Review for Introduction to Data assimilation

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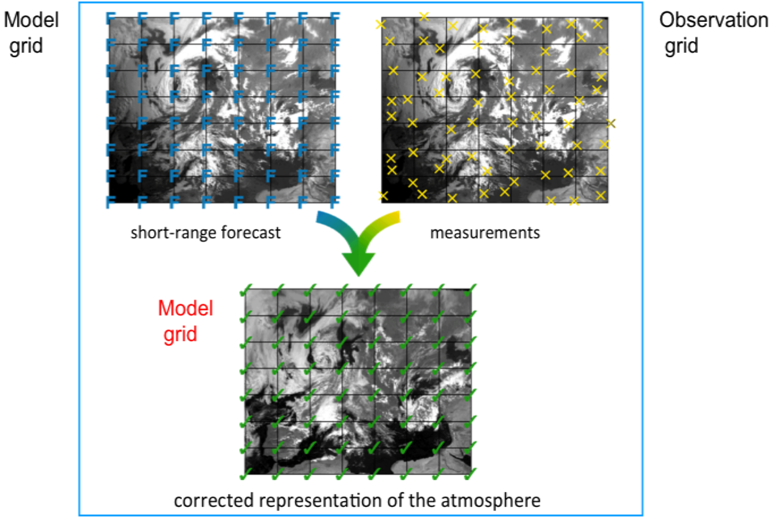
Content

- What is data assimilation and why do we need it?
- Components of data assimilation system

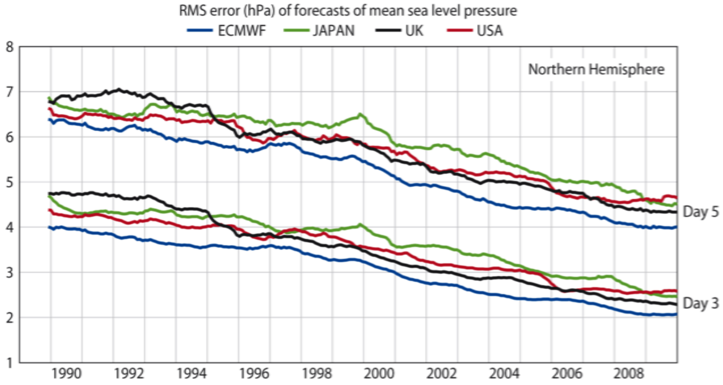
  Observing system
  (Numerical models)
  Data assimilation algorithms

- Basics of data assimilation methods
- Data assimilation methods in practice

What is data assimilation?



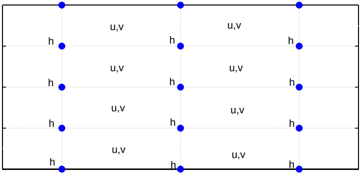
Data assimilation matters



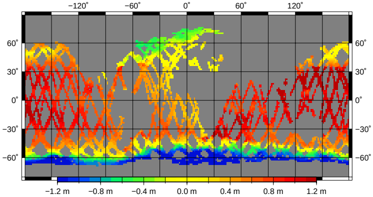
- NWP is continuously improving
- ECMWF has been leading for decades
- This is to a large extent due to efforts for data assimilation (other scores less drastic, but generally consistent)
- The computing time for data assimilation is nowadays often larger than for the deterministic forecast

Data assimilation algorithm combine forecast and observations to produce the best analysis

$\mathbf{w}_k^f$



$\mathbf{y}_k^o$



Analysis systems are dependent on appropriate statistics for observation and background errors.

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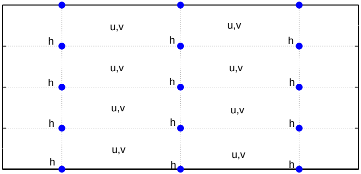

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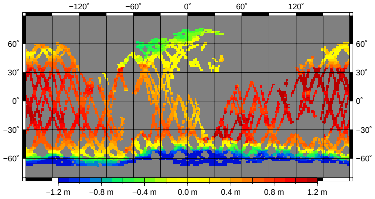
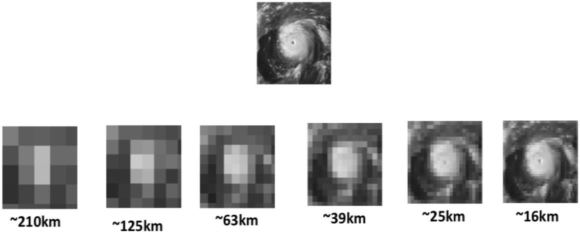


Analysis systems are dependent on appropriate statistics for observation and background errors.

