Cloud microphysics

Claudia Emde

Meteorological Institute, LMU, Munich, Germany

WS 2011/2012
Energy balance of the Earth

IPCC report, 2007
Cloud-aerosol interactions

- **Twomey, 1977**: High concentrations of aerosols reduce droplet size and increase cloud albedo for a constant amount of liquid water.

- **Albrecht, 1989**: High aerosol concentrations narrow the size distribution, suppressing precipitation and prolonging cloud lifetime.

- Aerosol-induced changes of cloud microstructure have profound impact on precipitation, dynamic evolution and vertical disposition of latent heat release (e.g. Rosenfeld, 2006).

Figure from Wallace and Hobbs
Cloud-aerosol interactions

**basic processes** explaining cloud formation and evolution are well established

**but still many unresolved fundamental issues:**
- temporal and spatial evolution of clouds
- lack of fundamental understanding of the glaciation of clouds
- formation of rain in warm clouds
- convective clouds
Impact of clouds on climate change

Radiative Forcing Components

<table>
<thead>
<tr>
<th>RF Terms</th>
<th>CO₂</th>
<th>N₂O</th>
<th>CH₄</th>
<th>Halocarbons</th>
<th>RF values (W m⁻²)</th>
<th>Spatial scale</th>
<th>LOSU</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long-lived greenhouse gases</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.66 [1.49 to 1.83]</td>
<td>Global</td>
<td>High</td>
</tr>
<tr>
<td>Ozone</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stratospheric</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0.05 [-0.15 to 0.05]</td>
<td>Continental to global</td>
<td>Med</td>
</tr>
<tr>
<td>Tropospheric</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.35 [0.25 to 0.65]</td>
<td>Global</td>
<td>High</td>
</tr>
<tr>
<td>Stratospheric water vapour from CH₄</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.07 [0.02 to 0.12]</td>
<td>Global</td>
<td>Low</td>
</tr>
<tr>
<td>Surface albedo</td>
<td></td>
<td></td>
<td></td>
<td>Black carbon on snow</td>
<td>-0.2 [-0.4 to 0.0]</td>
<td>Local to continental</td>
<td>Med - Low</td>
</tr>
<tr>
<td>Land use</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.1 [0.0 to 0.2]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Direct effect</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0.5 [-0.9 to -0.1]</td>
<td>Continental to global</td>
<td>Med - Low</td>
</tr>
<tr>
<td>Cloud albedo effect</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0.7 [-1.8 to -0.3]</td>
<td>Continental to global</td>
<td>Low</td>
</tr>
<tr>
<td>Total Aerosol</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.01 [0.003 to 0.03]</td>
<td>Continental</td>
<td>Low</td>
</tr>
<tr>
<td>Linear contrails</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Natural</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.12 [0.06 to 0.30]</td>
<td>Global</td>
<td>Low</td>
</tr>
<tr>
<td>Solar irradiance</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total net anthropogenic</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.6 [0.6 to 2.4]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
“Cloud feedbacks (particularly from low clouds) remain the largest source of uncertainty.”

IPCC Report 2007, Technical Summary

Cloud feedback parameter: 2 X CO$_2$

AR4 Models

Ringer et al., GRL 2006
Overview of cloud physics

- Atmospheric thermodynamics
  - gas laws, hydrostatic equation
  - 1st law of thermodynamics
  - moisture parameters
  - adiabatic / pseudoadiabatic processes
  - stability criteria / cloud formation

- Microphysics of warm clouds
  - nucleation of water vapor by condensation
  - growth of cloud droplets in warm clouds (condensation, fall speed of droplets, collection, coalescence)
  - formation of rain, stochastical coalescence

- Microphysics of cold clouds
  - homogeneous, heterogeneous, and contact nucleation
  - concentration of ice particles in clouds
  - crystal growth (from vapor phase, riming, aggregation)
  - formation of precipitation, cloud modification

- Observation of cloud microphysical properties
  (Parameterization of clouds in climate and NWP models)
Literature


Additional publications and slides on website:
http://www.meteo.physik.uni-muenchen.de/~emde/doku.php?id=teaching:cloud_microphysics:cloud_microphysics
The ideal gas equation

- Equation of state: relation between p, V, T of a material
- Equation of state for gases ⇒ ideal gas equation
  \[ pV = mRT \quad p = \rho RT \quad p\alpha = RT \]

- R - gas constant for 1 kg of gas
- \( \alpha = 1/\rho \) - specific volume of gas (V occupied by 1 kg of gas at specific p and T)
- Boyle’s law (T=const.) and Charles’ laws (p=const., V=const.)
Definitions

- gram-molecular weight (mole), e.g. 1 mol H$_2$O = 18.015 g
- number of moles $n = \frac{m}{M}$
- number of molecules in 1 mole $N_A = 6.022 \times 10^{23}$ (Avogadro’s number)
- Avogadro’s hypothesis: gases containing the same number of molecules occupy the same volume
- universal gas constant $R^* = 8.3145 \text{JK}^{-1}\text{mol}^{-1}$ \Rightarrow $pV = nR^* T$
- Boltzmann’s constant $k = R^*/N_A$

