The Role of Cumulus Convection in Hurricanes and its Representation in Hurricane Models

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Abstract

This paper reviews our understanding of the role of cumulus convection in hurricanes as well as the various convective parameterization schemes that have been used in hurricane models. Elementary principles show that the primary (tangential) circulation of a vortex intensifies as rings of rotating air converge inwards while conserving their (absolute) angular momentum. Thus intensification requires a mechanism to produce enough flow convergence above the surface boundary layer to counter the divergence induced there by the boundary-layer itself. Typically, such convergence is associated with the inward branch of the secondary circulation and is produced by an unbalanced negative radial gradient of buoyancy above the boundary layer resulting from condensational heating in the inner region of the vortex. The fact that such buoyancy gradients are produced by all the parameterization schemes, as well as by explicit schemes for latent heat release, explains why all the models are able to simulate hurricane intensification with some degree of realism. In a weak vortex, the secondary circulation is dominated by buoyant forcing, but as the vortex intensifies the contribution from surface friction increases until, in the mature stage, the buoyantly-induced convergence must closely balance the frictionally-induced divergence just above the boundary layer.

Only a handful of the more recent hurricane models and only two of those in which convection is parameterized represent the effects of convective downdraughts. These downdraughts cool and dry the subcloud layer, tending to suppress further convection, so that acting alone they would serve as a brake on hurricane intensification. The strength of downdraughts decreases when the middle-tropospheric relative humidity increases as a result of sustained cumulus convection over an area. If surface wind speeds are high enough, surface fluxes of sensible and latent heat can more than counteract the cooling and drying effects of convective downdraughts, allowing continued warming of the troposphere in the inner region of the vortex so that vortex intensification can proceed. Accordingly, the surface fluxes provide the energy source for the hurricane while convection transfers this energy vertically through the troposphere, creating a suitable radial gradient of buoyancy to drive the secondary circulation of the vortex.

1. Introduction

Hurricanes are intense atmospheric vortices that develop over the warm tropical oceans. The name is given to those vortices that form in the Western Hemisphere; similar storms that occur over the North West Pacific are called typhoons, those in the Australasian and South West Pacific regions are called tropical cyclones and over the Indian Ocean they are referred to simply as cyclones. Hurricanes are the costliest of all natural disasters in the United States (Pielke and Landsea, 1998).

The mature hurricane consists of an annular region of extremely strong winds rotating

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about some central axis, sometimes referred to as the primary circulation, with an inner core region (the eye), where the winds are light. The primary circulation is strongest just above the surface and decreases progressively with height. The eye, so named because it is often devoid of high clouds, is surrounded by a region of deep clouds (the eye wall) extending through the depth of the troposphere and coinciding with the band of strongest tangential winds. The clouds are fed by warm moist air from near the sea surface, which spirals into the storm at low levels. High moisture values at low-levels result from locally high evaporation rates from the sea surface, which are a strongly increasing function of surface wind speed. Indeed, the latent heat supplied by the ocean is the primary energy source for storms and its central importance is reflected in the rapid demise in the intensity of storms after they make landfall. The low-level spiralling inflow and upper-level outflow constitute what is sometimes referred to the secondary circulation of the storm (Ooyama, 1982). Hurricanes are quite variable in size, but damaging winds are usually confined to within 100 km of their centre. An excellent account of these storms is given by Anthes (1982). For more recent reviews of hurricane dynamics, the reader is referred to papers by Emanuel (1991) and Willoughby (1995).

Moist convection has long been recognized as a process of central importance in the development of tropical depressions and hurricanes, which, in contrast to cyclonic vortices in the extra-tropics form in regions of weak horizontal temperature gradient. However, convective clouds occur on scales that are typically too small to be resolved by numerical models and their collective effects must be represented, or parameterized, in terms of variables defined on the model grid. The early cumulus parameterization schemes were motivated by their application to hurricane dynamics, but the need to represent convective processes has greatly widened and their parameterization is especially important for extended-range weather forecast models, general circulation models and climate models. While some authors view the parameterization as a technical problem in modelling, Arakawa (1993) regards it as a scientific question that cannot be avoided by refining the grid to the stage that cumulus clouds are effectively resolved. In that case, he points out, one would still require simplifications that involve various levels of "parameterization" to understand the results.

Over the years, a range of parameterization schemes has been developed, but all schemes have limitations and none has been entirely satisfactory. This is partly a reflection of deficiencies in our understanding of convective processes. There have been periodic reviews of the subject (Frank, 1983; Molinari and Dudek, 1992; Emanuel and Raymond, 1993; Kuo *et al.*, 1997; Smith, 1997a), but progress continues to be made and an update on the hurricane problem seems timely.

The present article focuses on the hurricane problem. There is an urgent need to improve hurricane intensity forecasts, which, at the present time have little or no skill (Elsberry *et al.*, 1992). Because of the recognized importance of moist convection in hurricane dynamics, it is possible that any major improvement in intensity forecasts will depend, *inter alia*, on improvements in the representation of convection in hurricane models. In turn, such improvements will require improvements in our understanding of convection in hurricanes. Motivated but humbled by these goals, we discuss a number of issues relating to the role of convection on hurricane dynamics and review the range of cumulus parameterization schemes that have been used in hurricane models. We give also an appraisal of these schemes. The idea is not to provide a comprehensive account of parameterization schemes in general, but rather to provide an overview of one specific, albeit important, application. The article is aimed primarily at readers who are interested in hurricane behaviour, but who are not experts on cumulus parameterization, and at readers who need a first introduction to cumulus parameterization schemes used in hurricanes.

We begin with a brief discussion of the dynamics of a mature hurricane-like vortex, considering first the spin-down of a vortex when there is no convective heating and going on to consider the role of convective forcing. We continue with an appraisal of schemes that have been used to represent this forcing in hurricane models, concentrating mainly on the thermodynamical effects of convection and touching on convective momentum transports only briefly. Finally, we comment upon the claim of most authors, irrespective of the convection scheme they employ, to have simulated a hurricane with some degree of realism.

2. Dynamics of vortex spin down and spin up

Consider a symmetric vortex in a stably-stratified fluid situated above a rough horizontal boundary (Fig. 1). The vortex is assumed to be in hydrostatic and gradient wind balance with an *initial* cyclonic tangential wind speed distribution V(r), where r is the radius from the rotation axis*. Surface friction leads to a breakdown of the force balance in a shallow boundary layer resulting in a net inwards force in this layer (see e.g. Smith, 1968). This force drives a secondary circulation with horizontal convergence in the boundary layer and horizontal divergence above as illustrated in Fig. 1. As rings of fluid diverge above the boundary layer, conservation of (absolute) angular momentum requires that they spin more slowly and the tangential wind speed declines. This mechanism of spin-down dominates the effect of vertical diffusion of angular momentum to the surface.

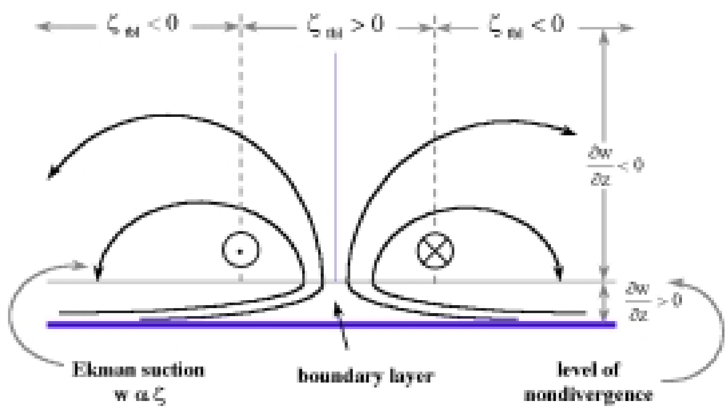

Figure 1. Illustration of the decay of balanced vortex associated with the frictionally-induced secondary circulation, indicated by arrows, which is divergent above the boundary layer. The vortex circulation is assumed to be initially independent of height. A solid vertical line marks the vortex axis. Regions where the initial vertical component of relative vorticity, ζ_{tbl} (delineated by broken vertical lines), and horizontal convergence, $\partial w/\partial z$, are positive or negative are indicated. Here w denotes the vertical velocity and z measures height above the surface. For weak vortices, where quasi-geostrophic balance holds, the vertical velocity at the top of the (Ekman) boundary layer is proportional to the vertical component of relative vorticity at that level. Clearly the vortex circulation cannot remain independent of height in this configuration because the frictionally-induced secondary circulation weakens with height. Moreover, ascent in the region where $\zeta_{tbl} > 0$ will lead to cooling, and subsidence in the region where $\zeta_{tbl} < 0$ will lead to warming, thereby producing a horizontal temperature gradient in the free troposphere.

If the vortex is weak enough for quasi-geostrophic dynamics to apply, the boundary layer has the structure of a classical Ekman layer and the process of vortex decay is referred

* Note that the vortex cannot remain barotropic in the configuratio shown in Fig. 1 because the frictionally-induced secondary circulation weakens with height. Moreover, ascent in the region where $\zeta_{tbl} > 0$ will lead to cooling, and subsidence in the region where $\zeta_{tbl} < 0$ will lead to warming to produce a horizontal temperature gradient in the free troposphere.

to as Ekman spin down. In this case, the vertical velocity at the top of the boundary layer is proportional to the vertical component of relative vorticity, ζ_{tbl}, just at the top of the boundary layer (see e,g. Gill, 1982, section 9.6). Thus upflow out of the boundary layer occurs where $\zeta_{tbl} > 0$ and subsidence into the boundary layer occurs where $\zeta_{tbl} < 0$. Moreover, if the fluid is stably stratified, the divergent circulation above the boundary layer will have the vertical scale fL/N, where L is the horizontal scale of the vortex, N is the Brunt-Väisälä frequency and f is the Coriolis parameter.

The key point is that, in the absence of forcing, the level of non-divergence occurs at the top of the boundary layer; there is divergence above this layer, and the primary (tangential) circulation of the vortex decays with time. This must be true also in the more general case as an unforced, balanced, inviscid vortex cannot intensify in the presence of surface friction. If there is a strong inversion at the top of the boundary layer, the boundary layer may thicken rather than discharge fluid into the layer above. The result, however, would be the same: any thickening must be accompanied by divergence above the boundary layer, even though the layer in which the divergence occurs may be relatively shallow.

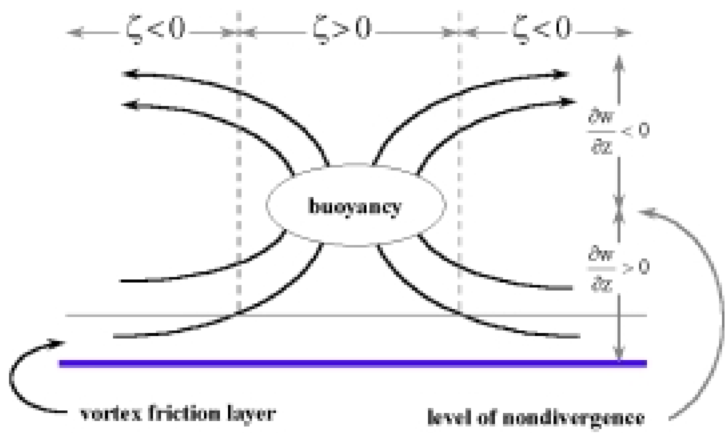

Figure 2. Vertical circulation induced by a region of positive buoyancy centred on the vortex axis and located above the vortex boundary layer. Regions where the vertical component of relative vorticity ζ (delineated by broken vertical lines) and horizontal convergence, $\partial w/\partial z$, are positive or negative are indicated. The horizontal line above the ground represents the top of the Ekman boundary layer. The ground is shown as a thick solid line.