Our goal: Best analysis for a prediction.

One major contributor to the forecast uncertainty is the model error.



Model resolution (slide ECMWF)

Model error

Unfortunately model error statistics are not perfectly known and their determination remains a major challenge in assimilation systems.

Reasons behind the model error:

- accuracy of numerical schemes
- unrepresented subgrid scale processes
- inaccurate forcing and boundary conditions
- representation of orography as well as parametrisation uncertainty.

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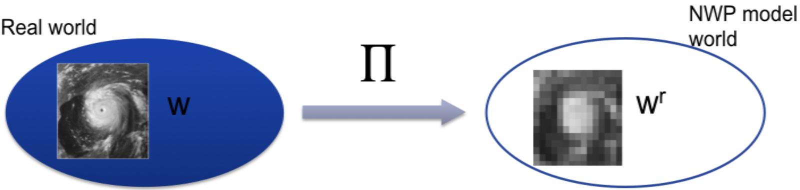
Model error statistics produced by use of multiple physics packages,inclusion of stochastic kinetic energy backscatter scheme, parameter variations, as well as use of deterministic stochastic dynamical models (Berner et al. 2011).

Model Error

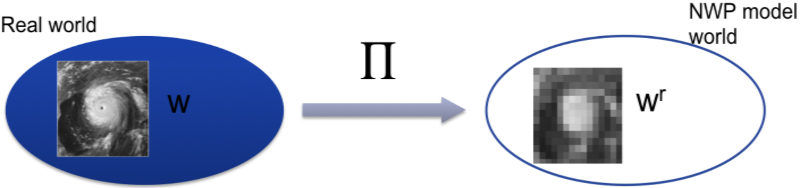


from time k to time k+1 atmosphere evolves without us knowing perfectly time propagator, F^c.

from time k to time k+1 numerical model, F, propagates w^r

Model error is the difference: $\Pi F^c(w) - F(w^r)$.

Observation Error



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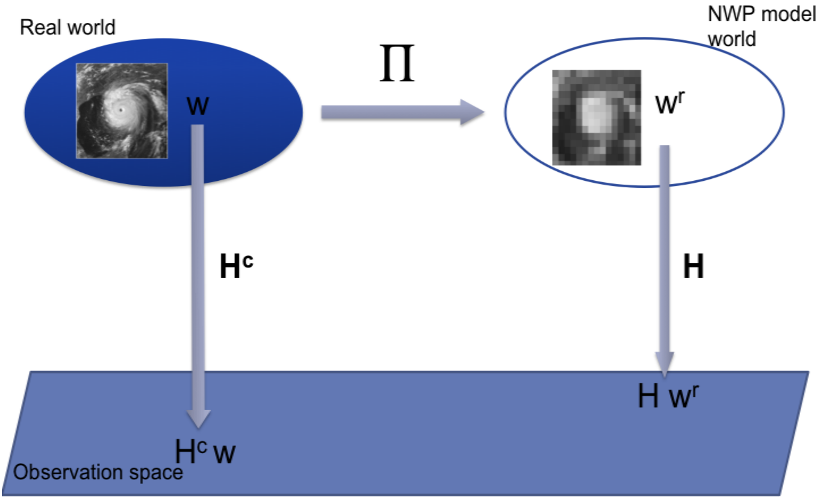