Amedeo Avogadro (1776–1856)

Ludwig Boltzmann (1844–1906)

Images from Wikipedia
Adiabatic processes

adiabatic = change in physical state without heat exchange \( \Rightarrow dq = 0 \)

\[ dq = du + pd\alpha \]

T rises in adiabatic compression
T=const. in isothermal process

\[ T_C > T_B \Rightarrow p_C > p_B \]

Fig. 3.5 An isotherm and an adiabat on a \( p-V \) diagram.
Figure from Wallace and Hobbs
Assumptions:

- *molecular mixing can be neglected* (in Earth’s atmosphere only important above \( \approx 105 \) km and for 1 cm layer above surface), i.e. mixing can be regarded as exchange of macroscale “air parcels”

- *parcel is thermally insulated from it’s environment*, i.e. T changes adiabatically as parcel rises or sinks, p always adapts to environmental air, which is assumed to be in hydrostatic equilibrium

- *parcel moves slow enough*, i.e. the macroscopic kinetic energy is a negligible fraction of the total energy
for adiabatic processes:

\[ d(c_p T + \phi) = 0 \Rightarrow -\frac{dT}{dz} \text{ dry parcel} = \frac{g}{c_p} \equiv \Gamma_d \]

\( \Gamma_d \) – dry adiabatic lapse rate (change of \( T \) with \( z \))

Example for Earth’s atmosphere:

- \( g=9.81 \frac{m}{s^2} \), \( c_p=1004 \frac{J}{K} \) \( \Rightarrow \Gamma_d=9.8 \frac{K}{km} \)

- Actual lapse rate (for moist air) is smaller than \( \Gamma_d \).
**Saturation vapor pressures**

**Fig. 3.8** A box (a) unsaturated and (b) saturated with respect to a plane surface of pure water at temperature $T$. Dots represent water molecules. Lengths of the arrows represent the relative rates of evaporation and condensation. The saturated (i.e., equilibrium) vapor pressure over a plane surface of pure water at temperature $T$ is $e_s$ as indicated in (b).

Figure from Wallace and Hobbs
evaporation rate from ice less than from water: 
\[ e_s(T) > e_{si}(T) \]
⇒ ice particle in water-saturated air grows due to deposition of water vapor on it (important for formation of precipitation)

**Fig. 3.9** Variations with temperature of the saturation (i.e., equilibrium) vapor pressure \( e_s \) over a plane surface of pure water (red line, scale at left) and the difference between \( e_s \) and the saturation vapor pressure over a plane surface of ice \( e_{si} \) (blue line, scale at right). Figure from Wallace and Hobbs
**Saturation vapor pressure**

**Clausius-Clapayron equation**

\[
\frac{de_s}{dT} = \frac{L_v}{T(\alpha_v - \alpha_l)}
\]

- integration yields \( e_s(T) \), approximate because \( L_v \) depends on \( T \)

**Magnus formula (empirical)**

- water \((0^\circ C - 100^\circ C)\) : \( e_s = 6.1078 \exp\left(\frac{17.0809T}{234.175 + T}\right) \)
- subcooled water : \( e_s = 6.1078 \exp\left(\frac{17.8436T}{245.425 + T}\right) \)
- ice : \((-50^\circ C - 0^\circ C)\) : \( e_s = 6.1071 \exp\left(\frac{22.4429T}{272.44 + T}\right) \)

\( T \) in °C and \( e_s \) in hPa.
Moisture parameters

- **mixing ratio:** \( w = \frac{m_v}{m_d} \)
  - typically a few g/kg in mid-latitudes to 20 g/kg in tropics

- **specific humidity:** \( q = \frac{m_v}{m_v + m_d} = \frac{w}{w + 1} \)
  - \( w \approx 0.01 \rightarrow q \approx w \)

- **Saturation mixing ratio** \( w_s \):
  \[
  w_s = \frac{m_{vs}}{m_d} = \cdots = \epsilon \frac{e_s}{p - e_s} \approx 0.622 \frac{e_s}{p} \]
  - (since for atmospheric T: \( p \gg e_s \))

- **Relative humidity** RH:
  \[
  RH = 100 \frac{w}{w_s} = 100 \frac{e}{e_s} \% \]

- **Dew point** \( T_D \):
  - temperature to which air must be cooled at \( p=\text{const.} \), so that air becomes saturated w.r.t. water (equivalent def. for frost point)

measurement of \( T_D \) yields
\[
RH = \frac{e_s(T_D, p)}{e_s(T, p)}
\]
Lifting condensation level (LCL)

- LCL: level to which moist air parcel can be lifted adiabatically before it becomes saturated w.r.t. water
- during lift: $w = \text{const.}, \theta = \text{const.}, w_s$ decreases until $w_s = w$ at LCL

**Fig. 3.10** The lifting condensation level of a parcel of air at A, with pressure $p$, temperature $T$, and dew point $T_d$, is at C on the skew $T - \ln p$ chart.