Suppose now that for some reason the air above the boundary layer has positive buoyancy (Fig. 2). In a developing hurricane the buoyancy arises from the release of latent heat in a region undergoing deep convection, but we focus on buoyancy rather than the heating, partly because much of the latent heat release in deep convective clouds is balanced by the adiabatic cooling of air rising in the clouds and the heating of the cloud environment is largely indirect, involving adiabatic compression due to subsidence in the clear air surrounding clouds (see e.g. section 2c). If the vortical flow is sufficiently weak it is clear that the gross* buoyancy force above the boundary layer will give rise to low-level convergence and upper-level divergence (see Fig. 2), in addition to the circulation induced by friction in the vortex boundary layer. Moreover, the strength of low-level convergence associated with the buoyancy distribution can be much larger than that induced by surface

* Gross buoyancy refers to a region of air, possibly including a field of clouds, in which the average buoyancy summed over all parcels is positive.

friction, especially for weak vortices, and the depth of the atmosphere through which the convergence occurs need not be confined to the vortex boundary layer. The proviso that the vortex is sufficiently weak is made because a part of the buoyancy distribution will be in thermal-wind balance with the vortex circulation and in mature hurricanes, most if not all of the buoyancy field will be in such balance. Clearly, if the vortex is to intensify as a result of the *unbalanced* part of the buoyancy gradient, the convergence produced by the latter must occur at least in part *above* the boundary layer (Ooyama, 1982) and it must be strong enough to more than offset the frictionally-induced divergence associated with the vortex. Then vortex spin-up occurs as rings of air above the boundary layer converge while conserving their absolute angular momentum. Obviously, the buoyancy distribution must be suitably located within the vortex to cause convergence towards the vortex axis.

The fact that positive (unbalanced) buoyancy in the middle troposphere must always produce divergence in the upper troposphere suggests that the vertical transport of angular momentum is necessary to account for the spin-up of the cyclonic circulation of a hurricane in the upper troposphere.

The foregoing remarks are based on general principles of fluid dynamics, but to build them into a model for tropical cyclogenesis or hurricane intensification, we need to represent the production of buoyancy by moist convection in the model. In the next sections we examine the way in which convective clouds generate buoyancy and in subsequent sections we review the methods that have been developed to represent moist convection in the models and comment briefly on their strengths and weaknesses.

3. Buoyancy generation by clouds

Convective clouds are produced in a conditionally-unstable atmosphere when, for any reason, parcels of moist air are lifted to a level at which their density becomes less than that of their surroundings, the so-called level of free convection (LFC). Conditionally unstable means that such a level exists and that small parcel displacements, smaller than the distance to their LFC, are stable. Typically, the LFC lies above the level at which condensation occurs, which is called the lifting condensation level (LCL). Above their LFC, air parcels have positive buoyancy relative to their environment and continue to rise while mixing with their environment until their buoyancy becomes negative. Because of their upward kinetic energy, rising air parcels overshoot their level of neutral buoyancy (LNB) before decelerating and falling back towards this level. Mixing with their environment continues during the period of overshoot and return.

As an unsaturated air parcel rises, it expands adiabatically on account of the reducing pressure and its temperature and vapour pressure fall, but the water vapour mixing ratio, which is related to the vapour pressure and parcel pressure, and the potential temperature remain constant. As the temperature falls, so does the saturated mixing ratio, which decreases rapidly with decreasing temperature. The saturated mixing ratio characterizes the maximum amount of water vapour that an air parcel can hold. Condensation first occurs when the mixing ratio falls to the saturated mixing ratio, which defines the LCL. As the parcel rises above its LCL, latent heat is released and water vapour condenses, but the parcel continues to cool, the latent heat release being more than counterbalanced by the adiabatic cooling. Additional latent heat is released if and when cloud droplets freeze. On account of the latent heat release, the potential temperature of the air parcel rises. Except in intense convective systems over land, the temperature of a rising air parcel is typically no more than a few degrees larger than the local environmental temperature. Mixing between cloudy air and environmental air leads to evaporation and local cooling, which, in turn, leads to the generation of downdraughts in and around the cloud. The frictional drag and evaporation of falling precipitation in the cloud leads also to downdraughts.

If a cloud does not precipitate during its lifetime, the latent heat released by condensation is all reconsumed as the cloud mixes with its environment and evaporates. Accordingly there is no net heating of the air. However, the removal of water substance by precipitation leads to a net heating and the question is: how is this heating manifest as net buoyancy and how is the buoyancy distributed?

Some of the buoyancy, both positive and negative, is converted to kinetic energy of updraughts and downdraughts, respectively, and this energy is dissipated locally as heat as the cloud decays. Thought experiments which idealize a deep convective cloud as a transient buoyancy source in a stably-stratified non-rotating environment at rest suggest that the subsidence occurs at the leading edge of a horizontally-propagating internal gravity wave that spreads out from the cloud (Bretherton and Smolarkiewicz, 1989; Nicholls *et al.*, 1991, Bretherton, 1993; Mapes, 1993). This subsidence leads to an adiabatic temperature rise in the cloud environment, which tends to adjust the cloud-free environment towards the moist adiabatic lapse rate found in the cloud (Fig. 3). If the buoyancy source is switched off after a time τ^*, a gravity wave spreads out from it, this time with ascent at the leading edge, and adjusts the fluid back to its unperturbed state. Thus transient heating leaves no permanent effect near the heat source, but produces a pair of outward-moving disturbances separated by a distance proportional to τ^*, between which the air is warmed through adiabatic subsidence.

The calculations suggest that the atmosphere in a region of active convection is warmed by the collective contribution of intra-cloud subsidence. They imply also that downdraughts that form during the cloud life cycle will also generate gravity waves associated with negative buoyancy perturbations and these must produce ascent and adiabatic cooling in the environment to offset the warming by subsidence. *The warming and/or cooling produced by spreading gravity waves represents a buoyancy perturbation relative to the far environment on a horizontal scale much larger than the horizontal scale of individual clouds.* In a rotating atmosphere, neglecting the effects of radiative cooling, this buoyancy perturbation must spread out on the order of a Rossby deformation radius, which is typically 1000 km or more in the undisturbed tropics, but may be much smaller in cyclonic disturbances. Generally the deformation radius decreases as the circulation strength increases and may be only a few tens of kilometres in the core of a hurricane (Ooyama, 1982). In the presence of rotation, some of the spreading gravity-wave energy remains as balanced rotational motion. The balanced rotational motion produced by an individual convective cloud in an environment of enhanced rotation has been investigated by Shutts and Gray (1994).

It is evident from the foregoing discussion that factors which determine the permanent effects of net latent heating by an individual precipitating cloud or cloud field are complicated and it seems fair to say that many details of the processes involved are still not well understood. The amount of energy per unit mass required to lift an air parcel to its LFC is called the convective inhibition (CIN) and the net amount per unit mass that can be released if the parcel reaches the LFC and rises to its LNB is called the convective available potential energy (CAPE). The total amount of convective energy in a column of air is characterized by the mass weighted CAPE summed over all parcels in the column that have any CAPE, the so-called Integrated-CAPE, or ICAPE (Mapes and Houze, 1992). Typically it is only air parcels in the lowest few hundred metres of the atmosphere that have any CAPE. One thing that can be said is that convective clouds consume CAPE or ICAPE from their environment. A precipitating cloud does so partly by laying down a carpet of cool air near the surface associated with the precipitation-cooled downdraught. A field of precipitating clouds may cover a large area with cool near-surface air, which is absolutely stable to further

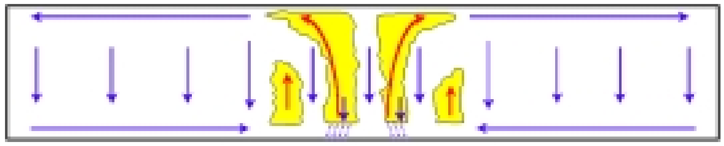

Figure 3. Schematic depiction of the vertical circulation associated with a localized region of deep convection in an otherwise quiescent environment. Deep precipitating clouds have positively-buoyant updraughts extending through the depth of the troposphere and negatively-buoyant downdraughts in the lower troposphere. The mean flow is divergent in the upper troposphere and convergent in the lower troposphere and subsidence occurs in the region between clouds and in the environment. The subsidence, which occurs near the leading edge of gravity-wave disturbances generated by individual clouds, leads to compression and adiabatic warming in the clear air, adjusting the virtual temperature profile in the clear-air towards the profile in the cloud, i.e. approximately towards a moist adiabat characteristic of subcloud air. While the subsidence is depicted as a deep circulation, air-parcel displacements are small in the clear air (which is stably-stratified) on time-scales less than those on which radiative cooling operates. Displacements through a deep layer of the troposphere can occur only if the adiabatic warming is approximately balanced by radiative cooling.

lifting. Thus to explain the intensification of a hurricane in terms of buoyancy produced by the collective effects of deep convection, we must explain how the buoyancy field (and convection) can be sustained and confined. Clearly the convection can be sustained only if its energy source is sustained, i.e. if the environmental CAPE (or ICAPE) is sustained. Observations over the tropical oceans show that day-to-day variations in CAPE are much smaller than the rate at which CAPE is produced by large-scale processes, such as the radiative cooling of the troposphere or the moistening of the oceanic subcloud layer by sea-surface fluxes. As a result one may hypothesize that CAPE is removed by convection approximately at the rate that it is generated by large-scale processes. These processes include surface fluxes of sensible and (predominantly) latent heat from the ocean, radiative cooling of the troposphere and by large-scale ascent, and various advective processes (see e.g. Emanuel, 1994, section 14.2). Based on the foregoing hypothesis that $\partial(CAPE)/\partial(t) \approx 0$, one can construct a theory for the interaction between convection and its environment without considering the details of individual clouds (see e.g. Emanuel, 1994, section 15.3). In particular, one can relate the buoyancy production by deep convection, characterized by horizontal gradients of saturated moist entropy in the troposphere, to large- scale processes that produce CAPE. At the same time one can determine the convective mass flux required to produce the buoyancy distribution. This type of approach has been applied to hurricane models by Emanuel (1986, 1989, 1995) as discussed later in section 8.

4. Cumulus parameterization schemes

The primary objective of a convective parameterization scheme in a numerical model is to specify the amount of water condensed and the amount of latent heat released in unsaturated grid boxes in terms of quantities that are resolved by the model. Two questions one should ask in assessing a particular scheme are how convection is initiated in the scheme and, in the case of so-called mass-flux schemes, how the cumulus mass flux is determined. In many schemes, the cloud-base mass flux is equated to the resolved low-level vertical motion in the model, i.e. by the boundary-layer convergence and convection is assumed to occur in such regions if the subcloud air is conditionally-unstable. But as discussed in section 2, the vortex-induced boundary-layer convergence may be small compared with that induced by convectively-produced heating aloft, especially in the early stages of development when the vortex is weak. Moreover, in reality, convection will occur only if mechanisms exist to enable air parcels to overcome their CIN. Such mechanisms, for example lifting produced along sea-breeze fronts, along the boundaries of spreading cold pools, and especially at the intersection of such boundaries, are typically subgrid-scale in models and their parameterization is normally not attempted (see Mapes, 1997). Therefore, it is generally assumed that convection will be initiated in regions of local convective instability (i.e. positive CAPE), caused for example by radiative cooling aloft and large surface fluxes of latent and/or sensible heat. This assumption may be adequate over the tropical oceans where values of CIN are typically small. However schemes that restrict convection only to conditionally-unstable regions where there is low-level convergence cannot allow convection to occur in a quiescent environment, even when there is large conditional instability and low or zero CIN. This is clearly not realistic in general, but may not be a major issue in hurricane models where low-level convergence is prevalent in the inner region! However, the prohibition of convection where there is low-level divergence may prevent the formation of outer rainbands, concentric eyewalls etc., that may important in hurricane evolution (Emanuel, personal communication).

