Klausur: Advanced Atmospheric Dynamics

03.08.2011

1. The cost function for 4DVAR is given by

$$J(\delta\mathbf{x}_0) = \delta\mathbf{x}_0^T\mathbf{B}^{-1}\delta\mathbf{x}_0 + \sum_{k=0}^{K}(\mathbf{d}_k - \mathbf{H}_k\mathbf{M}_{k-1,k}...\mathbf{M}_{0,1}\delta\mathbf{x}_0)^T\mathbf{R}_k^{-1}(\mathbf{d}_k - \mathbf{H}_k\mathbf{M}_{k-1,k}...\mathbf{M}_{0,1}\delta\mathbf{x}_0)$$

a) [10 Points] Name the symbols that appear in this equation.
b) [10 Points] Explain why the value of $\mathbf{x}_0$ that minimises J will lead to a good analysis of the atmospheric state.

2. a) [10 Points] Define the background error covariance matrix $\mathbf{B}$ in terms of the true state of the atmosphere $\mathbf{x}_t$ and the background state $\mathbf{x}_b$.

b) [5 Points] What is the meaning of the diagonal elements of $\mathbf{B}$?

c) [5 Points] What is the meaning of the off-diagonal elements of $\mathbf{B}$?

d) [10 Points] Why are most of the off-diagonal elements of $\mathbf{B}$ equal to zero? (There is more than one reason.)

3. [20 Points] The famous Lorenz 3-variable dynamical model is defined by:

$$\begin{aligned}\frac{dx_1}{dt} &= \sigma\,(x_2 - x_1)\\ \frac{dx_2}{dt} &= x_1\,(\rho - x_3) - x_2\\ \frac{dx_3}{dt} &= x_1x_2 - \beta x_3\end{aligned}$$

where the model state is given by $\mathbf{x} = (x_1, x_2, x_3)^T$, and σ, ρ and β are constants. State the mathematical definition of the tangent linear model, and compute it for the Lorenz system, linearised about a background state $\mathbf{x}_b$.

4. [30 Points] We wish to construct a data assimilation system to produce an analysis of soil moisture based on a single observations type, namely satellite measurements of microwave emmission from the surface. Suggest an appropriate data assimilation method, and explain your choice, taking into consideration the nature of the observational errors, the physical processes that must be modeled, and the background error characteristics.