Figure from Wallace and Hobbs
Latent heats

- If heat is added to system ⇒ change in T or change in phase
- phase transition: $\Delta u$ completely used for changes in molecular configuration in presence of intermolecular forces

- **Latent heat of melting $L_m$:** heat that is required to convert unit mass of a material from solid to liquid phase without change in T, equal to **latent heat of freezing**
- **melting point:** T at which phase transition occurs
- for water at 1013hPa, 0°C ⇒ $L_m = 3.34 \cdot 10^5 \frac{J}{kg}$

- **latent heat of vaporization or evaporation $L_v$** defined equivalently
- for water 1013hPa, 100°C (boiling point) ⇒ $L_v = 2.25 \cdot 10^6 \frac{J}{kg}$
Saturated adiabatic and pseudoadiabatic processes

- air parcel rises $\Rightarrow$ $T$ decreases with $z$ until saturation is reached
- further lifting $\Rightarrow$ condensation of liquid water (or deposition on ice) $\Rightarrow$ release of latent heat $\Rightarrow$ rate of decrease in $T$ reduced

**Saturated adiabatic process**

All condensation products remain in parcel, process still adiabatic and reversible

**Pseudoadiabatic process**

Condensation products fall out, process is irreversible. Not adiabatic since products carry out small amount of heat.
Saturated adiabatic lapse rate

\[ \Gamma_s = - \frac{dT}{dz} \approx \frac{\Gamma_d}{1 + \frac{L_v}{c_p} \left( \frac{dw}{dT} \right)_p} \]

- \( \Gamma_s \) varies with \( p, T \); in contrast to \( \Gamma \)
- since condensation releases heat: \( \Gamma_s < \Gamma \)
- typical values:
  - 4 K/km near ground in warm humid airmasses
  - 6-7 K/km in middle troposphere
  - near tropopause, \( \Gamma_s \) only slightly smaller than \( \Gamma \) (\( e_s \) very small, no condensation)
Static stability for unsaturated air

- Atmospheric layer with actual lapse rate $\Gamma$ less than dry adiabatic lapse rate $\Gamma_D$ \[ \Rightarrow \text{stable stratification, positive static stability} \]
- $\Gamma > \Gamma_D \Rightarrow \text{unstable stratification, positive static stability}$ (not persistent in free atmosphere due to strong vertical mixing)
- $\Gamma = \Gamma_D \Rightarrow \text{neutral}$
- Same for saturated air, when $\Gamma_S$ is used instead of $\Gamma_D$

**Fig. 3.12** Conditions for (a) positive static stability ($\Gamma < \Gamma_D$) and (b) negative static instability ($\Gamma > \Gamma_D$) for the displacement of unsaturated air parcels. Figure from Wallace and Hobbs
Gravity waves

For stably stratified layers, so called gravity waves may form.

**buoyancy oscillation of air parcel**

\[ z'(t) = z'(0) \cos Nt \]

**Brunt-Väisälä frequency**

\[ N = \left( \frac{g}{T} (\Gamma_d - \Gamma) \right)^{1/2} \]
Gravity waves

from Wikipedia
Conditional and convective stability

- atmospheric layer with actual lapse rate between $\Gamma_s$ and $\Gamma_d$ ⇒ conditional instability
- Level of free convection (LFC) ⇒ from this level parcel is unstable, is carried upward in absence of forced lifting
- vigorous convective overturning can occur if vertical motions are large enough to lift air parcel beyond LFC

Fig. 3.16  Conditions for conditional instability ($\Gamma_s < \Gamma < \Gamma_d$). $\Gamma_s$ and $\Gamma_d$ are the saturated and dry adiabatic lapse rates, and $\Gamma$ is the lapse rate of temperature of the ambient air. LCL and LFC denote the lifting condensation level and the level of free convection, respectively. Figure from Wallace and Hobbs
Phase transitions

vapor ↔ liquid  condensation, evaporation
liquid ↔ solid  freezing, melting
vapor ↔ solid  deposition, sublimation

Changes from left to right:
⇒ increasing molecular order, “free energy barrier” to overcome
⇒ cloud forming processes

**saturation** = equilibrium condition for thermodynamic system consisting of vapor (ice) and liquid
Why do droplets form?

- at equilibrium (saturation): 
  rate of condensation = rate of evaporation
- energy barrier of small droplets: generally no phase transition at saturation (homogeneous nucleation unlikely)
- when air parcels ascent without condensation \( \Rightarrow \) supersaturation
- energy barrier may be decreased by cloud condensation nuclei
  \( \Rightarrow \) heterogeneous nucleation
  - hygroscopic particles serve as centers of condensation
  - supersaturation in clouds not much larger than 1%
- when air parcel including cloud droplets ascend to \( T < 0^\circ \)
  - droplets become supercooled
  - freeze when ice nuclei are present
Energy difference due to formation of droplet

$\Delta E = \text{surface energy of droplet} - \text{Gibbs free energy due to condensation}$

$\Delta E = 4\pi R^2 \sigma - \frac{4}{3} \pi R^3 n k T \ln \frac{e}{e_s}$

- **blue curve**: subsaturated conditions, formation of droplets not possible
- **red curve**: supersaturated conditions, droplets grow above radius $r$

**Fig. 6.1** Increase $\Delta E$ in the energy of a system due to the formation of a water droplet of radius $R$ from water vapor with pressure $e$; $e_s$ is the saturation vapor pressure with respect to a plane surface of water at the temperature of the system.