Parameterization schemes *assume* also that the large-scale flow, and in particular the associated large-scale vertical motion, is known at a given time, or more specifically that it is resolved by the model. Thus part of the circulation depicted in Fig. 3, which is entirely a consequence of the presence of deep convective clouds, would be assumed to be known (this is the resolvable part) and the remaining part would have to be determined as a by-product of the parameterization scheme. We return to this issue in section 5d. Parameterization schemes for representing cumulus convection in numerical models of hurricanes may be divided into four broad groups:

1. those based on a closure constrained by moisture convergence into a column, the prototype being the scheme introduced by Kuo (1965);
2. those whose closure involves the use of (usually simple) cloud models, many of which are based on the scheme proposed by Ooyama (1969);
3. those based on the idea of moist convective adjustment, the prototype being the scheme used by Kurihara (1973); and
4. schemes based on the idea of quasi-equilibrium, including the rather complex scheme proposed by Arakawa and Schubert (1974), the simple scheme proposed by Emanuel (1986), and the more sophisticated ones proposed by Emanuel (1989, 1995). Both the Arakawa-Schubert scheme and the 1989 Emanuel-scheme also use a cloud model in their formulation.

The various types of schemes are described in sections 5 - 7. In section 8 we discuss models that seek to represent the explicit release of latent heat without parameterization and discuss the idea of Conditional Instability of the Second Kind (CISK) introduced by Charney and Eliassen (1964), which emerged from the early failures to simulate hurricanes with explicit representations of latent heat release. A brief discussion and outlook follows in section 9.

5. Early cumulus parameterization schemes

The earliest schemes to parameterize cumulus convection in numerical models include those of Kuo (1965), Pearce and Riehl (1968), Yamasaki (1968a,b), Rosenthal (1969) and

Ooyama (1969). Prior to these studies, a number of attempts were made to represent latent heat release explicitly; these attempts are reviewed in section 8. The Kuo-scheme formed a basis for many other schemes and slightly modified forms of it were suggested by Krishnamurti (1968), Krishnamurti and Moxim (1971) and Sundqvist (1970). The following discussion follows closely on that of Ceseleski (1973) who carried out a careful comparison of some of the early schemes when applied to an easterly wave. A similar comparison of the Kuo-, Pearce and Riehl-, and Rosenthal-schemes was made by Elsberry and Harrison (1972). The early schemes fall into three broad groups, which are examined in subsections (a) - (c) below.

(*a*) *Kuo-scheme and its derivatives*

The Kuo-scheme, illustrated schematically in Fig. 4, is based on five assumptions about the perceived nature of deep convection:

1. Deep cumulus convection occurs in regions where the stratification is conditionally-unstable, but only if there is low-level moisture convergence;
2. The cumulus clouds are formed from boundary-layer air and cloudy air can be characterized by a pseudo-moist adiabat typical of the boundary layer;
3. Clouds extend from the lifting condensation level of boundary-layer air to the level of neutral buoyancy for this air; and
4. Cumulus clouds exist only momentarily before they mix totally with their environment.
5. The cumulus mass flux is proportional to the moisture convergence.

Specifically, in regions of conditional instability and where there is moisture convergence at a rate C into a vertical column, heating is applied at the rate

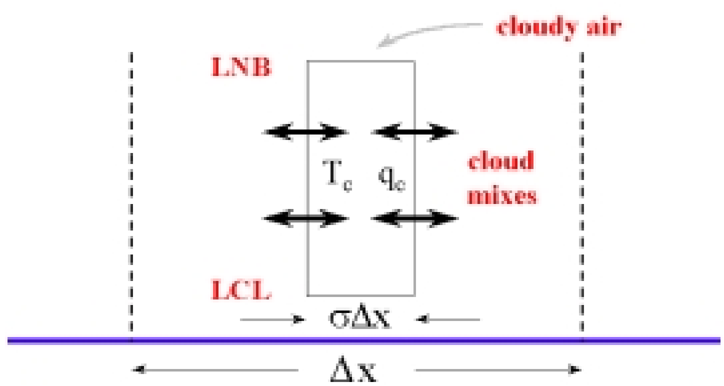

Figure 4. Schematic illustration of Kuo's 1965 cumulus parameterization scheme. LNB and LCL denote the level of neutral buoyancy and lifting condensation level, respectively. If a grid column, delineated by vertical dashed lines, is conditionally-unstable and if the rate of moisture convergence into it, C, is positive, the amount of moisture required to saturate the column, denoted by $W_1 + W_2$, is calculated. Here W_1 and W_2 are the amounts of moisture required to raise the environment temperature and mixing ratio over the depth of the unstable-layer to the moist-adiabatic values T_c and q_c, respectively. The fractional area of cloud, σ, that can be produced in a time step Δt is then $C\Delta t/(W_1 + W_2)$. The cloud is assumed to mix instantaneously with the environment, heating it by an amount $L\sigma\Delta t W_1$, where L is the latent heat of condensation, and moistening it by an amount $\sigma\Delta t W_2$. The precipitation produced is simply the amount of water vapour converted to heat, $\sigma\Delta t W_1$.

$$\dot{Q}_c = \begin{cases} \frac{\sigma}{\Delta t} c_p \, (T_c - T_o) & for \qquad C < 0 \quad and \quad T_c > T_o, \\ = 0 & otherwise \end{cases} \tag{1}$$

where T_c and T_o are the temperatures of the cloud and its environment at a particular level, c_p is the specific heat at constant pressure, Δt is the cloud time scale, normally taken to be the time step in the numerical model; and σ is a measure of the fractional area of a grid cell covered by active cumulus clouds. The moisture convergence is given by,

$$C = -\frac{1}{g} \int_{P_{LNB}}^{p_{LCL}} \nabla_p \cdot (\mathbf{u}q) \, dp - \frac{\omega_b q_b}{g} + F_{qs}, \tag{2}$$

where $\mathbf{u}$ is the horizontal wind vector, ω_b is the vertical p-velocity at the top of the boundary layer, generally assumed to be at 900 mb, and F_{qs} is the surface evaporation rate. In the revised scheme by Krishnamurti (1968), the contribution to C from the first term in Eq. (4) is neglected in comparison with the second term, which is the cloud base moisture flux. The parameter σ is the ratio of the resolved-scale moisture convergence out of the boundary layer to the moisture convergence that would be required to fill an entire grid column with convective cloud and is typically less than 0.02

$$\sigma = C\Delta t/(W_1 + W_2), \tag{3}$$

where

$$W_1 = \frac{1}{g\Delta t} \int_{P_{LNB}}^{p_{LCL}} \frac{c_p}{L} (T_c - T_o) \, dp, \qquad W_2 = \frac{1}{g\Delta t} \int_{P_{LNB}}^{p_{LCL}} (q_c - q_o) \, dp, \tag{4}$$

represent the amount of moisture required to raise the environment temperature and mixing ratio to the moist-adiabatic values T_c and q_c, respectively, p_{LCL} and p_{LNB} are the pressures of the lifting condensation level (LCL) and the level of neutral buoyancy (LNB), L is the latent heat of vaporization, and g is the acceleration due to gravity.

In vertical grid columns where the model atmosphere is found to be conditionally unstable and the moisture convergence is positive, a fractional area σ of deep cumulus clouds rises to p_{LNB} and mixes completely with the environment. The rate of moistening due to mixing, $\dot{W}_c$, is given by

$$\dot{W}_c = \frac{\sigma(q_c - q)}{\Delta t}, \tag{5}$$

and the precipitation rate is just the rate of heating given by (1), divided by the latent heat.

Sundqvist (1970) described simulations of a tropical cyclone using essentially a version of Kuo's scheme in a balanced, 10-level, axisymmetric model. He showed that a cyclone of realistic strength and structure could be obtained, but found that the intensification rate depends sensitively on the vertical distribution of the diabatic-heating rate.

A problem with the Kuo scheme is its propensity to excessively moisten the atmosphere (see e.g. Kitade, 1980, p475), i.e. too much of the water vapour that converges in a grid column is used to moisten the atmosphere, whereupon too little is available to heat the atmosphere and precipitate. An extended version of the scheme was presented by Kuo (1974), in which the rate of moistening integrated over a vertical column of the atmosphere is equal to a small fraction b of the precipitation rate, P, i.e.

$$\int_0^\infty \frac{\partial \overline{\rho} q}{\partial t} \, dz = bP. \tag{6}$$

The vertical integral of the moisture conservation equation [see e.g. Emanuel, 1994, Eq. (16.2.1)] is

$$\int_0^\infty \frac{\partial \overline{\rho} q}{\partial t}\, dz = -\int_0^\infty \nabla \cdot (\overline{\rho}\mathbf{u}_h q)\, dz - P + E_S, \tag{7}$$

whereupon, using (6),

$$P = \frac{E_s - \int_0^\infty \nabla \cdot (\overline{\rho}\mathbf{u}_h q)\, dz}{1+b}. \tag{8}$$

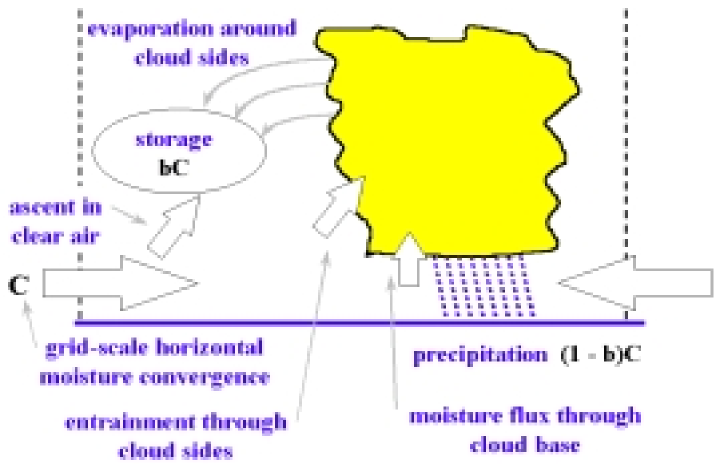

Figure 5. Schematic showing the moisture cycle in a column that contains convection in the 1974 Kuo-scheme (Adapted from Anthes, 1977a). In a region of horizontal moisture convergence in a grid box, and where the boundary-layer air in the grid box is conditionally unstable, the rate of moisture convergence C is partitioned into a part bC that moistens the air in the grid box and a part (1 - b)C that precipitates out, releasing latent heat in the grid box.

Then $\dot{Q}_c$ and $\dot{W}_c$ are obtained in terms of P from formulae equivalent to (1) and (2). The partition of moisture in the scheme is illustrated in Fig. 5.

The 1974 Kuo-scheme has enjoyed much popularity because of its relative simplicity, and together with its predecessor it has formed the basis of many of the schemes that have been used in hurricane models. However, its scientific basis has been seriously challenged in recent years (Raymond and Emanuel, 1993; Emanuel, 1994, Section 16.2). For one thing, there seems to be little scientific or observational basis for the assumption expressed by Eq. (6) with constant b, and for another, the left-hand side of this equation is not Galilean invariant, while the precipitation on the right-hand-side is. Moreover, as it stands, the scheme would not be activated in a conditionally-unstable environment with no basic flow (and hence no moisture convergence) and zero CIN. There have been numerous attempts to remove the many problems with the scheme as described by Emanuel (1994, p 531), but these have not removed the most basic limitations. For example Anthes (1977a,b) related the parameter b to a measure of the relative humidity in a grid column and used a cloud model in an effort to improve the vertical distribution of heating and moistening in the scheme. A further limitation of the scheme is its inability to produce a realistic moistening of the atmosphere in a radiative-convective calculation for zero basic flow, in which the surface radiative and turbulent heat fluxes balance the longwave radiation to space. Emanuel *op. cit.* notes that under these circumstances, the scheme will necessarily lead to a saturated atmosphere.

