Observation Nudging

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Observation Nudging

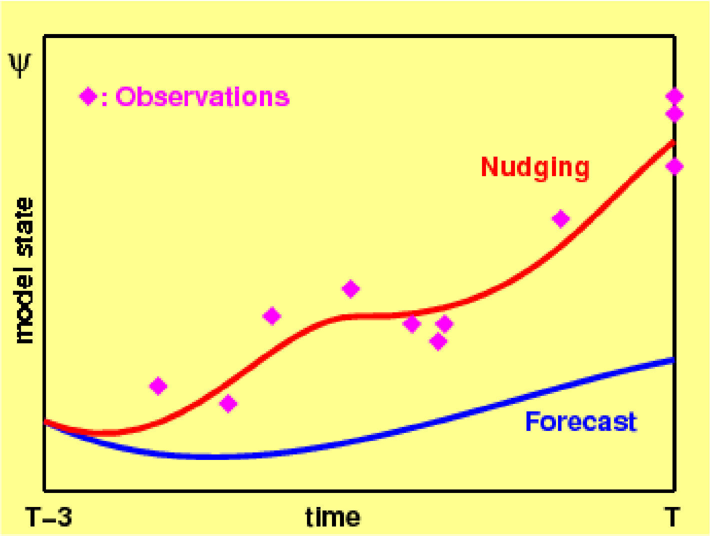
$$\frac{\partial}{\partial t}\psi(\underline{x},t) \;=\; F(\underline{\psi},\underline{x},t) \;+\; G_\psi \cdot \sum_{k_{(obs)}} W_k \cdot \left[\psi_k - \psi(\underline{x}_k,t)\right]$$

G_ψ determines the characteristic time scale for the relaxation

The weight W_k for the model grid point $(\underline{x},t)$ depends on

- temporal distance to the observation (w_t)
- spatial distance to the observation (w_{xy}, w_z)
- observation errors (ε_k)

$$W_k \;=\; \frac{w_k}{\sum_j w_j} \cdot w_k \qquad w_k \;=\; w_t \cdot w_{xy} \cdot w_z \cdot \varepsilon_k$$

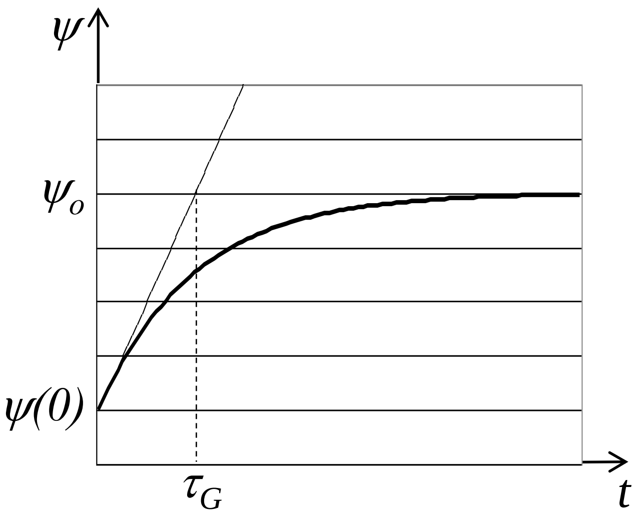


Relaxation Time Scale

- Scalar variable, only dependent on t: $\psi = \psi(t)$
- No physical forcing: $F = 0$
- Constant weights: $W = 1$
- Time-invariant observation: ψ_o