Figure from Wallace and Hobbs
Energy difference due to formation of droplet

![Graph showing energy difference due to formation of droplet.](Image)

- Plot includes a range of values for $\frac{e}{e_s}$, with annotations for specific values:
  - $\frac{e}{e_s} = 0.990$
  - $\frac{e}{e_s} = 1.000$
  - $\frac{e}{e_s} = 1.005$
  - $\frac{e}{e_s} = 1.010$
  - $\frac{e}{e_s} = 1.020$
  - $\frac{e}{e_s} = 1.050$
  - $\frac{e}{e_s} = 1.100$

- The x-axis represents the radius $R$ in micrometers ($\mu$m), with values ranging from $10^{-2}$ to $10^0$.
- The y-axis represents the energy difference $\Delta E$ in J, ranging from $10^{-17}$ to $10^{-12}$.

- The graph shows the relationship between the radius of droplets and the energy difference for various $\frac{e}{e_s}$ values.
Kelvin equation

\[ r = \frac{2\sigma}{nkT \ln \frac{e}{e_s}} \]
Heterogeneous nucleation

Surface tension is reduced when soluble aerosol is added to droplet.

Calculation for 0.5% supersaturation, $T=293$ K.
Raoult’s law

Vapor pressure of an ideal solution depends on mole fraction of the component present in the solution

\[
\frac{e'}{e} = f
\]

- \(e'\) – saturation water vapor pressure adjacent to solution droplet containing a mole fraction \(f\) of pure water
- \(e\) – saturation water vapor pressure adjacent to pure water droplet
- \(f\) – number of moles of pure water divided by total number of moles

⇒ saturation water vapor pressure is reduced when aerosol is solved in droplet
Köhler curves

\[
e / e_s = \left( \exp \frac{2\sigma'}{n'kTr} \right) \left( 1 + \frac{imM_w}{M_s \left( \frac{4}{3} \pi r^3 \rho' - m \right)} \right)^{-1}
\]

**Fig. 6.3** Variations of the relative humidity and supersaturation adjacent to droplets of (1) pure water (blue) and adjacent to solution droplets containing the following fixed masses of salt: (2) $10^{-19}$ kg of NaCl, (3) $10^{-18}$ kg of NaCl, (4) $10^{-17}$ kg of NaCl, (5) $10^{-19}$ kg of (NH$_4$)$_2$SO$_4$, and (6) $10^{-18}$ kg of (NH$_4$)$_2$SO$_4$. Note the discontinuity in the ordinate at 100% relative humidity. [Adapted from H. R. Pruppacher, “The role
Droplet activation

Droplets grow along Köhler curve

- Case 1: when ambient supersaturation is higher than maximum
  ⇒ activated droplets

- Case 2: when ambient supersaturation is lower than maximum, droplets grow to equilibrium state, where ambient supersaturation equals supersaturation adjacent to droplet
  ⇒ unactivated/haze droplets
Efficiency of cloud condensation nuclei

- small subset of atmospheric aerosols serve as CCN
- CCN are most efficient when droplets can grow at supersaturations as low as possible
  - the larger the size the lower the required supersaturation
  - the greater the solubility the lower the required supersaturation
Growth of droplets in warm clouds

1. Growth by condensation

2. Growth by collision and coalescence
Droplet growth by condensation

- air parcel rises, expands, cools adiabatically and reaches saturation
- further lifting produces supersaturation
- as supersaturation rises, CCN are activated (most efficient first)
- supersaturation reaches maximum when:

  \[
  \text{rate of water vapor in excess of saturation made available by adiabatic cooling} = \text{rate of water vapor which condenses on CCN and droplets}
  \]

  \[
  \text{concentration of cloud droplets} = \text{concentration of CCN activated by attained peak supersaturation}
  \]
Growth rate and size distribution

- growing droplets consume water vapor faster than it is made available by cooling and supersaturation decreases
- haze droplets evaporate, activated droplets continue to grow by condensation

**growth rate of water droplet**

$$\frac{dr}{dt} = G_i S \frac{1}{r}$$

- smaller droplets grow faster than larger droplets
- sizes of droplets in cloud become increasingly uniform, approach **monodispersed** distribution

**Fig. 6.16** Theoretical computations of the growth of cloud condensation nuclei by condensation in a parcel of air rising with a speed of 60 cm s\(^{-1}\). A total of 500 CCN cm\(^{-1}\) was assumed with \(im/M_s\) values [see Eq. (6.8)] as indicated. Note how the droplets that have been activated (brown, blue, and purple curves) approach a monodispersed size distribution after just 100 s. The variation with time of the supersaturation of the air parcel is also shown (dashed red line). [Based on data from *J. Meteor.* 6, 143 (1949).]

Figure from Wallace and Hobbs.
Sizes of cloud droplets

- growth by condensation alone can not produce raindrops with radii of several mm!

