"Why must hurricanes have eyes?" – revisited

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In a recent thought-provoking article, Pearce (2005) poses the question "Why must hurricanes have eyes"? In the article he explains aspects of the inner-core dynamics of a mature hurricane in terms of a simple axisymmetric model in which the eye is non-rotating and less dense than the vortex surrounding it. He describes a calculation in which the eye always has a finite radius at the surface and the inference appears to be that a similar dynamical constraint applies to a mature hurricane. My aim here is to review some of the important issues raised by Prof. Pearce and to present a slightly different view of the inner-core dynamics of a hurricane and an alternative explanation for the hurricane eye. In particular, I will argue that the foregoing calculation is unrealistic in one important respect and show how it can be repaired. Even so, its relevance to the hurricane eye remains unclear.

Prof. Pearce makes a commendable attempt to simplify the concepts required to understand the dynamics involved by considering parts of the problem separately. For example he explains

- how azimuthal vorticity is produced in a rising thermal;
- why the azimuthal wind speed must decrease with height above the surface friction layer; and
- how this decrease leads to an azimuthal vorticity tendency that balances the tendency associated with the negative radial temperature gradient observed in a hurricane.

He argues that subsidence must occur in the eye so that the radial temperature gradient in the eye can balance the production of azimuthal vorticity by vortex tilting in the eyewall. However there are some misleading aspects of the discussion that I will try to explain below.

An analysis of Pearce's simple model

First let me examine the simple model that always predicts the existence of an eye of finite radius at the surface. The model considers two layers of immiscible incompressible fluid with densities $\rho - \Delta\rho$ and ρ, where $\Delta\rho > 0$. The lighter fluid that represents the eye is at rest while the heavier one representing the surrounding vortex core is rotating. In the model, an expression is derived for the shape of the surface, h(r), separating the eye from the vortex outside it. The assumption is made that Coriolis forces can be neglected, which is not unreasonable for the rapidly rotating inner core of a hurricane. It is shown that with these assumptions, h(r) satisfies the ordinary differential equation $dh/dr = v^2/(g'r)$, where v is the azimuthal (tangential) wind speed, r is the radius, $g' = g\Delta\rho/\rho$ is the reduced gravity and g is the acceleration due to gravity. It is then assumed, for simplicity, that the vortex outside the eye has uniform angular momentum, i.e. the azimuthal wind profile $v(r)$ satisfies $rv(r) = V_R R$, where V_R is the wind speed at some radius R. Then the equation for h can be integrated to give $h = H[1 - (r_e/r)^2]$, where $H = h(\infty)$ is the value of h at large r and $r_e = V_R R/\sqrt{(2g'H)}$. It follows that h is zero when $r = r_e$, which is always finite. In other words, the eye region has a finite width at the surface irrespective of the prescribed strength of the angular momentum $V_R R$ and the density difference between the air inside and outside the eye. Our intuition tells us that this result is physically unreasonable, since by making the density contrast larger and larger and the rotation weaker and weaker, there must be a regime in which the surface between the air in the 'eye' and that outside is elevated above the surface, even at the rotation axis. The reason preventing this behaviour in Pearce's calculation is the possibility that v can become arbitrarily large if r becomes sufficiently small*. It follows that this calculation does not provide an acceptable explanation for the universal existence of an eye and therefore for the existence of deep convection in an annulus rather than at the hurricane centre. This is unfortunate since Pearce's subsequent arguments about the dynamics of the inner core and the eye itself are based on the assumed existence of an annular distribution of convection.

* The same limitation holds for alternative profile $v(r) = V_R(R/r)^{\alpha}$ (α a constant < 1) in the extended theory described by Pearce (2004).

An alternative calculation

The foregoing model behaviour does not arise if we choose a more realistic azimuthal wind profile that tends to zero as r tends to zero. Consider, for example, the profile

$$v(r) = v_m\left(\frac{r}{r_m}\right)\exp\left[\frac{1}{2}\left\{1-\left(\frac{r}{r_m}\right)^2\right\}\right],$$

which has the desirable properties† that $v(0) = 0$, v increases steadily to attain a maximum, v_m, at $r = r_m$ and then declines such that $rv \to 0$ as $r \to \infty$. Then $h(r) = H[1 - C\exp\{-(r/r_m)^2\}]$, where $C = (e/2)[v_m^2/(g'H)]$. In this case $h(0) > 0$ for $C < 1$, which conforms with our intuition that for sufficiently strong stability (g' relatively large) and sufficiently weak rotation (v_m relatively small) the interface is not depressed as far as the surface at the axis. In contrast, if $C > 1$, corresponding with the regime of relatively weak stability and strong rotation, the

† The profile has the desirable property also, that that the radially-integrated kinetic energy and angular momentum are bounded as r increases. That is not to say there is an initial state that would lead exactly to this profile, but the same remark is true of the profile with $v \propto 1/r$. Indeed, while Pearce talks of his profile as applying to a 'spun-up' vortex, it is not clear how such a vortex can evolve through inviscid dynamics from anything other than itself, since (relative) angular momentum rv would be conserved in any rearrangement of the flow, i.e. v would always remain proportional to $1/r$ with the same constant of proportionality! Furthermore, the vr = constant profile does not satisfy the constraints that the radially-integrated kinetic energy and angular momentum are bounded for large r.

dividing surface between the two fluids does intersect the surface at a finite radius ($r_m\sqrt{\ln C}$) again as expected. One might argue that with such a modification, Pearce's theory can be repaired, but there remains the difficulty of deciding what feature of the hurricane eyewall corresponds with the dividing surface in the model. Nevertheless, the calculation is certainly suggestive that as the rotation increases in strength, the formation of an eye-like feature is likely. Moreover, if and when the eye begins to form, it could be expected to push convection away from the axis of rotation and the accompanying subsidence (see below) would contribute to determining the size of the eye.