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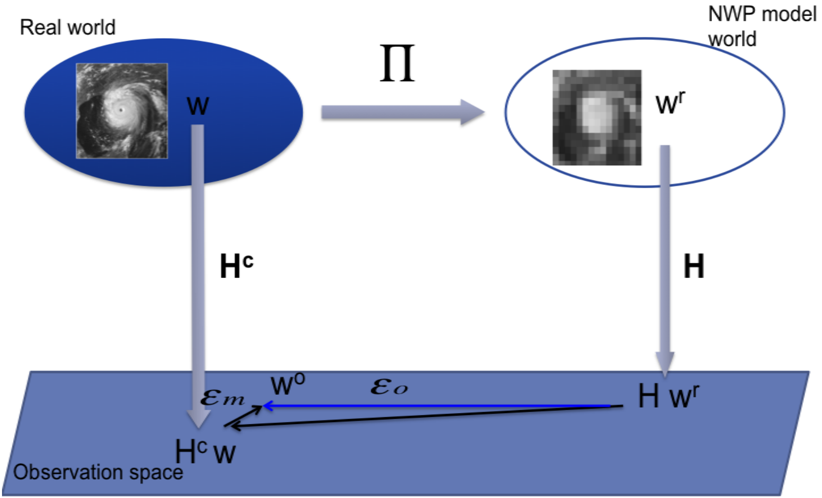
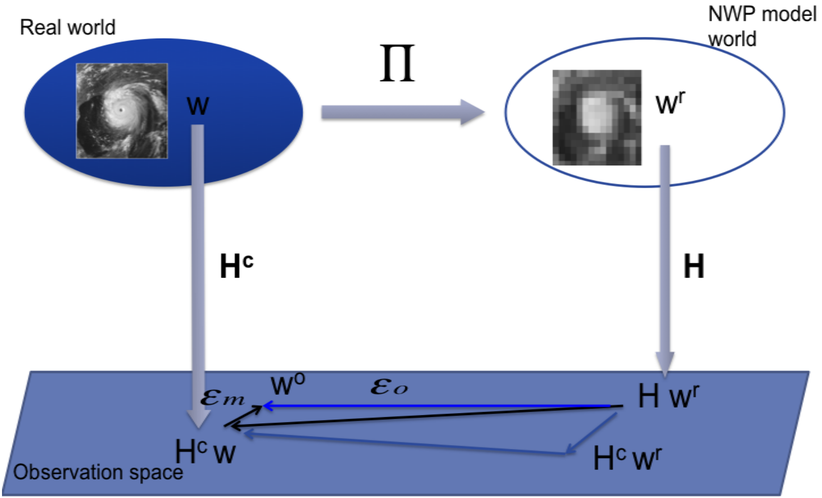


Observation Error



Atmospheric Data Assimilation

The state $\mathbf{w} \equiv \mathbf{w}(\mathbf{x}, t)$ of the atmosphere at time t_k:

$$\mathbf{w}(\mathbf{x}, t_k) \equiv \begin{bmatrix} w_1(\mathbf{x}, t_k) \\ \vdots \\ w_q(\mathbf{x}, t_k) \end{bmatrix}$$

where $w_i : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}$, $\forall i = 1, \ldots, q$, $w_i \in \mathcal{B}$.
($\mathcal{B}$ is a vector space of scalar valued, continuous functions.)

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Discrete problem: Find an estimate of some projection $\mathbf{\Pi w}$ of $\mathbf{w}$ on the space of the dynamical model.

$$\mathbf{\Pi w}(\mathbf{x}, t_k) \equiv \begin{bmatrix} \Pi w_1(\mathbf{x}, t_k) \\ \vdots \\ \Pi w_q(\mathbf{x}, t_k) \end{bmatrix}$$

where $\Pi w_i : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}$, $\forall i = 1, \ldots, q$.
$\Pi w_i(\mathbf{x}, t_k) \in \mathcal{B}_N$, where $\mathcal{B}_N$ is an N-dimensional subspace of $\mathcal{B}$.

Observation error

ϵ_k^o consists of measurement error and representativeness error. It can be divided into three parts:

$$\epsilon_k^o = \epsilon_k' + \epsilon_k'' + \epsilon_k^m$$

where

$$\begin{aligned} \epsilon_k' &\equiv \mathbf{H}_k^c \mathbf{w}(\cdot, t_k) - \mathbf{H}_k^c \mathbf{\Pi} \mathbf{w}(\cdot, t_k) \\ &= \mathbf{H}_k^c (\mathbf{I} - \mathbf{\Pi}) \mathbf{w}(\cdot, t_k) \end{aligned}$$

ϵ_k' – will be called *error due to unresolved scales*.

$$\begin{aligned} \epsilon_k'' &\equiv \mathbf{H}_k^c \mathbf{\Pi} \mathbf{w}(\cdot, t_k) - \mathbf{H}_k \mathbf{\Pi} \mathbf{w}(\cdot, t_k) \\ &= [\mathbf{H}_k^c - \mathbf{H}_k] \mathbf{\Pi} \mathbf{w}(\cdot, t_k) \end{aligned}$$

ϵ_k'' – will be called *forward interpolation error*.