Figure from Wallace and Hobbs
Collision efficiency

\[ E = \frac{y^2}{r_1^2 + r_2^2} \]

- \( E \) increases when size of collector drop \( r_1 \) increases
- \( E \) small for \( r_1 < 20\mu m \)
- \( r_1 \gg r_2 \): \( E \) small because small droplets follow streamlines around collector drop
- \( E \) increases with increasing \( r_2 \) until \( r_2/r_1 \approx 0.6–0.9 \)
- \( r_2/r_1 > 0.6–0.9 \): \( E \) decreases because relative velocity between droplets becomes small
- \( r_2/r_1 \approx 1 \): strong interaction between droplets, \( E \) increases again

Fig. 6.20  Calculated values of the collision efficiency, \( E \), for collector drops of radius \( r_1 \) with droplets of radius \( r_2 \). [Adapted from H. R. Pruppacher and J. D. Klett, *Microphysics of Clouds and Precipitation*, Kluwer Academic Pub., 1997, Fig. 14-6, p. 584, Copyright 1997, with kind permission of Springer Science and Business Media. Based on *J. Atmos. Sci.* 30, 112 (1973).] Figure from Wallace and Hobbs
Coalescence: Droplet is captured when it collides with larger droplet

Fig. 6.21  (a) A stream of water droplets (entering from the right), about 100 μm in diameter, rebounding from a plane surface of water. (b) When the angle between the stream of droplets and the surface of the water is increased beyond a critical value, the droplets coalesce with the water. [Photograph courtesy of P. V. Hobbs.]

Figure from Wallace and Hobbs
Coalescence efficiency

$E' = \text{fraction of collisions that result in coalescence}$

- $E'$ large for $r_2 \ll r_1$
- $E'$ initially decreases as $r_2$ increases
- as $r_2$ approaches $r_1$, $E'$ increases sharply

**Collection efficiency**

$$E_c = E \cdot E'$$

**Figure 6.22** Coalescence efficiencies $E'$ for droplets of radius $r_2$ with collector drops of radius $r_1$ based on an empirical fit to laboratory measurements. [Adapted from J. Atmos. Sci. 52, 3985 (1995).]

Figure from Wallace and Hobbs
Continuous collection model

- collector drop with radius $r_1$ and terminal velocity $v_1$
- drop falls in still air through cloud of equal sized droplets with $r_2$ and $v_2$
- droplets are uniformly distributed and collected uniformly by all collector drops of a given size

**Fig. 6.23** Schematic illustrating the continuous collection model for the growth of a cloud drop by collisions and coalescence. Figure from Wallace and Hobbs
Gap between condensational and collectional growth

- **condensational growth**
  - slows appreciably as droplet radius approaches $\sim 10\,\mu m$
  - tends to produce monodisperse size distribution
  - droplets then have similar fall speeds $\Rightarrow$ collisions become unlikely

- **collectional growth**
  - conditions: a few reasonably efficient collector drops (i.e. $r > 20\,\mu m$)
    - cloud deep enough and contains sufficient amount of water

- **Question 1**: How do the collector drops initially form
Question 2:
How do the broad size distributions form that are commonly measured?

Fig. 6.7  (a) Percentage of marine cumulus clouds with indicated droplet concentrations. (b) Droplet size distributions in a marine cumulus cloud. (c) Percentage of continental cumulus clouds with indicated droplet concentrations. (d) Droplet size distributions in a continental cumulus cloud. Note change in ordinate from (b). [Adapted from P. Squires, “The microstructure and colloidal stability of warm clouds. Part I—The relation between structure and stability,” Tellus 10, 258 (1958). Permission from Blackwell Publishing Ltd.]
Possible answers

- Giant cloud condensation nuclei (GCCN)
- Turbulence (enhances collision efficiency, fluctuations in supersaturation)
- Radiative broadening (droplet loses heat, saturation vapor pressure lower, faster growth)
- Stochastic collection
Shape of raindrops

- as raindrop size increases it becomes flattened, gradually changes shape from spherical to increasingly parachute
- if initial radius > 2.5mm parachute becomes inverted bag with toroidal ring of water around lower rim
- when drop bursts to produce fine spray of droplets, toroidal ring breaks up into large drops

Figure from Wikipedia
Size distribution of raindrops

Measurements of the size distribution of raindrops that reach the ground can often be fitted to the same size distribution function:

**Marshall-Palmer distribution**

\[ N(D) = N_0 \exp(-\Lambda D) \]

- \( N(D)dD \) – number of drops per unit volume with diameters between \( D \) and \( D + dD \)
- \( N_0 \) and \( \Lambda \) – empirical fitting parameters
- \( N_0 \) almost const., \( \Lambda \) varies with rainfall rate
Microphysics of cold clouds

- **cold cloud**: cloud that extends above 0°C level
- **mixed cloud**: clouds containing liquid droplets and ice crystals
- **glaciated cloud**: pure ice cloud
Homogeneous nucleation

- Water droplets become super-cooled when air parcel ascends and cools down.
- **Homogeneous nucleation**: pure water droplet freezes.
  - Phase transition: liquid $\rightarrow$ solid.
- Process analogue to nucleation of liquid droplet from vapor phase.
Homogeneous and heterogeneous nucleation

- measured median freezing temperatures
- homogeneous freezing
- heterogeneous freezing

**Fig. 6.29** Median freezing temperatures of water samples as a function of their equivalent drop diameter. The different symbols are results from different workers. The red symbols and red line represent heterogeneous freezing, and the blue symbols and line represent homogeneous freezing. [Adapted from B. J. Mason, *The Physics of Clouds*, Oxford Univ. Press, Oxford, 1971, p. 160. By permission of Oxford University Press.]

