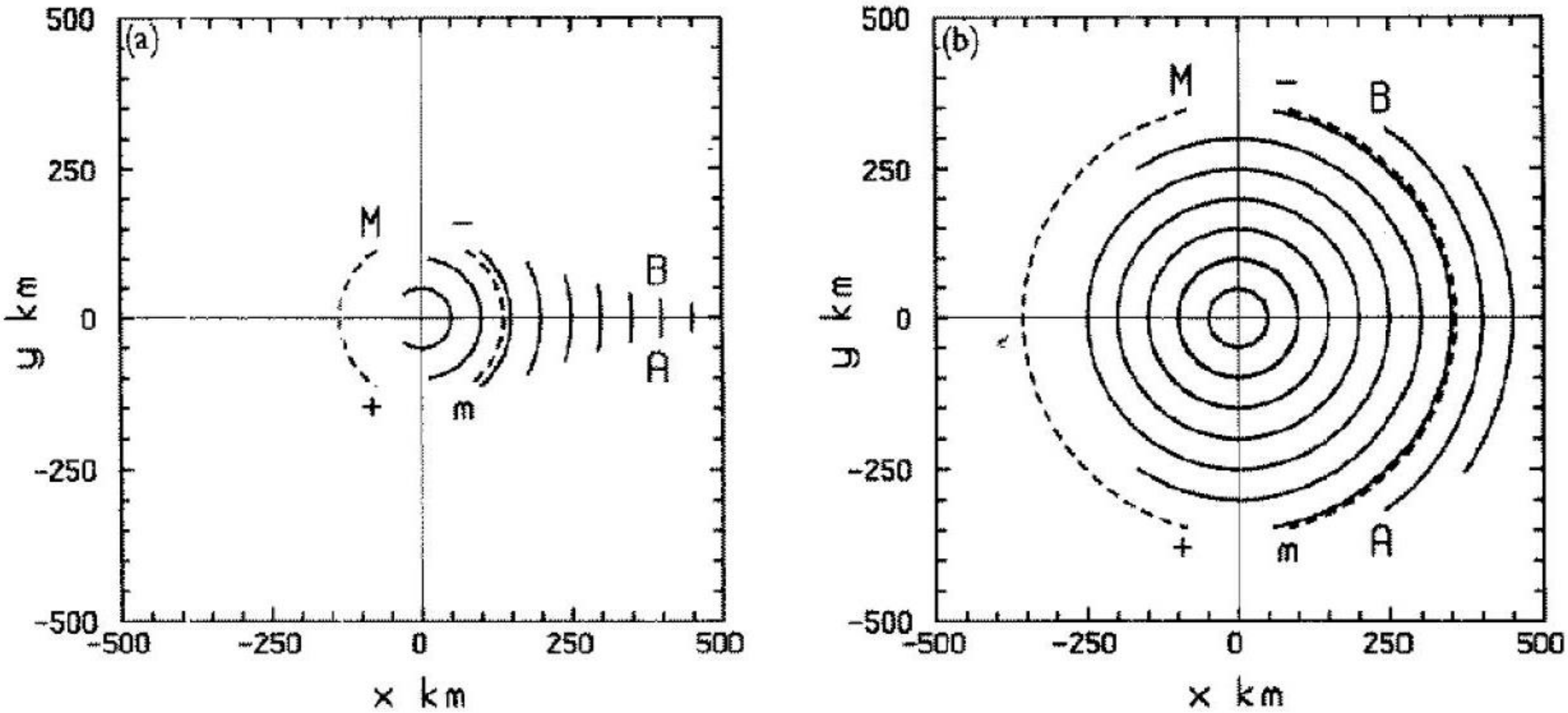

Figure 1.8: Approximate trajectories of fluid parcels which, for a given radius, give the maximum asymmetric vorticity contribution at that radius. The figures refer ro the case of motion of an initially-symmetric vortex on a β-plane with zero basic flow at (a) 1 h, (b) 24 h. The particles arc assumed to follow circular paths about the vortex centre (e.g. AB) with angular velocity $\Omega(r)$, where Ω decreases monotonically with radius r. Solid lines denote trajectories at 50 km radial intervals. Dashed lines marked 'M' and 'm' represent the trajectories giving the overall axisymmetric vorticity maxima and minima, respectively. These maxima and minima occur at the positive and negative ends of the relevant lines.

References

- Some papers on this subject are:
  - Smith et al. (1990) *Q. J. Roy. Met. Soc.*
  - Smith & Ulrich (1990) *J. Atmos. Sci.*
  - Smith & Ulrich (1993) *Q. J. Roy. Met. Soc.*