hybrid method (varETKF)

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hybrid methods (varETKF)

Here we describe a hybrid (3dvar/Ensemble filter) method following *Buehner et al*, in the following called varETKF.

- to obtain the *incremental* formulation of 3dVar we define $\Delta\mathbf{x} \equiv \mathbf{x} - \mathbf{x}^b$ and rewrite the cost function as

$$\begin{aligned} J(\Delta\mathbf{x}) &= \frac{1}{2}\Delta\mathbf{x}^T\mathbf{B}^{-1}\Delta\mathbf{x} + \frac{1}{2}(\mathbf{y}' - \mathbf{H}\Delta\mathbf{x})^T\mathbf{R}^{-1}(\mathbf{y}' - \mathbf{H}\Delta\mathbf{x}) \\ &= J_b + J_o, \end{aligned}$$

  where $\mathbf{y}' = \mathbf{y} - H(\mathbf{x}^b)$

- Now use the variable transform (*preconditioning*) $\Delta\mathbf{x} = \mathbf{B}^{1/2}\gamma$.
- Then the cost function can be further rewritten as

$$\tilde{\mathbf{J}}(\gamma) = \frac{1}{2}\gamma^T\gamma + (\mathbf{y}' - \mathbf{HB}^{1/2}\gamma)^T\mathbf{R}^{-1}(\mathbf{y}' - \mathbf{HB}^{1/2}\gamma)$$

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- we now augment the state vector and write:

$$\begin{aligned}\Delta \mathbf{x} &= \beta_1 \mathbf{B}_1^{1/2} \gamma_{\mathbf{1}} + \beta_2 \mathbf{B}_2^{1/2} \gamma_{\mathbf{2}} \\ &= \beta_1 \Delta \mathbf{x}_1 + \beta_2 \Delta \mathbf{x}_2\end{aligned}$$

  with static *full rank* $\mathbf{B}_2$ and ensemble generated *low rank* $\mathbf{B}_1$.
- In the minimization the augmented control vector $\gamma = [\gamma_1 \quad \gamma_2]^T$ is used. It has dimension $N + N \cdot N_{ens} = (N_{ens} + 1)N$, where N is the model dimension. Compared with 3dVar where the dimension of the control vector is N this significantly increases the numerical cost.
- The ensemble part of the control vector can be written as

$$\Delta \mathbf{x}_1 = \frac{1}{\sqrt{N_{ens}}} \sum_{n=1}^{N_{ens}} \mathbf{e}_n \circ \left(\mathbf{L}_g^{1/2} \gamma_n \right)$$

- here, $\mathbf{L}_g$ is a *localization operator*. The γ_n are vectors of length N.

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- the basic idea is to use a mixture of a *full rank*, but *static* matrix $\mathbf{B}_2$ and a *low rank*, but ensemble generated and thus *dynamic* matrix $\mathbf{B}_1$. Thus, the corrections made by the analysis are not restricted to the subspace spanned by the ensemble perturbations, but at the same time use dynamic informations.
- the weights β_1 and β_2 determine the relative influence of the ensemble part and the deterministic part, respectively. Their squares should add up to 1, $\beta_1^2 + \beta_2^2 = 1$, to keep the total variance. The actual values used for β_1 and β_2 are tuning factors.
- localization is - as in the LETKF - necessary to supress spurious correlations. Here, localization is performed on the $\mathbf{B}$ matrix, not on the observations as in the LETKF. This is an advantage in the case of nonlocal observations as radiances.
- The varETKF just gives a *deterministic analysis*; to obtain a *analysis ensemble* e.g. a LETKF has to be run. The LETKF analysis ensemble is then centered around the varETKF analysis.

References

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