Figure from Wallace and Hobbs
Heterogeneous nucleation

- water molecules in droplet collect onto surface of particle contained in droplet (freezing nucleus) ⇒ ice like structure is formed ⇒ growth starts at larger crystal size ⇒ freezing occurs
- heterogeneous nucleation occurs at much higher T than homogeneous nucleation
Ice nucleating crystals

<table>
<thead>
<tr>
<th>crystal substance</th>
<th>lattice constants [Å]</th>
<th>reported nucleation threshold [°C]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a</td>
<td>b</td>
</tr>
<tr>
<td>Ice</td>
<td>4.52</td>
<td>7.37</td>
</tr>
<tr>
<td>AgI</td>
<td>4.58</td>
<td>7.49</td>
</tr>
<tr>
<td>PbI(_2)</td>
<td>4.54</td>
<td>6.86</td>
</tr>
</tbody>
</table>

**Table:** adapted from Byers, Elements of cloud physics

AgI and PbI\(_2\) have hexagonal structures and are insoluble in water. The lattice structure is very similar to ice \(\Rightarrow\) AgI and PbI\(_2\) are active nucleation agents for ice crystals.
Further nucleation processes

**Contact nucleation**
Freezing starts when suitable particle *(contact nucleus)* comes into contact with super-cooled droplet.

**Deposition**
Some particles *(deposition nuclei)* serve as centers where ice forms directly from vapor phase. Conditions: air supersaturated w.r.t. ice and T sufficiently low.

When air is supersaturated w.r.t. ice and water, some particles may act as freezing nucleus *(vapor ⇒ liquid ⇒ ice)* or as deposition nucleus *(vapor ⇒ ice).*
Onset of ice nucleation

- Onset of ice nucleation as a function of temperature and supersaturation
- Onset occurs at higher T under water-supersaturated conditions
- Lower T required under water-subsaturated conditions, when only deposition is possible

**Fig. 6.30** Onset of ice nucleation as a function of temperature and supersaturation for various compounds. Conditions for condensation-freezing and ice deposition are indicated. Ice nucleation starts above the indicated lines. The materials are silver iodide (red), lead iodide (blue), methaldehyde (violet), and kaolinite (green). [Adapted from *J. Atmos. Sci.* 36, 1797 (1979).]

Figure from Wallace and Hobbs
Measurements of ice nucleus concentrations

Empirical relationship

\[ \ln N = a(T - T_1) \]

- \( T_1 \) – temperature at which 1 nucleus/liter is active (typically \( \approx 20^\circ C \))
- \( a \) – constant between 0.3 and 0.6, depending on conditions

e.g. \( a=0.6 \) \( \Rightarrow \) \( N \) increases by factor of 10 for every 4\(^\circ\) decrease in \( T \)

**Fig. 6.31** Measurements of average ice nucleus concentrations at close to water saturation in the northern and southern hemispheres. Southern hemisphere, expansion chamber (red); southern hemisphere, mixing chamber (blue); northern hemisphere, expansion chamber (green); northern hemisphere, mixing chamber (black square); Antarctica, mixing chamber (brown). Vertical lines show the range and mean values (dots) of ice nucleus concentrations based on Millipore filter measurements in many locations around the world.

Figure from Wallace and Hobbs
Effect of supersaturation on ice nucleus concentration

The greater the supersaturation the more particles act as ice nuclei.

empirical fit:

\[ N = \exp(a + b(100(S_i - 1))) \]

\(a=-0.639, \; b=0.1296\)
Maximum concentration of ice particles

Fig. 6.34 Maximum concentrations of ice particles versus cloud top temperature in mature and aging marine cumuliform clouds (blue dots) and in continental cumuliform clouds (red dots). Note that on the abscissa temperatures decrease to the right. Symbols along the abscissa indicate ice concentrations \( \leq 1 \text{ liter}^{-1} \), which was the lower limit of detection. The green line shows ice nucleus concentrations predicted by Eq. (6.33) with \( a = 0.6 \) and \( T_1 = 253 \text{ K} \). The black line shows ice nucleus concentrations from (6.35) assuming water-saturated conditions. [Data from J. Atmos. Sci. 42, 2528 (1985); and Quart. J. Roy. Meteor. Soc. 117, 207 (1991) and 120, 573 (1994). Repro.

- empirical relation from laboratory measurements corresponds to minimum values of maximum concentrations
- concentrations in natural clouds can be several orders of magnitude larger!
Explanations for high ice crystal concentrations

- Measurement techniques in laboratory cannot be applied to natural clouds (conditions too different).
- Ice multiplication or ice enhancement process:
  - Some crystals are fragile and may break up in several splinters when colliding with other particles.
  - Super-cooled droplet freezes in isolation (e.g. free fall), or after it collides with an ice particle (i.e. riming – freezing of droplet on ice crystal).
  - Freezing in 2 stages, particle may explode in 2nd stage of freezing.
Riming

Freezing of droplet on ice crystal.

