3dVar

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DWD-HErZ winterschool on data assimilation, Offenbach

13-17. February 2012

3dVar

The analysis can be obtained by numerically minimizing a *cost function J*. This cost function and its gradient are given by

$$J(\mathbf{x}) = \frac{1}{2}[(\mathbf{x}-\mathbf{x}^b)^T\mathbf{B}^{-1}(\mathbf{x}-\mathbf{x}^b) + (\mathbf{y}-\mathbf{H}\mathbf{x})^T\mathbf{R}^{-1}(\mathbf{y}-\mathbf{H}\mathbf{x})]$$
$$\nabla J(\mathbf{x}) = \mathbf{B}^{-1}(\mathbf{x}-\mathbf{x}^b) - \mathbf{H}^T\mathbf{R}^{-1}(\mathbf{y}-\mathbf{H}\mathbf{x}),$$

where $\mathbf{B}, \mathbf{R}$ are the background and observation error covariance matrices, $\mathbf{H}$ is the observation operator.
The method to obtain the analysis by minimizing a cost function is called **3dVar**.
Setting ∇J to 0 leads to the analysis update equation:

$$\mathbf{x}^a = \mathbf{x}^b + \mathbf{K}(\mathbf{y}-\mathbf{H}\mathbf{x}^b)$$
$$\mathbf{K} = \mathbf{B}\mathbf{H}^T(\mathbf{H}\mathbf{B}\mathbf{H}^T+\mathbf{R})^{-1}$$

$\mathbf{K}$ is called the *Kalman gain* matrix. This equation can also be obtained by using *Bayes' Theorem* and assuming gaussian pdf's or by a least square estimation.

3dVar

- the solution is obtained by minimizing the cost function numerically using e.g. the *conjugate gradient* method
- advantages of 3dVar:
  - variational scheme, allows e.g. outer loops (necessary to deal with nonlinearities), varQC of observations
  - radiances can be assimilated directly (no 1dVar needed)
  - no adjoint model as in 4dVar required
  - solves global problem, no localization issues
- disadvantages of 3dVar:
  - the **B** matrix is constant in time.
  - comparison between observations and model equivalents not at appropiate time (different in *fgat-3dVar*); no 4D method
  - adjoint of observation operator needed
- a method to speed up convergence is the *preconditioning*; this will be discussed in the context of the hybrid method.

References