Observations for global models

Conventional observations

- *Observations of model variables*

  (u, v, q, p)

Radiosondes, pilot, dropsondes

Surface stations, ships

Buoys

Aircraft

Atmospheric Motion Vectors

Wind profiler

Non-conventional observations

- Complex observation operators

  (for mapping from model to observation space)

- Mainly from satellite

Passive instruments (mainly T, q and O3 information)

- Microwave radiance
- Infrared radiance

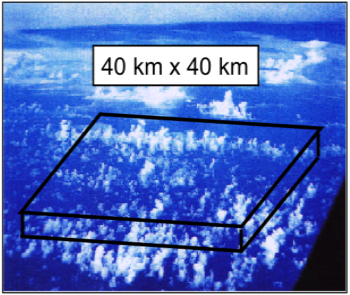
(clouds are usually seen as contamination)

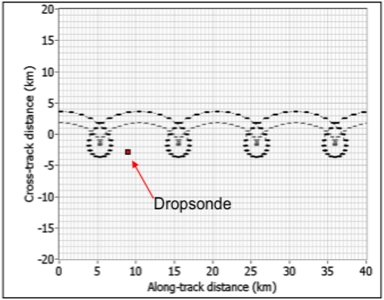
(often only data over oceans is used)

Active instruments

- Radar (scatterometer surface winds)
- GPS radio-occultation
- Lidar (not operational)

Representativeness error





Observational error lidar: 0.75-1 m/s

Rep. error < 0.5 m/s (Frehlich and Sharman 2004)

Assigned error: 1-1.5 m/s

Radiosonde/Dropsonde: (most accurate operational wind observation)

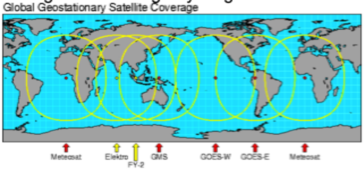
Observational error <0.5 m/s

Assigned error: 2-3 m/s

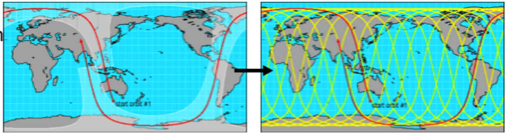
Assigned error AMV: 2-5 m/s

Satellite observations

	Advantages	Disadvantages
GEO (high) MSG GOES MTSAT	- large regional coverage	- no global coverage by single satellite



	Advantages	Disadvantages
GEO	- very high temporal resolution > short-range forecasting/nowcasting > feature-tracking (motion vectors) > tracking of diurnal cycle (convection)	- moderate spatial resolution (VIS/IR) > 5-10 km for VIS/IR > much worse for MW
LEO Low Earth Orbit NOAA Metop Terra, Aqua	- global coverage with single satellite - high spatial resolution >best for NWP!	- low temporal resolution



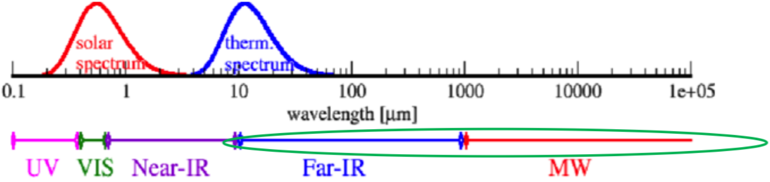
What do satellites measure?