(b) *Rosenthal's and related schemes*

Rosenthal (1970a) describes a numerical model formulated in σ-coordinates and having 7 vertical levels including the surface. In this, a heating function is prescribed when $w_b > 0$ and $T_c > T_1$, where T_c is the temperature of a surface air parcel rising with constant equivalent potential temperature, T_1 is the temperature at level 1 and w_b is the cyclone-scale vertical motion at level 2, which is located at about 900 mb and is assumed to be representative of the flow entering or leaving the boundary layer. The heating function has the form:

$$\dot{Q}_c = \overline{\rho}_2 w_b \overline{q} \times L(T_c - T_o) / \int_{z_{LCL}}^{z_{LNB}} \overline{\rho}(T_c - T_o)\, dz. \tag{9}$$

If the above criterion is not satisfied then $\dot{Q}_c = 0$. Here $\overline{\rho}_2$ is a standard density at level 2, $\overline{q}$ is the average specific humidity for the layer between levels 1 and 2, z_{LCL} is the LCL of the surface air, and z_{LNB} is the LNB for this air. As in the Kuo schemes, convection occurs only in the presence of low-level convergence ($w_b > 0$) and conditional instability for surface air parcels ($T_c > T_1$). Moreover, *all the water vapour that converges in the boundary layer rises in convective clouds, condenses, and falls out as precipitation - there is no moistening of the atmosphere. Thus all the latent heat released is made available to the large-scale flow.* Also, the vertical distribution of this heating is such that the large-scale lapse rate is adjusted towards a pseudo-adiabat appropriate to ascent from the surface. The same scheme was used in an asymmetric hurricane model by Anthes *et al.* (1970a,b), Rosenthal (1970b, 1971), and in a slightly modified form by Anthes (1972).

Yamasaki (1968a,b) used a similar scheme in a four layer model, with again the total heating in a column proportional to the vertical velocity at the top of the friction layer and its vertical distribution prescribed. In addition all moisture entering the column falls out as precipitation. Yamasaki (1968a) examined the development of cyclones as the time-independent ratio of heat release between the upper troposphere and lower troposphere was varied and found a sensitivity to this ratio. This sensitivity was explored further in the second paper Yamasaki (1968b), where the ratio was allowed to vary with time in proportion to the difference between the potential temperature of the mean field and that of the parameterized clouds. Similar schemes to Rosenthal (1969) were used by Kitade (1980), who modified the rate at which the convection moistens the atmosphere, and Madala and Piacsek (1975) who developed a three-dimensional three-layer hurricane model on a beta plane.

(c) *Ooyama's 1969-scheme*

Ooyama (1964, 1969) considered a flow configuration with two layers of homogeneous fluid of different densities overlying a shallow boundary layer of uniform thickness and having the same density as the layer above. He represented the heating effects of deep cumulus clouds in terms of a mass flux from the boundary layer into the upper layer wherever there is resolved-scale boundary-layer convergence (see Fig. 6). The representation is based on the idea that the deep cumulus clouds that form in such a region will entrain ambient air from the middle layer as they rise through this layer and detrain into the upper layer. It is assumed that for each unit of mass transferred from the boundary layer into the upper layer, $\eta - 1$ units of mass are entrained from the middle layer and transferred to the upper layer also. The total mass flux entering the upper layer is then ηw_b, where w_b is the resolved-scale vertical velocity at the top of the boundary layer. The entrainment rate, which is proportional to $\eta - 1$, is determined as a function of time so as to satisfy the conservation of moist static energy, with the assumption that the air detrained into the

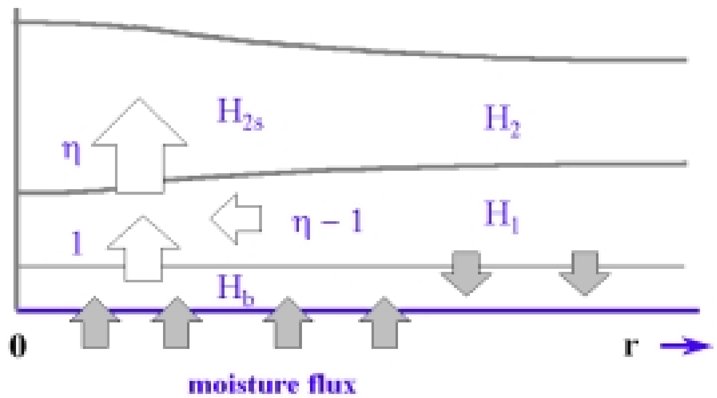

Figure 6. Flow configurations in Ooyama's problem. The moist static energy of air in the lower and upper troposphere and the surface boundary layer are H_1, H_2 and H_b, respectively, and H2s denotes the saturated moist static energy of the upper layer. For each unit mass of air that exits the boundary layer, in a convective updraught,
$eta - 1$ units are entrained into the updraught and η units are deposited in the upper layer. The amount of entrainment is determined by the requirement that cloudy air entering the upper layer has zero buoyancy there. Accordingly, this air has the saturated moist static energy of the upper layer and energy conservation then requires that

$$\eta = 1 + (H_b - H_{2s})/(H_{2s} - H_1).$$

The mass flux into the upper layer is equal to ηw_b, where w_b is the vertical velocity at the top of the boundary layer. Surface moisture fluxes are present in the model also.

upper layer has the saturation moist static energy of ambient air in that layer. The moist static energy budget gives

$$\eta = 1 + \frac{H_b - H_{2s}}{H_{2s} - H_1} \tag{10}$$

where H_b, H_1, and H_{2s} are respectively the moist static energy of the air in the boundary layer and the middle layer (layer-1) and the saturated moist static energy in the upper layer (layer-2). Note that the parameter η depends on the degree of instability of the flow to deep convection, i.e., on the CAPE, which is proportional to $H_b - H_{2s}$. In the nonlinear version of the model (Ooyama, 1969), both H_b and H_{2s} are taken to be dependent variables of the problem, whereupon η varies with time. The variation of H_b is influenced by sensible and latent heat fluxes from the ocean. Ooyama *op. cit.* shows that, as the hurricane vortex develops a warm core structure aloft, the inner region becomes more neutral to convection whereupon η tends to unity and the entrainment declines to zero. The entrainment is a crucial factor in Ooyama's model as it leads to convergence in the middle layer which, as noted in section 2, is required for vortex spin-up (Ooyama, 1982). As the entrainment declines to zero there is no further intensification.

It should be remarked that Ooyama's model consists of three layers of homogeneous fluid and includes no thermodynamic equation as such: the diabatic heating and cooling are represented by mass transfer between layers. The coupling between thermodynamics and dynamics arises from the specification of the entrainment η, the value of which is determined from thermodynamic considerations. Angular momentum is transported vertically between the middle layer and upper layer by the diabatic mass transfer, but there is no such transfer between the boundary layer and the middle layer because the radial profile of tangential velocity in these two layers is taken to be the same.

The numerical integrations of the nonlinear equations are able to produce hurricane-like vortices with a considerable degree of realism, including their radial scale, growth rate and mature strength. Ooyama *op. cit.* (p38) noted that latent and sensible heat transfer from a warm ocean are a crucial requirement for vortex intensification in the model.

Ooyama's 1969-scheme has been used in slightly modified forms in many subsequent modelling studies, which have generally included prognostic equations for momentum in the boundary layer. DeMaria and Schubert (1984) developed a three-dimensional version of Ooyama's (1969) hurricane model on a mid-latitude beta-plane, using the original convective scheme and a spectral method of solution. DeMaria and Pickle (1988) reformulated the original (axi-symmetric) Ooyama model to include three layers of constant potential temperature instead of three homogeneous layers, arguing that the modifications facilitated the representation and interpretation of thermodynamic processes. The governing equations in the adiabatic case turn out to be mathematically equivalent to those in Ooyama's model, except for an extra term that appears in the pressure gradient force. Shapiro (1992) studied vortex motion using a three-dimensional finite-difference version of DeMaria and Pickle (1988) model, but with angular momentum transport proceeding directly from the boundary layer to the upper layer in regions of deep convection. Dengler and Reeder (1997) also studied vortex intensification and motion in a three-dimensional finite-difference version of Ooyama's model on a mid-latitude beta-plane, using Shapiro's method for the vertical transport of angular momentum. They showed, *inter alia*, that the vortex motion is sensitive to the representation of angular momentum transport by convection and that with Shapiro's method, the rate of vortex intensification is larger than with DeMaria and Pickle's method. They showed also that vortex intensification can occur even if the initial sounding is neutrally-stable to convection (i.e. $\eta = 1$). In this case, surface fluxes soon destabilize the profile making $\eta > 1$ and surface convergence then activates the convection scheme in the inner core region. The findings corroborate those of Emanuel (1986) and Rotunno and Emanuel (1987) discussed in section 7(b).

Dengler and Smith (1998) used Dengler and Reeder's model to investigate the development of a monsoon depression centred on a coast. In this case regions develop where there is boundary-layer convergence, but where $0 \leq \eta < 1$. The parameterization scheme is unrealistic in such regions and convection was not permitted to occur if $\eta < 1$. The calculations expose additional problems of representing convection over land.

Neither the original Ooyama scheme, nor the numerous modifications thereof consider the effects of precipitation-cooled downdraughts.

(*d*) *Ooyama's 1971-theory*

Ooyama (1971) noted that the earliest cumulus parameterization schemes are largely intuitive and empirical and do not provide a description of the actual physical mechanisms through which a field of cumulus clouds interacts with larger scale flows. Accordingly he set out to provide a general theoretical framework for cumulus parameterization. In his formulation, he idealizes a field of cumulus clouds as an ensemble of buoyant elements, each of which is dynamically independent of the others. These "clouds' are assumed to be governed by a steady one-dimensional entraining plume model. Ooyama shows that, with the assumption of immediate fall-out of liquid water, the effective heating rate associated with cumulus convection can be expressed as

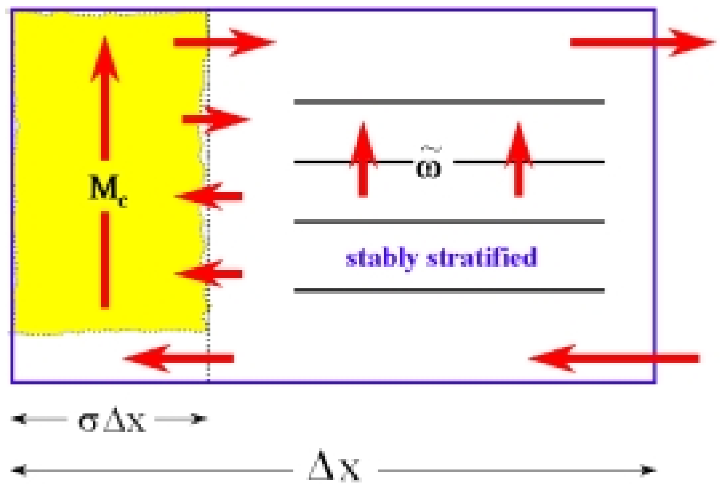

Figure 7. Idealization of convection in a grid column of width Δx in the x-direction and unit width in the y-direction as a single entraining cloud, without downdraughts, surrounded by clear air which is stably stratified. The "bulk" cloud covers a small fractional area σ of the grid column, to the left of the vertical dotted line. The vertical p-velocity in the cloud is M_c, and that in the clear air is $\tilde{\omega}$, both expressed per unit horizontal area. Of course, the cloud does not have to be at the left side of the grid box, but placing it there and assuming that all exchanges between neighbouring grid boxes takes place at the right hand edge simplifies the discussion and the mathematics that goes with it without loss of generality.