$$\dot{\psi}(t) = G_\psi \cdot [\psi_o - \psi(t)]$$

$$\psi(t) = \psi_o + [\psi(0) - \psi_o] \cdot e^{-G_\psi t}$$



- Exponential approach to observation with $\tau_G = 1/G_\psi$

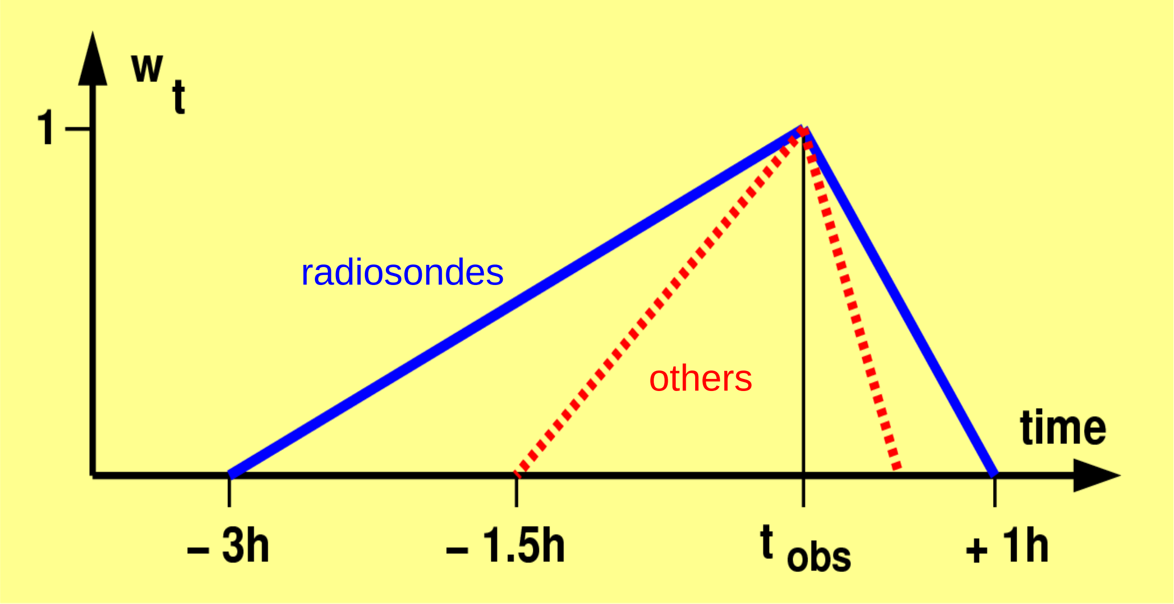
Nudging Constant G_ψ

- Determines relative magnitude of the nudging term to all other processes contained in F
- Nudging time scale τ_G usually matches the time scale of the slowest physical adjustment process of the system
- For meteorological systems typically $10^{-4}\ s^{-1} \leq G_\psi \leq 10^{-3}\ s^{-1}$ (15min $\leq \tau_G \leq$ 3h)
- Numerical stability criterion: $G_\psi \leq 1/\Delta t$
- Values of G_ψ which are
  - Too large: overruling and disruption of model physics
  - Too small: too small impact of observations
- G_ψ needs to be tuned…

