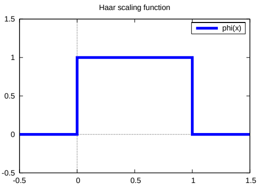
DWD Representation of $\mathbf{B}$ in the 3D-Var

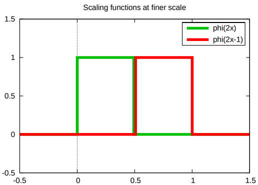
- Change of basis from grid-point space to wavelet space

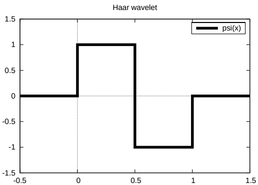
$$\mathbf{B} = \mathbf{W}\hat{\mathbf{B}}\mathbf{W}^T = \mathbf{W}\hat{\mathbf{L}}\hat{\mathbf{L}}^T\mathbf{W}^T$$

- Wavelets provide a versatile basis for multi-resolution analyses
  Example: Haar wavelet

$$\begin{aligned} \phi_{\text{Haar}}(x) &= \phi_{\text{Haar}}(2x) + \phi_{\text{Haar}}(2x-1)\,, \\ \psi_{\text{Haar}}(x) &= \phi_{\text{Haar}}(2x) - \phi_{\text{Haar}}(2x-1) = \begin{cases} 1 & \text{for } 0 \le x < \frac{1}{2} \\ -1 & \text{for } \frac{1}{2} \le x < 1 \\ 0 & \text{otherwise.} \end{cases} \end{aligned}$$







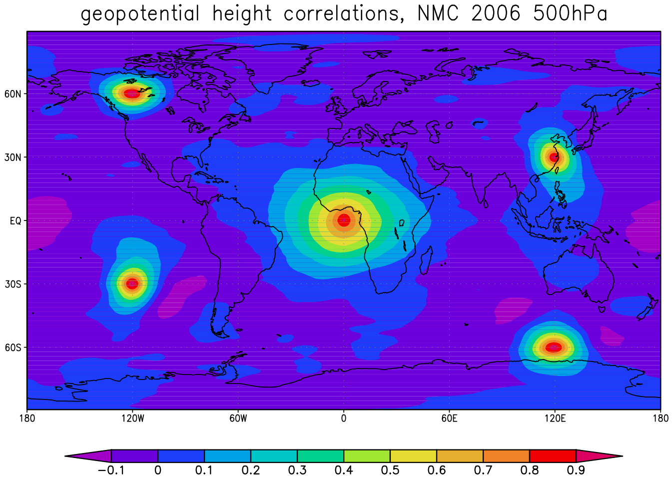
DWD Representation of **B** in the 3D-Var

- Higher-order wavelets allow for efficient compression of large matrices, only few coeffcents per grid-point necessary
  $\Rightarrow$ sparse representation of full correlation matrices possible!
  (However not yet implemented in 3 dimensions)
- Very fast transformation algorithms
  (much faster than spectral transform)
- Implementation of linear balance:

$$\hat{\mathbf{L}} = \begin{pmatrix} \hat{\mathbf{L}}_{hh} & & & \\ \hat{\mathbf{L}}_{\psi h} & \hat{\mathbf{L}}_{\psi_u \psi_u} & & \\ . & . & \hat{\mathbf{L}}_{\chi\chi} & \\ . & . & . & \hat{\mathbf{L}}_{rhrh} \end{pmatrix}$$

NMC derived covariances

Horizontal correlations for geopotential height in 500hPa (512 x 256 grid-points), reconstructed from truncated wavelet expansion



NMC derived covariances

Covariance with a location in 100 hPa height in vertical slice at the equator (512×64 grid-points, vertical axis from 1000 to 10 hPa equidistant in log p)