Why does the convection occur in an annulus?

An alternative reason to expect the convection to be confined to an annular region lies in the effects of surface friction, which leads to strong convergence of air in a shallow layer some 500 m to 1 km deep adjacent to the sea surface. This layer is referred to as the boundary layer, or friction layer. The convergence arises as follows. Observations show that above the boundary layer, the azimuthal winds in a hurricane are approximately in gradient wind balance (Willoughby 1990). This means that the inward-directed pressure gradient force is approximately balanced by the sum of the outward-directed centrifugal and Coriolis forces. Frictional stresses in the boundary layer reduce the tangential wind speed and thereby the centrifugal and Coriolis forces, while it can be shown that the pressure gradient force remains largely unchanged. As a result there is a net inward force in the boundary layer which drives the convergence. Calculations (e.g. Smith 1968, 2003; Kepert and Wang 2001) show that for azimuthal wind profiles characteristic of a mature hurricane, the inflow in the boundary layer turns upwards before it reaches the hurricane centre with the maximum upflow at the top of the boundary layer being near to the radius of maximum azimuthal wind speed. This inflowing air is very moist and as it rises out of the boundary layer, the water vapour soon condenses to form the clouds surrounding the eye.

The thermal structure of the convective region

Another difficulty with Pearce's explanations is that the example of a rising thermal illustrated in Fig. 6(a) of his article cannot be extended to the inner-core region of a mature hurricane depicted in Fig. 6(b). This is because the air in the hurricane eye is warmer than the air in the convective clouds surrounding the eye*, while the air surrounding the thermal is everywhere cooler. Thus the eyewall region depicted in his Fig. 6(b), the region between the curves B'C' and BC, is not a region of negative azimuthal vorticity *tendency*, even though the azimuthal vorticity in that region is negative (as indicated). The fact is that the negative vorticity shown emanates from the boundary layer where the radial gradient of vertical velocity is positive: it is not generated by the radial temperature gradient as in the thermal in Pearce's Fig. 6(a). How, then, can we understand the role of latent heat release in the clouds?

A series of seminal papers by Emanuel (1986, 1989, 1991, 1995, 1997) has taught us that the negative radial gradient of virtual† temperature in the cloudy region (the eyewall and convection region in Pearce's Fig. 6(b)) is closely related to the negative radial gradient of equivalent potential temperature in the boundary layer, which in turn is largely a result of the strong increase in the surface moisture flux as the wind speed of air converging in the boundary layer increases. When air ascends out of the boundary layer into the eyewall clouds, it approximately conserves its equivalent potential temperature, which for saturated air is a monotonically increasing function of virtual temperature. Thus at every height in the cloudy region, the radial gradient of virtual temperature is negative. It follows that no distinction needs to be made between the eyewall region and the convection region depicted in Pearce's Fig. 6(b), thereby simplifying Pearce's arguments for the fact that the mature hurricane must be everywhere in approximate thermal wind balance and that the warm eye is necessary prerequisite for such balance. In this picture, the role of the clouds in the generation of azimuthal circulation is not in providing local buoyancy to 'drive the circulation', but to simply maintain the radial distribution of equivalent potential temperature throughout the free troposphere (Emanuel 1971). This radial distribution of equivalent potential temperature, itself, is determined primarily by the radial distribution of surface moisture flux. Where, then, does the eye fit in?

* This fact raises the subtle question as to whether the clouds surrounding the eye have positive buoyancy, a question examined in detail by Smith *et al.* 2005. It calls into question also Pearce's statement on p. 22 of his article that "the positive circulation in the convection region is driven by latent heat release in the clouds".

† Strictly, the density of moist air is inversely proportional to the virtual temperature rather than the temperature itself.

An alternative view of eye dynamics

Above the boundary layer, the air that ascends in the convective region flows radially outwards (there is essentially nowhere else for it to go, although because of the high levels of turbulence, there must be some mixing across the inner edge of the eye). Indeed, observations show that the eyewall clouds tilt outwards with height (a good review of the structure of a mature hurricane is given by Willoughby 1995). As explained by Pearce, when the rising air leaves the friction layer it approximately conserves its (absolute) angular momentum and as it moves outwards, it spins more slowly. Moreover, the maximum azimuthal wind speed occurs at ever increasing radii. In his Fig. 6(b), Pearce shows how this leads to an azimuthal wind speed that decreases with height. Calculations by Emanuel (1997) show that the eyewall has some likeness to an atmospheric front and they suggest that the eye is formed as a passive response to processes outside it.