- riming might be most important for ice enhancement
- when ice particle falls through super-cooled cloud it is impacted by thousands of droplets, each may shed numerous ice splinters

**Laboratory measurement**

- **Setup:**
  - droplet concentration: 50/cm$^3$
  - droplet diameter: 5–35$\mu$m
  - liquid water content: 0.2 g/m$^3$
  - temperature: -4.5°C
  - impact speed: 3.6 m/s

- 300 splinters are produced for every $\mu$g of accumulated rime
Riming

from Avila et al., 2009
Growth from the vapor phase in mixed-phase clouds

- mixed-phase cloud is dominated by super-cooled droplets
- air is close to saturated w.r.t. liquid water
- air is supersaturated w.r.t. ice

**Example**

\[ T = -10^\circ C, \text{RH}_l \approx 100\%, \text{RH}_i \approx 110\% \]
\[ T = -20^\circ C, \text{RH}_l \approx 100\%, \text{RH}_i \approx 121\% \]
⇒ much greater supersaturations than in warm clouds

In mixed-phase clouds, ice particles grow from vapor phase much more rapidly than droplets.
Cloud microphysics

Growth of ice crystal in supercooled water droplets

Growing ice crystal lowers vapor pressure in its vicinity below saturation

⇒ droplets evaporate

Figure from Wallace and Hobbs
Cumulus turrets containing relatively large ice crystals and have fuzzy boundaries.

Turrets containing small water droplets have well-defined sharper boundaries.

Figure from Wallace and Hobbs.
Cumulus turrets containing relatively large ice particles often have ill-defined, fuzzy boundaries, whereas turrets containing only small droplets have well-defined, sharper boundaries, particularly if the cloud is growing (Fig. 6.37). Another factor that contributes to the difference in appearance of ice and water clouds is the lower equilibrium vapor pressure over ice than over water at the same temperature, which allows ice particles to migrate for greater distances than droplets into the nonsaturated air surrounding a cloud before they evaporate. For the same reason, ice particles that are large enough to fall out of a cloud can survive great distances before evaporating completely, even if the ambient air is subsaturated with respect to ice; ice particles will grow in air that is subsaturated with respect to water, provided that it is supersaturated with respect to ice. The trails of ice crystals so produced are called fallstreaks or virga (Fig. 6.38).

The factors that control the mass growth rate of an ice crystal by deposition from the vapor phase are similar to those that control the growth of a droplet by condensation (see Section 6.4.1). However, the problem is more complicated because ice crystals are not spherical and therefore points of equal vapor density do not lie on a sphere centered on the crystal (as they do for a droplet). For the special case of a spherical ice particle of radius $r$, we can write, by analogy with (6.19),

$$\frac{dM}{dt} = \frac{4}{3} \pi r^3 \rho_v \left( \frac{v - v_c}{H^{9267}} \right)$$


where $\rho_v$ is the density of the vapor just adjacent to the surface of the crystal and the other symbols were defined in Section 6.4.1. We can derive an expression for the rate of increase in the mass of an ice crystal of arbitrary shape by exploiting the analogy between the vapor field around an ice crystal and the field of electrostatic potential around a charged conductor of the same size and shape. The leakage of charge from the conductor (the analog of the flux of vapor to or from an ice crystal) is proportional to the electrostatic capacity $C$ of the conductor, which is entirely

$$\frac{dM}{dt} = \frac{4}{3} \pi r^3 \rho_v \left( \frac{v - v_c}{H^{9267}} \right)$$

Figure from Wallace and Hobbs

---

*Fallstreaks of ice crystals*

- since equilibrium vapor pressure over ice is lower than over water, ice crystals evaporate slower and may migrate for larger distances into subsaturated air surrounding the cloud
- large ice crystals may fall out of clouds and survive great distances before they evaporate completely, even if ambient air is subsaturated w.r.t. ice
- trails of ice are called *fallstreaks* or *virga*
Shapes of ice crystals

Fig. 6.40 Ice crystals grown from the vapor phase: (a) hexagonal plates, (b) column, (c) dendrite, and (d) sector plate. [Photographs courtesy of Cloud and Aerosol Research Group, University of Washington.] (e) Bullet rosette. [Photograph courtesy of A. Heymsfield.]
Mass growth rate of an ice crystal

- Diffusional growth of ice crystal similar to growth of droplet by condensation.
- More complicated, mainly because ice crystals are not spherical ⇒ points of equal water vapor do not lie on a sphere centered on crystal.

\[ \frac{dM}{dt} = 4\pi CD (\rho_v(\infty) - \rho_{vc}) \]
Mass growth rate of an ice crystal

Approximate form:

\[
\frac{dM}{dt} = 4\pi CG_i S_i
\]

- Maximum growth rate at about -14°C

**Fig. 6.39** Variation of \(G_i S_i\) [see Eq. (6.37)] with temperature for an ice crystal growing in a water-saturated environment at a total pressure of 1000 hPa.