$$\dot{Q} = c_p \pi M_c \frac{\partial \overline{\theta}}{\partial p} + \mathcal{D}\left[c_p(T - \overline{T})\right] \tag{11}$$

where $\dot{Q} = c_p \pi D\overline{\theta}/Dt$, π is the Exner function, M_c is the cloud mass flux*, D/Dt is the material derivative, an overbar denotes a horizontal average over a grid element and $\mathcal{D}$ denotes the *detrainment* (conversion) of cloud air to the cloud-free environment. It is insightful to derive a result, analogous to Eq. (11), for a grid column in a numerical model. The convection in the grid column is idealized as a single "bulk" entraining cloud, without downdraughts, surrounded by clear air which is stably stratified (Fig. 7). It is sufficient to assume a two-dimensional configuration with unit horizontal width in the y-direction since the generalization to three dimensions is straightforward. The cloudy air is taken to cover the region $(0, \Delta X_c)$ and the clear air covers the region $(\Delta X_c, \Delta X)$. Further, it is assumed that cloud covers only a small fractional area σ of the total grid area, taken for simplicity here to be independent of height (a more general derivation is presented by Fraedrich, 1973). Accordingly $\Delta X_c = \sigma \Delta X$. In pressure coordinates the mean vertical p-velocity averaged horizontally over the column is $\overline{\omega}$, while that averaged over the clear air is $\tilde{\omega}$. Mass continuity at any level in the grid column requires that

$$\overline{\omega} = M_c + \tilde{\omega}. \tag{12}$$

Since motion in the clear air is approximately adiabatic, the potential temperature change is governed by the equation $D_h\theta/Dt + \omega(\partial\theta/\partial p) = 0$. Writing this equation in flux form and integrating horizontally over the clear air gives

* Strictly the cloud mass flux is $-M_c/g$ when expressed in pressure coordinates, as M_c is then negative for upward motion. However, it is common in the literature to refer to M_c, with units of pressure change per unit time, as the mass flux. We adhere to this convention except in section 7, where the units are vertical velocity.

$$\frac{\partial \tilde{\theta}}{\partial t} + \tilde{\omega}\frac{\partial \tilde{\theta}}{\partial p} + \tilde{\theta}\frac{\partial \tilde{\omega}}{\partial p} + \frac{1}{\Delta X - \Delta X_c}\left[u\theta\right]_{\Delta X_c}^{\Delta X} = 0, \tag{13}$$

where u is the horizontal wind speed and the tilde denotes a horizontal average over the clear air region. Setting $\theta = \tilde{\theta} = 1$ in this equation gives the integrated form of the continuity in that region and, in particular, an expression for $\partial\tilde{\omega}/\partial p$. Then Eq. (13) may be written as

$$\frac{\partial \tilde{\theta}}{\partial t} + \tilde{\omega}\frac{\partial \tilde{\theta}}{\partial p} + \frac{1}{\Delta X(1-\sigma)}\left[u(\theta - \tilde{\theta})\right]_{\Delta X_c}^{\Delta X} = 0. \tag{14}$$

Assuming now that $\sigma << 1$, it follows that $\tilde{\theta} \approx \overline{\theta}$, whereupon, using (12) we obtain an expression for the $\partial\overline{\theta}/\partial t$, namely

$$\frac{\partial \overline{\theta}}{\partial t} = -\overline{\omega}\frac{\partial \overline{\theta}}{\partial p} + M_c\frac{\partial \overline{\theta}}{\partial p} - \frac{1}{\Delta X}\left[u(\theta - \overline{\theta})\right]_{X=\Delta X} + \frac{1}{\Delta X}\left[u(\theta - \overline{\theta})\right]_{X=\Delta X_c}. \tag{15}$$

The first term on the right-hand-side of this equation represents the mean vertical advection of mean potential temperature averaged over the grid box. If the air is stably-stratified ($\partial\overline{\theta}/\partial p < 0$) and there is mean ascent ($\overline{\omega} < 0$), the term contributes to mean cooling in the grid box, while for an upward the cumulus mass flux ($M_c < 0$), the second term is positive and like the first term on the right-hand-side of (11) may be interpreted as the effect of warming by adiabatic compression associated with 'compensating subsidence' in the cloud environment (Ooyama, *op. cit.*, p751). The third and fourth terms on the right-hand-side of Eq. (15) represent the horizontal advection of perturbation potential temperature across the grid box boundary ($x = \Delta x$) and the boundary between the cloud and the grid box ($x = \Delta x_c$), respectively. Typically, in a finite-difference formulation of the equations, only grid-averaged quantities* are calculated. Then the third and fourth terms are zero if air leaves the clear air region (i.e. if $u > 0$ at $x = \Delta x$ and $u < 0$ at $x = \Delta x_c$) because in both these cases $\theta = \overline{\theta}$. If the flow is inwards towards the clear air at $x = \Delta x$, θ takes the value of $\overline{\theta}$ at the adjacent grid box and this term is analogous to the term $\overline{u}(\partial\overline{\theta}/\partial x)$ in the continuous form of the equation for $\overline{\theta}$. If the flow is inwards towards the clear air region at $x = \Delta x_c$, the fourth term represents the detrainment of cloud air with potential temperature $\theta = \theta_c$ into the region and is analogous to the second term on the right-hand-side of (11). Similar terms in the horizontally-averaged moisture equation represent a moistening of the cloud environment due to detrainment and a drying due to subsidence in this region.

The system of equations for the grid-averaged temperature and moisture is closed when the cloud-base mass flux is determined. Ooyama considered the "bulk cloud" to be composed of a collection of steady entraining plumes fed by a statistical ensemble of air bubbles of different sizes released per unit time. He called the mass flux distribution of these bubbles the "dispatcher function". The optimum method for relating the dispatcher function to the large- scale model variables was left for future research, although an example was explored in which a particular size distribution of bubbles was prescribed with the net cloud-base mass flux set equal to the mean (resolvable-scale) vertical velocity at the top of the subcloud layer.

Recently, Mapes (1997) pointed out that it is only strictly correct to interpret the first term on the right-hand-side of Eq. (11) as "compensating subsidence" when $\overline{\omega} = 0$, in which case $\tilde{\omega} = -M_c$. Then *all* the convective mass flux descends within a grid box. For example, in the situation depicted in Fig. 3 it is clear that, depending on the grid size in relation to the region covered by clouds, a fraction of the convective mass flux will be manifest as

* Various methods of averaging across the grid are discussed by Anthes (1977a). The present method involving "top-hat" profiles in the clear air and cloud regions suffices for the purpose of illustration here.

clear-air subsidence inside the grid box and the remainder will occur on the resolvable scale outside the grid box. Mapes argues that in the configuration shown, the large-scale vertical motion $\overline{\varpi}$ is determined entirely by the convection, itself, and may not be thought of as externally imposed. This poses a dilemma because a requirement for parameterizability is that the convection heating and moistening rates and vertical momentum transfers can be uniquely associated with the prevailing large-scale conditions. If these large-scale cannot be externally imposed, it is not clear that the convection can be *parameterized* in terms of it. However, provided that the convection is in quasi-equilibrium with the large-scale conditions, it is conceivable that even if the large-scale conditions evolve in response to the convection, a knowledge of these conditions at a particular time is sufficient to determine the convective mass flux consistent with them.

Ooyama *op. cit.* [Eq. (45)] showed that if the cloud updraughts originate from a single level, p^*, the total convective mass flux at this level, M_c^*, is equal to the large-scale convergence of air in the planetary boundary layer below p^*, say $\overline{\varpi}^*$. Then, the first term on the right-hand-side of (11) can be written in the form

$$\dot{Q} = c_p \pi \eta \overline{\varpi}^* \frac{\partial \overline{\theta}}{\partial p}, \tag{16}$$

where

$$\eta(p) = M_c(p)/M_c^*. \tag{17}$$

This expression for $\eta(p)$ provides a generalization of that defined in Ooyama's 1969-model, where it represents the ratio of mass transfer from the middle layer into the upper layer to that which enters the middle layer from the boundary layer. Ooyama points out that the Kuo scheme incorporates only the effects of detrainment, represented by the second term on the right-hand-side of (11), while his own 1969-scheme considers only the effects of compensating subsidence, represented by the first term. Strictly, $\overline{\theta}$ is not defined in the 1969-model, but an analogous result holds. The difference between the local large-scale mass convergence $\rho_1 u_1$ in the middle layer and the local rate of mass extracted from that layer, $\rho_1 w_b(\eta - 1)$, determines whether the depth of this layer falls, equivalent to heating, or whether it deepens, equivalent to cooling. As before, w_b is the vertical velocity at the top of the boundary layer and ρ_1 is the density in layer-1 *and* in the boundary layer.

Rosenthal (1979) applied Ooyama's 1971-scheme to an axisymmetric hurricane model and compared cases in which the dispatcher function consists either of a single cloud or of five cloud sizes with equal mass flux. He concluded that the growth of a hurricane is sensitive to the assumed spectrum of convective elements and their associated cloud-base mass flux. Furthermore he showed that the growth rate is strongly affected by the manner in which the resolvable-scale supersaturation is treated. A form of the scheme was applied also to a tropical-cyclone model by Yamasaki (1986a,b), who related the cumulus mass flux to the resolved-scale vertical velocity at a low level and to the degree of conditional instability.

A theoretical treatment with some similarities to Ooyama's was described by Fraedrich (1973).

(*e*) *Moist convective adjustment*

Adjustment schemes, which date back to Manabe *et al.* (1965) and Kurihara (1973), are based on the simple idea that convection tends to adjust the virtual temperature of the atmosphere back to some equilibrium state that is nearly neutral to convection. Moist convective adjustment is applied to any model layer that becomes saturated and in which the lapse rate exceeds the moist adiabatic lapse rate. The moist static energy of the layer is held fixed during the adjustment. In a sense, the scheme is not a representation subgrid-scale convection, but rather an adjustment to neutral stability in an explicitly simulated cloud (which lasts only momentarily). Further details of the implementation of the scheme are described succinctly by Emanuel (1994, section 16.3), who also provides a critique of the method. For large-scale model grids, the scheme is slow to respond to large-scale forcing as the air must first become saturated on the grid scale and because the rate of ascent is limited by the model's explicit vertical velocity, which is also limited by the grid scale. As a result, significant instability can accumulate during the time it takes for an explicit cloud to form. Emanuel *op. cit.* shows also that the fractional area of clouds using the scheme is larger than in nature.

A moist adjustment scheme is used in the Geophysical Fluid Dynamics Laboratory (GFDL)'s hurricane prediction model, integrations of which are described by Kurihara and Tuleya (1974), Kurihara (1975, 1976), Tuleya and Kurihara (1975, 1981), Kurihara (1976), Kurihara and Bender (1980), Kurihara *et al.* (1990).