Pearce notes correctly that the subsidence that gives rise to the eye occurs during the developing phase of the hurricane and he attributes the occurrence of this subsidence to 'vortex tilting and gravity-wave propagation'. An alternative and arguably simpler reason for the subsidence may be given in terms of the pressure forces following Smith (1980). The existence of gradient wind balance above the boundary layer implies that the pressure, $p(0, z)$, on the vortex axis is less than that in the far environment at the same height, $p(\infty, z)$. If Coriolis forces are included, this difference is expressed mathematically by a radial integral of the gradient wind equation, i.e.

$$p(0,z) - p(\infty,z) = -\int_0^\infty \rho\left(\frac{v^2}{r} + fv\right)dr, \quad (1)$$

where f is the Coriolis parameter. The quantity $p'(0,z) = p(0,z) - p(\infty,z)$, which is negative throughout the vortex, is called the perturbation pressure. Equation (1) shows that the magnitude of the pressure difference at a given height increases with the maximum azimuthal wind speed and the density at that height. The weakening and radial spreading of the azimuthal wind field with height and the decline in density with height imply that $p'(0,z)$ increases with height* (i.e. its magnitude decreases) so that the lowest *perturbation pressures* occur at low levels on the axis. This negative vertical gradient of perturbation pressure tends to

* Note that the integral in Eq. (1) is just the area under the curve representing the function $\rho(v^2/r + fv)$ plotted as a function of radius. This function is always positive and the area beneath it decreases as both v and ρ decrease.

drive subsidence along and near to the axis to form the eye. However, as this air subsides, it is compressed and warms relative to air at the same level outside the eye and thereby becomes locally buoyant (i.e. relative to the air outside the eye). This upward buoyancy approximately balances the downward directed (perturbation) pressure gradient so that the actual subsidence results from a small residual force. In essence, the flow remains close to hydrostatic balance.

As the vortex strengthens, the downward pressure gradient must increase and the residual force must be downwards to drive further subsidence. On the other hand, if the vortex weakens, the residual force must be upwards, allowing the air to re-ascend. As noted by Pearce, in the steady state, the residual force must be zero and there is no longer a need for up- or down-motion in the eye, although, in reality there may be motion in the eye associated with turbulent mixing across the eyewall or with asymmetric instabilities within the eye.

Smith *op. cit.* showed that the extent to which gravity waves are generated during this process as depends on the rapidity with which the subsidence is initiated, but is probably small. Indeed, Shapiro and Willoughby (1982) showed that a heat source near the radius of maximum tangential wind speed such as that produced by latent heat release in the eyewall, leads to subsidence in the eye in a balanced model that doesn't support gravity waves. Their calculations are consistent with the foregoing description.

Radiative cooling and eye subsidence

A reviewer of this note queried the possible role of radiative cooling on the subsidence in the eye. It is well known that in the cloud-free regions of the tropical atmosphere the mean radiative cooling rate is 1 to 2 degC/day from the surface to 10 km (≈ 250 mb) and decreases to about zero at the tropopause. In a region of multi-layer clouds, however, there is practically no radiative cooling in the clouds, but there is strong cooling at their top (see e.g. Anthes1982). The result of differential radiative heating between the cloud-free eye and the cloudy cirrus canopy of a hurricane would be to generate a direct circulation, with sinking motion in the eye and enhanced rising motion in the cloudy air. Assuming that the cooling is approximately balanced by the warming associated with the adiabatic compression of subsiding air gives an estimated* subsidence rate in the middle troposphere of 0.75 cm s^{-1} (about 650 m per day).

Concluding remarks

It should be realized that the 'big picture' of the dynamics and thermodynamics of the hurricane's inner-core provided by the foregoing arguments is based on consistency. Since the azimuthal and meridional circulations of a vortex are intimately coupled by the pressure field, it is difficult (indeed dangerous) to construct arguments based on cause and effect. For example, it would be dangerous to argue that either the subsidence in the eye or the boundary-layer-induced upflow alone set the radial scale of the eye for a mature hurricane. Both effects must contribute and both the eye and the boundary layer must evolve in a consistent way that satisfies all of the coupled dynamical and thermodynamical constraints.

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* The estimate is obtained as follows. Taking the potential temperature, θ, at the surface as 300 K and at the tropical tropopause as 360 K, and assuming the tropopause depth to be 16 km, the mean vertical potential temperature gradient is $60 \div (1.6 \times 10^4)$ K m^{-1}. If the subsidence rate is w m s^{-1}, the rate of increase in potential temperature at a given height in the eye on account of vertical advection is w times this gradient. From the first law of thermodynamics, a local radiative cooling rate of 2 K/day would give rise to a decrease in potential temperature of $(2 \div (3600 \times 24)) \times \theta/T$, where θ and T are the potential temperature and temperature at the height in question. Taking typical mid-tropospheric values of these quantities to be 330 K and 270 K, respectively and equating the two expressions gives the value cited for w, but in m s^{-1}.

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