Figure from Wallace and Hobbs
Mass growth rate of an ice crystal

Maximum growth rate at about $-14^\circ\text{C}$

$\Rightarrow$ difference between saturated pressures over water and ice is maximal at this temperature

$\Rightarrow$ ice crystals grow most rapidly

**Fig. 3.9** Variations with temperature of the saturation (i.e., equilibrium) vapor pressure $e_s$ over a plane surface of pure water (red line, scale at left) and the difference between $e_s$ and the saturation vapor pressure over a plane surface of ice $e_{si}$ (blue line, scale at right).

Figure from Wallace and Hobbs
Morphology diagram

Figure from Libbrecht 2005
Growth by accretion

- ice crystals falling through cloud of supercooled water droplets and other ice crystals may grow by **accretion** of water or of other ice crystals
- leads to rimed structures and graupel

**Fig. 6.41** (a) Lightly rimed needle; (b) rimed column; (c) rimed plate; (d) rimed stellar; (e) spherical graupel; and (f) conical graupel. [Photographs courtesy of Cloud and Aerosol Research Group, University of Washington.]

Figure from Wallace and Hobbs
Growth by aggregation

Fig. 6.44  Aggregates of (a) rimed needles; (b) rimed columns; (c) dendrites; and (d) rimed frozen drops. [Photographs courtesy of Cloud and Aerosol Research Group, University of Washington.]

Figure from Wallace and Hobbs

Snowflakes are formed by aggregation.
Collection efficiency for accretion

collection efficiency = collision efficiency $\times$ coalescence efficiency

- can be determined theoretically for simple ice plates (Pitter and Pruppacher, 1974)
  - aerodynamic calculation of trajectories of water droplets relative to ice crystals
  - coalescence efficiency $\approx 1$, because ice crystals are relatively small
Collection efficiency for aggregation

- not yet determined theoretically
- observations:
  - open structures (e.g. dendrites) more likely stick to other ice crystals
  - sticking more likely at higher temperatures
- $\Rightarrow$ significant aggregation only at $T > -10^\circ C$
Mass growth rate for accretional and aggregational growth

\[
\frac{dm}{dt} = \bar{E} w_l \pi R^2 (v(R) - v(r))
\]

- $\bar{E}$ – mean collection efficiency
- $w_l$ – cloud liquid water content
- $v$ – fall speed of crystals / droplets
- $R$ – radius of collector crystal
- $r$ – radius of supercooled droplets

Same approach for aggregation, with $w_l$ replaced by $w_i$ (ice water content).
Formation of precipitation in cold clouds

- 1789: Franklin suggested that “much of what is rain, when it arrives at the surface of the Earth might have been snow, when it began its descent ...”
- 1911: Wegener stated that ice particles grow preferentially by deposition from the water phase in mixed phase clouds.
- 1933: Bergeron, 1938: Findeisen
  First quantitative studies of formation of precipitation in cold clouds

Bergeron-Findeisen Process

1. Deposition from vapor phase
2. Riming / aggregation

⇒ precipitation sized particles can be produced in reasonable time periods.
Field experiments

- several field experiments were performed in the last years: e.g. CRYSTAL-FACE, INCA, TC4 ...
- provide information about cloud microphysics at specific points in the cloud, usually no measurements of vertical structure
- can not characterize evolution of cloud microphysics spatial and temporal structure, and link these characteristics to environmental factors (available CCN)
Remote sensing methods

- **Precipitation radar**: vertical development of precipitation sized droplets in clouds, information on thermodynamic phase of hydrometeors
- **Cloud radar (millimeter wavelength)**: cloud boundaries (bottom and top), small droplets not measured
- **Satellite images (visible, NIR)**: Provide information about optical thickness and particle size (at cloud top).
  - Polar orbit (e.g. MODIS): relatively good spatial resolution (1km), but poor temporal resolution
  - Geostationary orbit (e.g. MSG): good temporal resolution (up to 5 min), but poor spatial resolution (5km)
Conceptual diagram of microphysical stages

The diagram describes 5 microphysical stages (droplet growth by diffusion, collision-coalescence, warm rainout, ice-water mixed phase, glaciated phase).

- **Bottom curve**: maritime environment with low CCN concentration (possibility of warm rainout).
- **Middle curve**: continental case, large number of CCN suppress warm rain, glaciation starts at slightly lower T.
- **Top curve**: polluted environment where very large number of CCNs produce numerous small droplets at cloud base, supressing collision-coalescence, freezing starts at even lower T.

Figure from Martins et al., 2011, adapted from Rosenfeld and Woodley, 2003.
Cloud side remote sensing

- **vertical profile of effective radius**: very sensitive to aerosol environment
- **brightness temperature profile**: can directly be associated with thermodynamic phase, provides information on the glaciation temperature
- **high temporal resolution**: Evolution of cloud microphysics can be observed
First cloud side measurements

Figure from Martins et al., 2011
Retrieval of effective radius and cloud phase

Figure from Martins et al., 2011
MYSTIC simulation of cloud scanner observation

Figure from Zinner et al., 2008
Cloud observation system at MIM

Figure from Zinner et al., Poster EGU 2009