6. The Betts-Miller scheme

A different type of adjustment scheme was introduced by Betts (1986). The scheme is applied by adding terms F_T and F_q, given by

$$F_T = \frac{T_{ref} - T}{\tau} \qquad and \qquad F_q = \frac{q_{ref} - q}{\tau}, \tag{18}$$

to the thermodynamic and water vapour continuity equations, respectively, in layers that are convectively-unstable at a particular grid point and time. Here T and q are the temperature and water vapour mixing ratio at a grid point, T_{ref} and q_{ref} are reference values to which these quantities are adjusted and τ is the adjustment time scale. The reference profiles, T_{ref} and q_{ref}, depend on the cloud top and are different for deep and shallow convection. The reference profiles for shallow convection are constructed from a mixing line, subject to an energy constraint and no latent heat release, while for deep convection, they are constructed empirically on the basis of observational data.

This scheme has been implemented in a hurricane model by Baik *et al.* (1990a,b) and predictions using it are compared with those using the Kuo-scheme by Baik *et al.* (1992). The model, itself, is described by Baik *et al.* (1990a); it is an axisymmetric, primitive equation model with 15 vertical levels and a horizontal resolution of 20 km. They show that the convection scheme is capable of simulating the developing, rapidly intensifying, and mature stages of a hurricane from a weak vortex. Their model includes an explicit representation of latent heat release when saturation occurs on the grid-scale. Noteworthy is the result that during the early stage, the latent heat release is from the convective parameterization, but at the mature stage the latent heat release is mainly due to the grid-scale phase change. Presumably, as the mature stage is approached, frictional convergence in the boundary layer is sufficient to force saturated ascent on the grid scale in an annular region in the inner vortex core. This forced inner-core convection has an outward slope that would tend to stabilize the flow to upright convection. It is stated that the order of application of the grid-scale and convective scale schemes does not appear to be important, but the reasons for this surprising result are not discussed. In the companion paper Baik *et al.* (1990b) investigate, *inter alia*, the sensitivity of the model to the three convective adjustment parameters that have to be specified. These include the adjustment time scale, the stability weight on the moist adiabat, and the saturation pressure departure. Increasing the stability weight from 0.9 to 1.0, for example, has little effect on the final storm intensity, but reduces the time at which rapid deepening occurs by more than a day, a factor which would have significant forecasting implications!

Emanuel (1994) points out that the Betts-Miller scheme has a great advantage over the Kuo-type schemes and moist adjustment schemes in that it contains no artificial constraints on the release of instability. As in nature, convection occurs when and where the atmosphere is unstable and the convection drives the atmosphere back to neutrality as observed. A weakness of the scheme is that there is no physical basis for a universal reference profile of relative humidity.

7. Schemes based on quasi-equilibrium hypotheses

(a) *The Arakawa-Schubert scheme*

Arakawa and Schubert (1974) devised a cumulus parameterization scheme based on the idea of quasi-equilibrium, which maintains that the collective effect of clouds is to remove the conditional instability of the large-scale flow at the rate that it is produced. More specifically, the consumption of energy (as opposed to moisture or mass) by convection is in equilibrium with its generation by large-scale processes. The quasi-equilibrium assumption assumes that the characteristic time scale for the large-scale flow is much larger than the time scale for convective clouds. In their scheme, cumulus clouds are represented by a spectrum of steady entraining plumes (Fig. 8), each plume having a different, but constant entrainment rate, λ. The model "clouds" all have the same base, whereas the top of each cloud is determined by its level of neutral buoyancy, which decreases with increasing entrainment rate. The amount of work done by buoyancy in each cloud type per unit cloud base mass flux is called the cloud work function*. This function depends on the thermodynamic structure of the cloud environment and increases as a result of large-scale processes that tend to destabilize the atmosphere, such as radiative cooling, vertical motion and surface heat and moisture fluxes. In contrast, convection tends to eradicate the instability by warming its environment through compensating subsidence, thereby reducing the cloud work function.

Closure in the Arakawa-Schubert scheme is obtained by setting the time rate-of-change of the cloud work function equal to zero for each cloud type, a condition which determines the cloud-base mass flux for each cloud type.

The Arakawa-Schubert scheme was implemented in a tropical-cyclone model by Wada (1979). The model, formulated in height coordinates, is axisymmetric and has five vertical layers, including a surface boundary layer. It has three different cloud types: high, medium and low with tops at 13, 9 and 5 km, respectively. Wada showed that realistic tropical-cyclone simulations could be obtained using this scheme. In particular, an eye and eyewall form in the rapid development stage when the tangential velocity near the centre becomes strong and the eyewall is located near the radius of maximum wind. High clouds occupy the eyewall in the later stages of development, while medium and low clouds coexist with the high clouds in the earlier stages. In line with our discussion in section 2, it was found that surface friction is indispensable to the formation of an eyewall and an eye, whereas it does not play an essential role in vortex development at the early stage when the vorticity is weak.

Rosenthal (1979) pointed out that the validity of using the Arakawa-Schubert scheme

* The cloud work function is equal to the CAPE if the entrainment rate is zero, but is otherwise less than the CAPE.

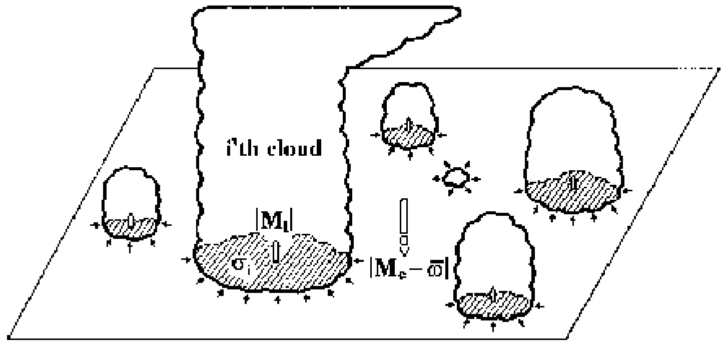

Figure 8. Schematic diagram of the configuration of clouds in the Arakawa-Schubert convection scheme showing a unit horizontal surface between cloud base and cloud top. The taller clouds are shown penetrating this level and entraining environmental air. A cloud that has lost buoyancy is shown detraining cloud air into the environment. The i'th cloud has fractional area, σ_i, and mass flux, M_i. The downward-pointing arrow denotes the compensating subsidence outside clouds, M_c, relative to the large-scale mass flux $\rho\overline{\omega}$, where M_c is the sum ΣM_i over all clouds. (Adapted from Arakawa and Schubert, 1974)

in a hurricane model is far from clear, because the 'large scale flow' is not the synoptic scale, but rather the mesoscale. Accordingly, the separation between large-scale and convective time scales that leads to closure is weakened and probably invalidated. A further difficulty is the assumption that the fractional area of convection in a grid area is small compared with unity, which is generally not satisfied in a hurricane model, especially in the eyewall region. These objections apply also to many of the schemes in use. In its earliest form, including that used by Wada *op. cit.*, the Arakawa-Schubert scheme does not take into account precipitation-driven downdraughts, which are recognized to have an important influence on the boundary layer of a hurricane (see below). More recent versions of the scheme as well as the concept of quasi-equilibrium are discussed by Randall *et al.* (1997a,b).

(*b*) *Emanuel's schemes*

A somewhat different approach to that of previous authors for representing the effects of moist convection in a hurricane model is adopted in a series of papers by Emanuel (1986, 1989, 1995), in which three different parameterization schemes are explored, again all based on the idea of quasi-equilibrium. All previous parameterization schemes are formulated for upright convection, but observations show a marked outward slope of the eye-wall convection in mature hurricanes (see e.g. Jorgensen, 1984) and at least in this region the convection is more "slantwise" than upright, it tends to be along or close to surfaces of absolute angular momentum*. To take account of this fact, the three hurricane models presented by Emanuel are formulated in potential radius coordinates in which the surfaces of absolute angular momentum are upright.

Emanuel (1986) formulated an idealized axisymmetric steady-state hurricane model in which diabatic processes are represented by equating the saturated moist entropy of the free troposphere, defined assuming a reversible process, to the moist entropy of the sub-cloud layer along an angular momentum surface. In essence, cumulus convection is assumed to redistribute heat acquired from the sea surface so as to keep the environment locally

* In a rotating fluid, the net force on a fluid parcel displaced from equilibrium includes an outward-directed centrifugal force as well as the vertically-oriented buoyancy force. As a result convection tends to occur along angular momentum surfaces, which, in a hurricane, slope radially-outwards with height (see e.g. Emanuel, 1983, 1986). This is referred to as slantwise convection.

neutral to slantwise moist convection, in the spirit of the quasi-equilibrium hypothesis. The representation is suggested by observations reported by Betts (1982, 1986) that the (vertical) thermodynamic structure of the tropical atmosphere in regions of deep convection is close to that of a reversible moist adiabat. The closure problem reduces to determining the radial distribution of the subcloud-layer entropy. Since this entropy increases with increasing wind speed, and hence with decreasing radius as far inwards as the radius of maximum tangential wind speed, the saturated moist entropy, and hence the virtual temperature of the free troposphere increase with decreasing radius, leading to a warm-cored vortex. The thermodynamic structure obtained from the formulation mimics that documented, for example, in Hurricane Inez (1966) by Hawkins and Imbembo (1976).

Important results of the study include the demonstration that a hurricane can be maintained exclusively by self-induced heat transfer (sensible and latent) from the sea surface in an atmosphere that is conditionally neutral to slantwise convection. Here, conditionally neutral is not supposed to mean that the atmosphere has no CAPE, which is of course a prerequisite for convection, but that the convection acts on a short time scale to produce a mean state in which the tropospheric temperature is close to that of a (reversible) moist adiabat. The result that hurricane growth can proceed in such an atmosphere is supported by numerical model integrations using an axisymmetric model with explicit convection by Rotunno and Emanuel (1987); see section 8. Emanuel *op. cit.* showed that, energetically, his steady-state model resembles a simple Carnot heat engine, in which latent and sensible heat are extracted from the ocean at the sea-surface temperature and ultimately given up in the upper-tropospheric outflow layer at a much lower absolute temperature. Finally, given these two temperatures, the latitude, and with some assumption about the magnitude of the boundary-layer relative humidity, the model predicts a minimum attainable central pressure.

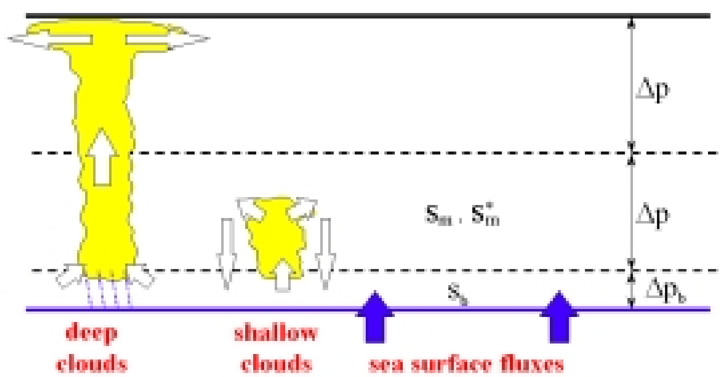

Figure 9. Vertical structure of the hurricane model formulated by Emanuel (1989). The moist entropy represents a vertically averaged value, but is assigned to the middle level; it characterizes the mean (virtual) temperature of the troposphere. Entropy, s, is calculated in the subcloud layer (s_b) and in the lower tropospheric layer (s_m). The saturated moist entropy in the lower tropospheric layer, (s_m^*) is used to predict shallow cumulus activity, which occurs when $s_b > (s_m^*)$. Deep clouds transfer mass from the subcloud layer to the top layer while shallow clouds exchange entropy between the lower troposphere and the subcloud layer without producing a net mass flux.