Satellite instruments measure: Radiances

Emitted radiation	- From earth & atmosphere (thermal radiation)
Reflected radiation:	- Solar radiation - Radar / Lidar radiation

Electromagnetic spectrum

VIS = visible light
IR = infrared radiation
MW = microwave radiation



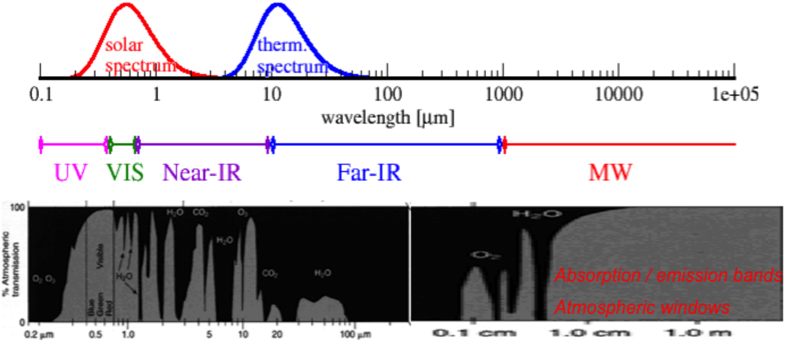
Typical wave-lengths for earth observation satellites

Measurement principles

Active: radar, lidar, radio-occultations

Passive: Radiance measurements

- Resolution
  - Sounders: vert. res.
  - Imagers: horiz. res.
- Wavelength
  - Visible
  - Infrared
  - Microwave
- Agency / purpose
  - R&D: ESA, NASA, JAXA
  - operational: EUMETSAT, NOAA, JMA

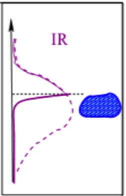


Comparison IR and MW

Infra-red	*Microwave*
Stronger emission → higher resolution	Weaker emission, typically lower resolution
Sensitive to clouds	Can see through clouds Can be used to correct cloud-contaminated IR
Meteosat SEV**IR**I GOES MTSAT	A**M**SU-A: Temperature SS**M**/I: Window channel, TCWV A**M**SU-B: Humidity **M**HS: Microwave Humidity Sounder A**M**SR-E
H**IR**S **I**ASI A**IR**S	

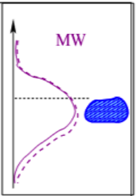
Clouds in MW and IR

- IR
  - strongly nonlinear (opaque)
  - good identification of cloud location
  - more than 80% of channels are affected by cloud
  - cloud impact on IR sounders is subtracted from RT computations.



usually opaque

- MW
  - multiple scattering important
  - sees only thick clouds and rain
  - less than 20% of channels are affected by cloud
  - cloud information from MW imagers is assimilated



signal modified by cloud

ECMWF now uses

- "all-sky" MW radiances
- contaminated IR-radiances for fully overcast scenes (in both observation and forecast)

One of the major contributions to forecast improvement in recent years

Issues: (1) cloud radiative transfer and (2) model clouds may not be realistic

Conclusion part 1

- Data assimilation algorithms require us to specify the statistical properties of the observation and model error.
- Both of these errors depend on the state of the atmosphere.
- Since we are searching for the best estimate for the scales that our model can represent,
- the unresolved scales are part of the model error as well as observation error.
- The error of unresolved scales is particularly large for the sonde observations, and the forward observation error for the satellite data.

Data assimilation methods

- 3DVar
- 4DVar (ECMWF)
- Kalman filter
- Ensemble Kalman filter (Enviroment Canada)
- Hybrid methods (NCEP)

3DVar

$$J = (\mathbf{x} - \mathbf{x}_b)^T \mathbf{B}^{-1} (\mathbf{x} - \mathbf{x}_b) + (\mathbf{y} - \mathbf{H}\mathbf{x})^{\mathbf{T}} \mathbf{R}^{-1} (\mathbf{y} - \mathbf{H}\mathbf{x})$$

or

$$J = \delta\mathbf{x}^T \mathbf{B}^{-1} \delta\mathbf{x} + (\mathbf{d} - \mathbf{H}\delta\mathbf{x})^T \mathbf{R}^{-1} (\mathbf{d} - \mathbf{H}\delta\mathbf{x})$$

where

$$\delta\mathbf{x} = \mathbf{x} - \mathbf{x}_b \text{ and } \mathbf{d} = \mathbf{y} - H(\mathbf{x_b})$$

with gradient given by

$$\nabla J = \mathbf{B}^{-1} \delta\mathbf{x} - \mathbf{H}^T \mathbf{R}^{-1} (\mathbf{d} - \mathbf{H}\delta\mathbf{x}).$$

- **B** needs to be specified from climatology.