Structure functions

$$w_k = w_t \cdot w_{xy} \cdot w_z \cdot \varepsilon_k$$

- Temporal structure function w_t

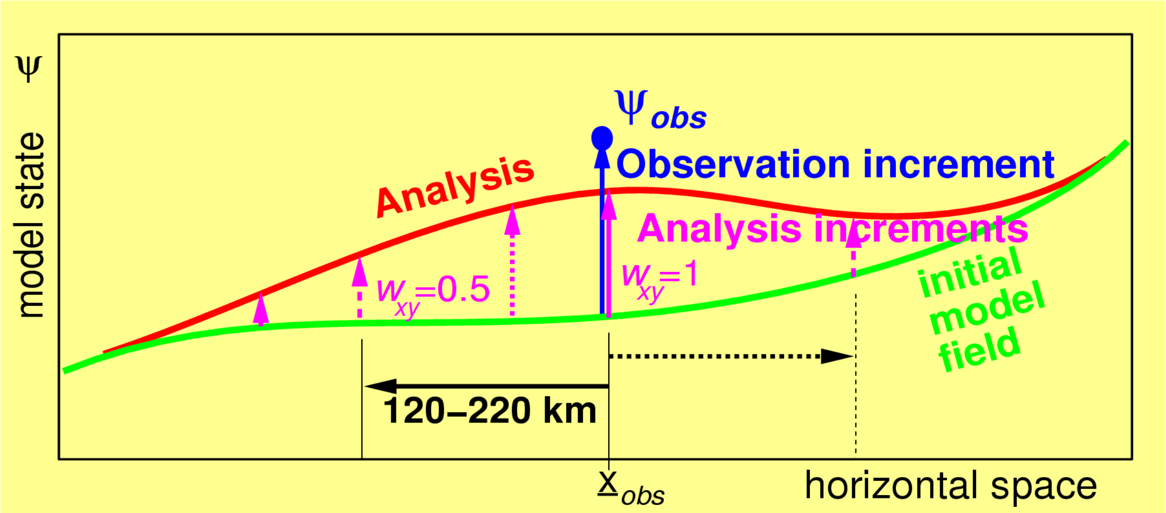


Structure functions

- Horizontal structure function

$$w_{xy} = (1 + \Delta r / s) e^{-\Delta r / s}$$

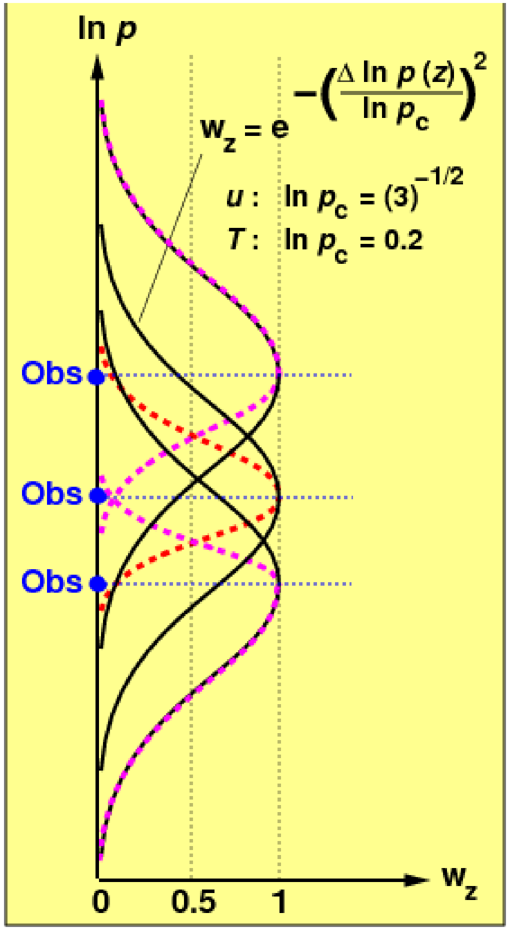
Δr: spatial distance from observation to model grid point
s : scale length (60 – 100km)



Structure functions

- Vertical structure function

  Weighted combination of incomplete vertical profiles and single-level data



Nudging: pro‘s and con‘s

- Advantages
  - 4D DA, applied every time step, strong coupling to model dynamics
  - Able to assimilate high-frequency observations at appropriate time
  - Computationally cheap
    - COSMO: < 10% more expensive than forecast
  - Conceptionally simple
- Disadvantages
  - No optimal combination of obs and model
    - Only crude treatment of model and obs errors
  - Can only assimilate prognostic model variables
  - Retrieval of prognostic variables from other observations
    - E.g. Latent Heat Nudging for Surface Rainfall
  - Needs explicit balancing (difficult/unknown for small scales)

Nudging in the L95 System I

- The nudging equation for the L95 system reads

$$\frac{dx}{dt} = L(x) + GW(y - x)$$

- x is the state vector, L is the L95 operator, G is the nudging constant, W is the weight function and y is the observation

- For simplicity, we set $W = \varepsilon = e^{\frac{-\sigma^2}{4}}$, i.e. the spatial and temporal weight is set to unity

Nudging in the L95 System II

Implicit time discretisation yields

$$\frac{x_j^{n+1} - x_j^n}{\Delta t} = L\left(x_j^n\right) + G\varepsilon\left[y^n - x_j^{n+1}\right]$$

and finally

$$x_j^{n+1} = x_{L,j}^n + \frac{G\varepsilon\Delta t}{1 + G\varepsilon\Delta t}\left[y^n - x_{L,j}^n\right]$$

$x_{L,j}^n$ is the state variable at location j after the L95 operator application.

References

- Nudging
  - Hoke and Anthes, 1976, MWR, 104, 1551-1556
  - Davies and Turner, 1977, QJRS, 103, 225-245
  - Stauffer and Seaman, 1990, MWR, 118, 1250-1277
  - Stauffer and Seaman, 1994, JAM, 33, 416-434
  - Schraff, 1997, MAP, 65, 21-50
  - COSMO Documentation

  http://www.cosmo-model.org/content/model/documentation/core/default.htm

- Latent Heat Nudging
  - Jones and MacPherson, 1997, MetApps, 4, 269-277
  - Leuenberger and Rossa, 2007, MAP, 98, 195-215
  - Stephan et al., 2008, QJRS, 134, 1315-1326