Deutscher Wetterdienst
Wetter und Klima aus einer Hand

The Lorenz95 system

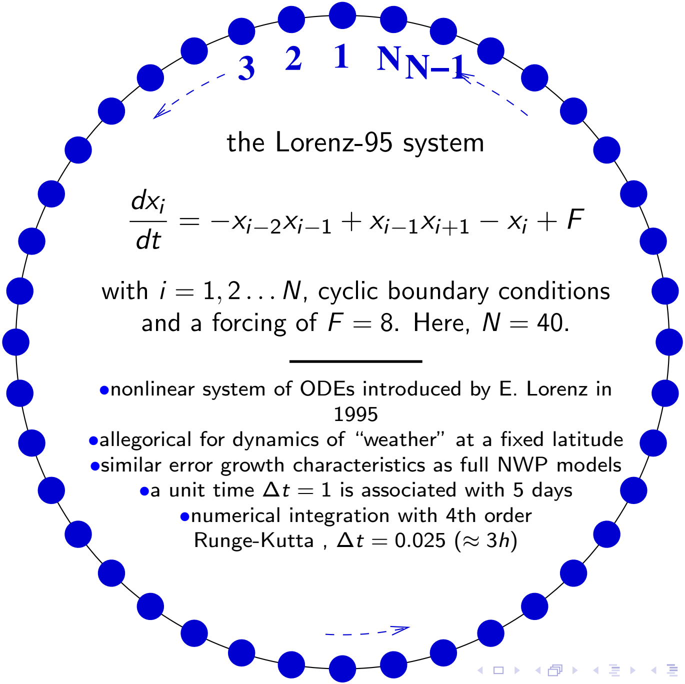
Hendrik Reich, Daniel Leuenberger, Michael Würsch

DWD-HErZ winterschool on data assimilation, Offenbach

13-17. February 2012

Acknowledgment

Some parts of the L95 introduction and the integration part of the scilab programs have been taken from the ECMWF data-assimilation tutorial by Martin Leutbecher



the Lorenz-95 system

$$\frac{dx_i}{dt} = -x_{i-2}x_{i-1} + x_{i-1}x_{i+1} - x_i + F$$

with $i = 1, 2 \ldots N$, cyclic boundary conditions and a forcing of $F = 8$. Here, $N = 40$.

- nonlinear system of ODEs introduced by E. Lorenz in 1995
- allegorical for dynamics of “weather” at a fixed latitude
- similar error growth characteristics as full NWP models
- a unit time $\Delta t = 1$ is associated with 5 days
- numerical integration with 4th order Runge-Kutta , $\Delta t = 0.025$ ($\approx 3h$)

L95 system

- With the forcing used here, $F = 8$, the systems behaves chaotic:

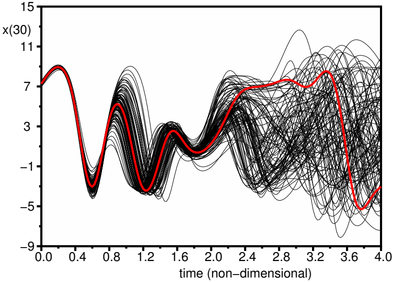


Fig. 1: gridpoint 30 as a function of time for slightly different initial conditions

- With this settings the system has 13 positive Lyapunov exponents, the largest one corresponds to a doubling time of 2.1 days.
- Variables fluctuate about the mean with a climatological standard deviation of $\sigma_{clim} \approx 3.6$

L95 system

- The dynamic is the same for each variable; the equation is invariant under a transformation $i \to i + 1$.
- A perturbation of the initial condition will grow and its leading edge propagates "eastward" (higher indices) at a speed of about 25 degrees/day - corresponding to 14 indices in a (non-dimensional) time unit.
- Due to these properties the L95 system can be used as a toy model for data assimilation; it has some of the features of realistic NWP systems but is low-dimensional
- limits of L95 system: e.g. *convectice scale*; here, more sophisticated toy models are needed (e.g. *shallow water* models)

References

Lorenz, E. N., 1995: Predictability: A problem partly solved. In *Seminar on Predictability*, volume Vol. I, ECMWF, Reading, UK, 1-18. Available at http://www.ecmwf.int/publications/library/ecpublications/pdf/seminar/1995

Lorenz, E. N., 2005: Designing chaotic models. *J. Atmos. Sci.*, **62**, 1574-1587.

Lorenz, E. N. and K. A. Emanuel, 1998: Optimal sites for supplementary weather observations: Simulation with a small model. *J. Atmos. Sci.*, **55**, 399-414.