3DVar

- Due to the large minimization problem (of the order 10^8) iterative techniques used for minimization
- To speed up the minimization process transformation of variables is used as well.
- Further approximation include number of iterations performed, simplifications of covariances and linearity of observation operator.
- analysis not consistent model state and timing of the observations ignored

4DVar

$$J(\delta \mathbf{x}_0) = \delta \mathbf{x}_0^T \mathbf{B}^{-1} \delta \mathbf{x}_0$$

$$+ \sum_{k=0}^{K} (\mathbf{d}_k - \mathbf{H}_k \mathbf{M}_{k-1,k} \ldots \mathbf{M}_{0,1} \delta \mathbf{x}_0)^T \mathbf{R}_k^{-1} (\mathbf{d}_k - \mathbf{H}_k \mathbf{M}_{k-1,k} \ldots \mathbf{M}_{0,1} \delta \mathbf{x}_0)$$

- **B** needs to be specified from climatology.
- needs tangent linear model and adjoint
- waits for the observations
- invalid for strong nolinearities

4DVar

- assimilates observations at correct time
- $\mathbf{B}$ is evolved according to dynamics
- Analysis close to consistent model state

Kalman filter

$$\mathbf{x}_k^a = \mathbf{x}_k^b + \mathbf{K}_k(\mathbf{y}_k^o - \mathbf{H}_k\mathbf{x}_k^b),$$

$\mathbf{K}_k$ is taken as

$$\mathbf{K}_k = \mathbf{P}_k^b\mathbf{H}_k^T(\mathbf{H}_k\mathbf{P}_k^b\mathbf{H}_k^T + \mathbf{R}_k)^{-1}$$

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$$\mathbf{P}_k^a = (\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)^T\mathbf{P}_k^b.$$

$$\mathbf{x}_k^b = \mathcal{M}\mathbf{x}_k^b$$

$$\mathbf{P}_k^b = \mathbf{M}\mathbf{P}_k^a\mathbf{M}^T + \mathbf{Q}.$$

Derived under assumptions that $q \sim \mathcal{N}(0, \mathbf{Q})$ and $r \sim \mathcal{N}(0, \mathbf{R})$ and $< r_k q_j >= 0$

Kalman filter

- Recursive filter. There is no need to store past measurements, all the information is embodies in the prior estimate.
- It is the optimal filter in case observation operator is linear, dynamics are linear, observation and model errors are Gaussian and uncorrelated.

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- However, over the same time interval under assumption that model is perfect and that both algorithms use the same data, then there is equivalence between final analysis produced by Kalman filter and final value of the optimal trajectory estimated by 4DVar.

Why ensemble Kalman filter

- The Kalman filter is difficult to implement in realistic systems because of:
  - computational costs,
  - the nonlinearity of dynamics and
  - poorly characterized error sources.
- The ensemble Kalamn filter (EnKF) (Evensen 1994) uses ensembles (a sample) to calculate the uncertainty of the background and analysis error covariance.
- Ensembles are propagated with full nonlinear numerical model. This can be done over long time period, and results in flow dependent covariances.

Ensemble Kalman filter

- Kalman filter equations are used with covariance calculated from the sample.
- Covariances are flow dependent and computationally algorithm is not expensive.
- Additional step to the calculation of the analysis is added, the resampling step, where new ensemble are generated.
- ETKF algorithm takes an advantage of the small number of ensemble members to have the equation written in reduced form.

Ensemble Kalman filter

- Only small number of ensembles can be evolved due to complexity of the dynamical systems;
- Due to the small ensemble numbers covariances are not representing correctly uncertainty, in particular long-distance correlations, and this effects the accuracy of the analysis.
- The analysis increment is restricted to the r dimensional subspace
- Localization is introduced to elevate the problem.
- Uses full nonlinear model
- evaluates its own $\mathbf{B}$.

Conclusion Part 2

- All data assimilation algorithms require us to specify the statistical properties of errors.
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- Master thesis http://www.meteo.physik.uni-muenchen.de/dokuwiki/doku.php?id=lscraig:herz:master