In a subsequent paper, Emanuel (1989) constructed an axi-symmetric prognostic model with a more sophisticated representation of moist processes that includes two categories of convection as illustrated in Fig. 9. The model has three vertical layers, a subcloud layer and lower- and upper-tropospheric layers, and the two convective cloud types include shallow non-precipitating clouds, which penetrate only into the lower tropospheric layer, and deep precipitating clouds that extend through the depth of the model troposphere. The latent heat acquired from the sea is mostly converted to latent heat within the deep clouds, the amount depending on the precipitation efficiency of the deep clouds, i.e. the fraction of the upward moisture flux that subsequently falls out of clouds as precipitation. In contrast, the shallow clouds have zero precipitation efficiency and produce no net heating, since all the condensed water is ultimately re-evaporated. The shallow clouds are important because they transport high-entropy boundary-layer air to the lower troposphere, where there is normally an entropy minimum, and carry the low entropy air into the boundary layer.

The model carries prognostic equations for the saturation moist entropy s^* of the deep troposphere, assigned to the mid-tropospheric level; the moist entropy s_m and saturation moist entropy s_m^* of the lower tropospheric layer; and the entropy of the subcloud layer, s_b. Shallow clouds occur whenever there is local convective instability in the lower troposphere (i.e. when $s_b > s_m^*$) and deep clouds occur when $s_b > s^*$. In the formulation of the dynamics, the quantity s^* is assumed to be a constant along an angular momentum surface as in Emanuel (1986), while the quantities s_m^* and $s_m^* - s_m$ are used to characterize the temperature and humidity of the lower troposphere. In particular, s_m^* is used for predicting the occurrence of shallow convection.

The prognoses of subcloud-layer and lower-troposphere moist entropies are based on budget equations, obtained by integrating the entropy equation through the depth of the subcloud layer and lower troposphere, respectively.

A simple cloud model is used to calculate the slantwise mass flux of clouds, based on the parcel buoyancy, and the fractional area of clouds in a radial grid interval is set equal to the ratio of the height of the troposphere divided by the radial grid size in physical space, if this ratio is less than unity. Otherwise this ratio is set equal to unity. In a sense, this allows the convection to be effectively explicit in the inner core region, with air parcels conserving their (reversibly-defined) moist entropy. Note that the radial grid interval is constant in potential radius coordinates, but varies with radius in physical space.

Figure 10 shows a schematic illustration of the airflow in a developing tropical cyclone as represented by the model. As explained earlier, for a cyclone to spin-up, lower tropospheric air above the boundary layer must flow inwards. Emanuel argues that this air has a relatively low moist entropy and if it were to ascend directly into the vortex core, the core entropy would reduce, the core would cool and the cyclone would decay. Instead, he argues, the air descends within shallow clouds, within precipitating downdraughts, and outside of clouds because of Ekman suction. This descent of relatively dry air reduces the entropy of the subcloud layer. Emanuel emphasizes that the vortex core can become warmer than its environment only if the surface fluxes are large enough to offset this drying effect on the subcloud layer. He notes that, in a developing storm, individual air parcels flow inwards in the lower troposphere, sink downwards in downdraughts, receive entropy from the ocean, and then ascend in deep convective clouds.

A noteworthy feature of the model behaviour is the threshold intensity of the initial vortex required for the vortex to amplify. This behaviour is attributed to the damping effects of subsidence and cumulus downdraughts into the subcloud layer, which reduce the entropy of this layer. Only if the surface entropy fluxes are large enough to outweigh this cooling effect (i.e. if the surface wind speeds are sufficiently large) can intensification occur. The threshold intensity is a decreasing function of mid-tropospheric relative humidity, an increase of which reduces the effects of downdraughts.

Emanuel (1995) reformulated the foregoing model, using a convection scheme based on the concept of boundary-layer quasi-equilibrium (see Raymond, 1997). The basic configuration is shown in Fig. 11. The thermodynamic variables include: the saturation entropy at

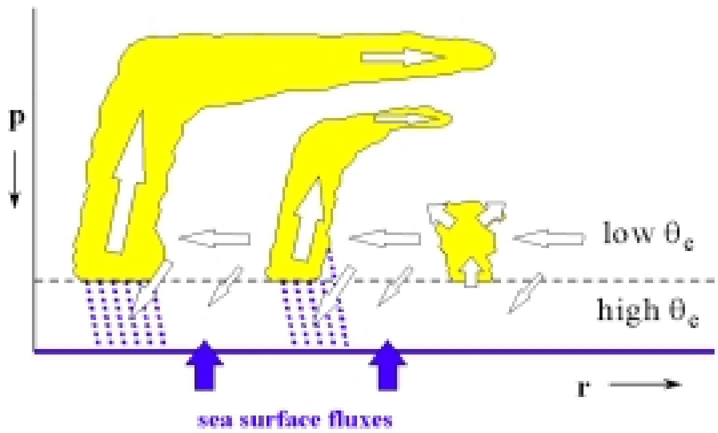

Figure 10. Schematic illustration of airflow in a developing tropical cyclone as represented by Emanuel's 1989 model. For a cyclone to spin-up, lower tropospheric dry-adiabatically air above the boundary layer must flow inwards. This air has a relatively low moist entropy and if it were to ascend directly into the vortex core, the core entropy would reduce, the core would cool and the cyclone would decay. Instead, the air descends within shallow clouds, within precipitating downdraughts, and outside of clouds because of Ekman suction. This descent of relatively dry air reduces the entropy of the subcloud layer. The vortex core can become warmer than its environment only if the surface fluxes are large enough to offset this drying effect on the subcloud layer. In a developing storm, individual air parcels flow inwards in the lower troposphere, sink downwards in downdraughts, receive entropy from the ocean, and then ascend in deep convective clouds.

the top of the boundary layer s_+^*, which is assumed to equal the saturation entropy through the depth of the troposphere, s^*; the lower-troposphere entropy s_m; and the subcloud-layer entropy s_b. The model includes prognostic equations for these quantities, but in regions of convection $(s_b > s_+^*)$, s_b is set equal to s_+^* on the assumption that convection establishes a neutral profile in s^*. A convective mass flux, M_{ueq} is calculated on the assumption that there is an exact balance between the ocean surface entropy flux and the downward flux of low entropy air through the top of the boundary layer, neglecting the radiative cooling of the boundary layer, i.e.,

$$M_d + w_e = -F_s/(s_b - s_m), \tag{19}$$

where F_s denotes the surface energy flux and w_e is the vertical velocity in clear air. The closure requires an assumption about the relationship between convective updraught mass flux*, M_u, and the downdraught mass flux, M_d. The quantities are related by the equation

$$M_d = -(1 - \epsilon_p)M_u, \tag{20}$$

where ϵ_p is a bulk precipitation efficiency. Emanuel points out that, in general, ϵ_p is a function of the distribution of cloud water and the environmental temperature and humidity structure. If all the rain evaporates, $\epsilon_p = 0$ and $M_d = M_u$, consistent with the fact that there is no net latent heat release. On the other hand, if $\epsilon_p = 1$, $M_d = 0$ and there is no evaporation to drive a downdraught. Then all the latent heat is available to warm the air. Emanuel specifies ϵ_p as a function of the relative humidity in the lower troposphere; i.e.

$$\epsilon_p = (s_m - s_{mi}/(s_b - s_{mi}), \tag{21}$$

* Emanuel's uses the term "mass flux" for the vertical velocity in cloudy air averaged over the area of a grid column. In his theory, the mass flux is positive when the motion is upwards.

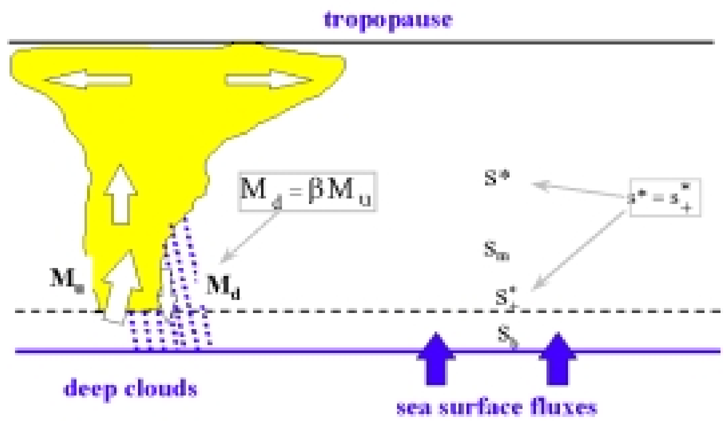

Figure 11. Vertical structure of the hurricane model formulated by Emanuel (1995). The notation is the same as in Fig. 7. In this scheme, the convective downdraught mass flux M_d is taken to be proportional to the updraught mass flux M_u. The constant of proportionality is related to the precipitation efficiency, ϵ_p by the formula $\beta = -(1 - \epsilon_p)$. An equilibrium value $M_u eq$ for M_u is calculated on the assumption that$D_{u.c.}/Dt$ is zero. Finally, M_u is obtained by relaxation to M_{ueq} on a time scale τ_c.

where s_{mi} denotes the value of s_m in the initial state. Finally the actual convective mass flux, M_u, is obtained by relaxation towards the equilibrium mass flux on a time scale τ_c, assumed to be a few hours; i.e.

$$\frac{D_h M_u}{Dt} = \frac{M_{ueq} - M_u}{\tau_c}, \tag{22}$$

where D_h/Dt is the horizontal part of the material derivative. Emanuel (1995, p3964) states that the sensitivity of the calculations to the choice of τ_c appears to be weak.

Unlike the earlier scheme (Emanuel, 1995), shallow clouds are not considered explicitly, but the moistening effects of convection are included by prescribing the detrainment of entropy in the lower troposphere as a function of M_u and ϵ_p.

The scheme can be understood in terms of Fig. 7 if M_c in that figure is associated with the net mass flux $M_u + M_d$ in the Emanuel scheme. Again, $M_u + M_d$ is related to the mean vertical velocity averaged over a grid interval just above the boundary layer, $\overline{w}_+$, by the relationship equivalent to (12), i.e.

$$\overline{w}_+ = M_u + M_d + w_c. \tag{23}$$

Eliminating M_u and M_d from Eqs. (19), (20) and (23) gives an expression for w_e, namely

$$w_e = -\epsilon_p F_s/(s_b - s_m) + (1 - \epsilon_p)\overline{w}_+. \tag{24}$$

Thus the direction of clear-air vertical motion depends on the magnitude of the surface entropy flux in relation to $\overline{w}_+$, which is assumed to be determined by the large-scale flow as before. Typically, if $\overline{w}_+ > 0$ and surface fluxes are sufficiently weak, there will be ascent in the clear air and cooling averaged over the grid box, at least just above the boundary layer. On the other hand, averaged over the grid box, there is descent and warming in the clear air if

$$F_s > (1 - \epsilon_p)(s_b - s_m)\overline{w}_+/\epsilon_p. \tag{25}$$

If the large-scale circulation is associated exclusively with the convection as depicted in Fig. 3, then $\overline{w}_+$ and the condition (25) is always satisfied if $s_b > s_m$. That is, the convection heats the atmosphere just above the boundary layer, and from the assumption that layer $s^*_+ = s^*$, the troposphere will warm also. The opposite case of tropospheric cooling occurs when $\overline{w}_+ > 0$ and is large enough to violate (25). It is conceivable that this might happen, for example, if a vortex moves rapidly over significantly cooler water, so that surface moisture fluxes are abruptly reduced. Then, for a time, the boundary-layer induced upflow in the vortex core may dominate any remaining convectively-induced subsidence, thereby cooling the core and allowing the vortex to weaken. This cooling may happen also at other times when the vortex is weakening.

The model behaviour using this scheme is very similar to that described by Emanuel (1989). In particular, the near saturation of a mesoscale column of the troposphere in the cyclone core is a prerequisite for intensification. Two limitations of all three models described in this subsection is the assumption of moist neutrality outside the convective region and the absence of an explicit water cycle.

