LETKF

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LETKF basics

- Implementation following *Hunt et al., 2007*
- basic idea: do analysis in the space of the ensemble perturbations
  - computational efficient, but also restricts corrections to subspace spanned by the ensemble
  - explicit localization (doing separate analysis at every grid point, select only certain obs)
  - analysis ensemble members are locally linear combination of first guess ensemble members
- LETKF belongs to the class of square root filters; no perturbedf observations required

LETKF equations

- let $\mathbf{w}$ denote gaussian vector in k-dimensional ensemble space with mean 0 and covariance $\mathbf{I}/(k-1)$
- let $\mathbf{X}^b$ denote the (background) ensemble perturbations
- then $\mathbf{x} = \bar{\mathbf{x}}^b + \mathbf{X}^b\mathbf{w}$ is the corresponding model state with mean $\bar{\mathbf{x}}^b$ and covariance $\mathbf{P}^b = (k-1)^{-1}\mathbf{X}^b(\mathbf{X}^b)^T$
- let $\mathbf{Y}^b$ denote the ensemble perturbations in observation space and $\mathbf{R}$ the observation error covariance matrix

LETKF equations

- do analysis in the k-dimensional ensemble space

$$\bar{\mathbf{w}}^a = \tilde{\mathbf{P}}^a (\mathbf{Y}^b)^T \mathbf{R}^{-1} (\mathbf{y} - \bar{\mathbf{y}}^b)$$

$$\tilde{\mathbf{P}}^a = [(k-1)\mathbf{I} + (\mathbf{Y}^b)^T \mathbf{R}^{-1} \mathbf{Y}^b]^{-1}$$

- in model space we have

$$\bar{\mathbf{x}}^a = \bar{\mathbf{x}}^b + \mathbf{X}^b \bar{\mathbf{w}}^a$$

$$\mathbf{P}^a = \mathbf{X}^b \tilde{\mathbf{P}}^a (\mathbf{X}^b)^T$$

- Now the analysis ensemble perturbations - with $\mathbf{P}^a$ given above - are obtained via

$$\mathbf{X}^a = \mathbf{X}^b \mathbf{W}^a,$$

where $\mathbf{W}^a = [(k-1)\tilde{\mathbf{P}}^a]^{1/2}$

LETKF: implementation

- Localization: do analysis at each gridpoint, use only obs within certain radius
- weight obs with distance-dependent weight $0 \leq w \leq 1$
- weight determined by *Caspari-Cohn* function: similar to Gaussian, but identical to zero at finite distances
- inflation factor ρ to increase spread; ad-hoc method to account for model error ("multiplicative" inflation; "additive" inflation: add model error at each time step)
- inflation factor ρ is applied when computing $\tilde{\mathbf{P}}^a$:

$$\tilde{\mathbf{P}}^a = [(k-1)\mathbf{I}/\rho + (\mathbf{Y}^b)^T \mathbf{R}^{-1} \mathbf{Y}^b]^{-1}$$

References

Hunt. et al 2007, Efficient data assimilation for spatiotemporal chaos: A Local Ensemble Transform Kalman Filter, *Physica D*, **230**, 112-126

Internet:
`http://www.weatherchaos.umd.edu/`