The use of potential-radius coordinates in all three models provides higher resolution in the vortex core and facilitates the parameterization of slantwise convection, including the angular momentum transport. However, the use of these coordinates complicates the generalization of the models to three dimensions. Moreover, in reality the convection may not be exactly along angular momentum surfaces; if the convection is not neutral but releases CAPE, the sloping updraughts will have a component towards lower absolute angular momentum.

8. Explicit representations and CISK

Early attempts to simulate hurricanes with an explicit representation of latent heat release were not successful (see Rosenthal (1978) for a review of these attempts). The calculations were rapidly contaminated by small-scale disturbances that were attributed to the release of conditional instability in model cumulus clouds. These early failures led many investigators to the conclusion that the growth of larger scale systems such as hurricanes could not be simulated by numerical integration of the hydrostatic equations for conditionally-unstable, saturated motions. It was this conclusion that led to the emergence of cumulus parameterization and the concept of CISK by Charney and Eliassen (1964), although Rosenthal *op. cit.* went on to show that the conclusion was not true. At least some of the failures occurred because convection was treated as a quasi-linear process and was not allowed to stabilize its environment to further convection (see e.g. Rosenthal, 1978, p259; Smith, 1997b, section 4).

The basic idea of CISK is that in a conditionally-unstable atmosphere, the low-level convergence of moisture in the surface boundary layer of an incipient vortex provides the moisture required to fuel convective clouds, while the buoyancy generated by these clouds leads to convergence above the boundary layer and thereby to vortex intensification. As the vortex intensifies, so does the moisture convergence and this feedback is hypothesized to lead to the so-called CISK-instability. While the concept of CISK has been highly influential in tropical meteorology, it has come under heavy criticism (Ooyama, 1982, Emanuel, 1994, Emanuel *et al.*, 1994, Craig and Gray, 1996, Smith, 1997b, Ooyama, 1997). Emanuel (1994, p518) asserts that "there is no convincing evidence that CISK occurs in nature" and presents arguments suggesting that, in the absence of a significant positive sea-surface entropy flux, the collective effects of deep convective clouds together with the large-scale ascent that forces them are to cool the free troposphere. In this case the net heating associated with deep convection is then smaller than the adiabatic cooling resulting from the forced large-scale ascent. According to the ideas presented in section 2, the negative buoyancy produced locally by the cooling would lead to divergence in the lower troposphere and thereby to vortex decay. As discussed in section 7, Emanuel attributes the cooling of the troposphere to the occurrence of precipitation-induced downdraughts, which cool the boundary layer. Even if these are not taken into account, one could argue that the increased convection associated with increased moisture convergence will heat the upper troposphere, reducing the amount of conditional instability. This is a nonlinear feedback, which is represented in Ooyama's (1969) model, but not in the linear CISK theory of Charney and Eliassen (1964), nor in the many modifications thereof. Indeed, increasing the moisture convergence in a homogeneously stratified atmosphere cannot lead indefinitely to increased tropospheric heating as the free troposphere virtual temperature cannot exceed that of a moist adiabat emanating from the subcloud layer, although there is some effect of decreasing pressure on the subcloud-layer equivalent potential temperature. However, increasing the moisture convergence does lead to an ever increasing heating rate in convection schemes where the heating rate is effectively proportional to the vertical motion, with no compensating factor to account for the reduced degree of conditional instability brought about by the convection (as in linear CISK theories). An interesting historical perspective of the emergence of the theory of CISK and the later disenchantment with it is contained in the article by Ooyama (1997).

An overall discomfort with the correctness of the early cumulus parameterization schemes together with improvements in computer technology in the mid-70s led a number of researchers to re-examine the possibility of allowing for the explicit release of latent heat in hurricane models and of doing away with cumulus parameterization altogether. Notable early successes were achieved by Yamasaki (1975, 1977) and Rosenthal (1978) using hydrostatic, axisymmetric models and Jones (1977, 1980), who used a three-dimensional hydrostatic model with triple grid nesting. More recent studies include the axisymmetric model calculations of Willoughby *et al.* (1984), Yamasaki (1983) and Rotunno and Emanuel (1987), all using non-hydrostatic axisymmetric models.

With the exception of Yamasaki (1975, 1977), in which the temperature and mixing ratio at the lowest model level were held fixed, all the foregoing models include a representation of sea-surface fluxes of sensible and latent heat. Yamasaki's calculations began from an initial state that is highly convectively unstable and on account of this and in the absence of surface heat fluxes, they produce what is more like a thunderstorm-scale vortex (Emanuel, 1991, p188). Rosenthal used a horizontal grid of 20 km, which he recognized to be too coarse to resolve the details of convective clouds, but despite this limitation, the model produced a hurricane-like vortex with a realistic strength and overall structure, including the development of an eyewall structure and eye-like feature. This early study, as well as that of Yamasaki (1983), pointed to the importance of convective downdraughts on hurricane development, which up to that time had not been incorporated into cumulus parameterization schemes. Jones (1980) extended Rosenthal's model to three dimensions.

Willoughby *et al.* (1984) and Rotunno and Emanuel (1987) both used different axisymmetric versions of the Klemp-Wilhelmson model (Klemp and Wilhelmson, 1978) to study hurricane evolution. Willoughby *et al.* used a horizontal grid of 2 km and compared two microphysical parameterizations, one that considers liquid water processes only and the other including ice processes. In the early stages of the calculations, the convection is organized into structures that superficially resemble tropical squall lines, but are modified in two respects by the presence of weak rotation. The updraughts lean outwards along angular momentum surfaces and as they propagate into the vortex, the primary inflow comes from the periphery of the circulation while the anvil streams outwards. This is similar to the situation in Rosenthal's simulations. Willoughby *et al.* note that when the tangential wind is still relatively weak, the inflow is largely a result of the horizontal buoyancy gradient associated with latent heat release in the inner region. However, as the wind increases, frictional inflow becomes more important and by the time tropical storm intensity (16 ms^{-1}) is reached, frictional inflow predominates and the convection increasingly resembles an eyewall. This behaviour corroborates the picture presented in section 2. When ice processes are included, convective rings form more readily on account of the greater convective instability and more active moist downdraughts. Moreover, melting of frozen hydrometeors leads to mesoscale downdraughts that preclude vortex-scale ascent such as happens in the model with liquid water only.

Rotunno and Emanuel (1987) use a different version of the Klemp-Wilhelmson model with simpler representations of microphysical and turbulent processes, a radiation boundary condition at the outer wall and a crude representation of radiative cooling. The radial grid size is 15 km, more than 7 times that used by Willoughby *et al.*. They consider only one class of liquid water and no ice, distinguishing between cloud water and rain by the size of the liquid- water mixing ratio*, q_l: when $q_l \leq 1$ g/kg, liquid water is assumed to move with the air, while for $q_l > 1$ g/kg, liquid water is assumed to fall with a terminal velocity of 7 ms^{-1}. The main thrust of the calculations is to test the hypothesis of Emanuel (1986) that the most important thermodynamic interaction in the hurricane, even in the developing stage, is between the vortex and the ocean, with cumulus convection rapidly redistributing heat acquired from the ocean upwards and outwards. Indeed, they show that a hurricane-like vortex can amplify in an atmosphere that is essentially neutral to cumulus convection and can attain an intensity in accordance with the predictions of Emanuel (1986). However, the hurricane "heat-engine" requires a finite-amplitude trigger to begin working; i.e. the surface wind speed must be high enough for surface heat fluxes to outweigh the cooling tendency of convectively-induced downdraughts as explained in section 7.

9. Discussion and Outlook

As discussed in section 2, the intensification of a hurricane requires the convergence of air above the surface boundary layer of an incipient vortex. This convergence can be produced by an unbalanced negative radial buoyancy gradient in the free troposphere located about the centre of rotation. Any mechanism that gives rise to such a gradient, such as latent heat release associated with cumulus convection can, in principle, lead to vortex intensification. It is not surprising, therefore, that all the parameterization schemes described here can be used to simulate tropical cyclones with some degree of realism, despite major differences in the construction of the schemes and despite the limitations of the particular scheme. Notwithstanding this fact, there have been important shifts in emphasis on the perceived role of convection in tropical cyclones. It had been long accepted that the primary energy source for the development and maintenance of a tropical cyclone is the latent heat released by condensation (see footnote 9) and the pioneering theoretical and observational studies of Malkus and Riehl (1960), Riehl and Malkus (1961) and Yanai (1961) suggested that this release takes place in tall convective clouds, which became known as "hot towers". However, the difference in scale between these cumulonimbus clouds and the primary circulation of the tropical cyclone was difficult to reconcile and led to the idea of CISK, proposed by Charney and Eliassen (1964) and discussed in section 8. The same idea was implicit in the linear theory for vortex intensification by Ooyama (1964), but Ooyama realized that a linear theory was inadequate to account for hurricane intensification. Nevertheless, Ooyama's theory was a forerunner of the nonlinear model published by Ooyama (1969). In the latter paper, Ooyama found that a prerequisite for hurricane intensification in the model is the inclusion of sea surface fluxes, primarily of moisture. In a series of

* Emanuel and Rotunno note that one disadvantage of this formulation is that the evaporation of falling rain is constrained to proceed at a rate sufficient to keep the air saturated which greatly exaggerates the evaporation.

more recent papers, Emanuel (1986), Emanuel and Rotunno (1987), Emanuel (1989, 1991, 1995) stressed the fundamental importance of these fluxes, especially their dependence on the surface wind speed, and relegated the role of the "hot towers" to distributing the latent heat acquired from the sea in the vertical and generating gross buoyancy in the inner part of the vortex[†]. Indeed, he argues forcefully that, *in the absence of sea-surface fluxes of sensible and latent heat*, the collective effects of deep convective clouds are to cool the troposphere rather than heat it, thereby producing negative buoyancy locally in the free troposphere (see e.g. Emanuel *et al.*, 1994, 1997). The arguments apply to convection that is forced by an externally-imposed vertical motion, such as that induced by an existing vortex boundary layer. It follows from the discussion in section 3 that (precipitating) convection always heats a column, but as shown in section 7, it is possible for this heating to be outweighed by the adiabatic cooling of forced ascent when the ascent is not driven solely by the convection. It is clear also that without surface fluxes, prolonged convection cannot be sustained, certainly in the tropics.

An important aspect of the explanation for hurricane intensification proposed by Ooyama (1982) is the progressive reduction of the local Rossby radius of deformation as the inertial stability of the vortex increases in the inner core region. This intrinsically nonlinear effect, he argues, continuously reduces the scale separation between the deep cumulus clouds and the balanced tangential circulation of the vortex in that region so that the individual clouds become more and more under the control of the balanced dynamics. Presumably, as this occurs, a representation of convective heating based on resolvable-scale mass-flux convergence becomes more reasonable for the inner core region. At the same time, the heating becomes more efficient in producing kinetic energy (see e.g. Schubert and Hack, 1982, Hack and Schubert, 1986).

Observational studies of tropical convection during the 1960's and 70's pointed to the importance of convective downdraughts[*], which have not been incorporated in most of the cumulus parameterization schemes used in hurricanes, but which have been shown to have an important influence on hurricane development in many of the models with explicit representations of moist processes. The important role of downdraughts is highlighted also in the simplified models with parameterized convection by Emanuel (1989, 1995). The strength of convective downdraughts, and hence the rate at which they decrease the subcloud layer entropy, is reduced as convection increases the relative humidity of the middle troposphere. At the same time, surface fluxes of sensible and latent heat, which raise the subcloud-layer entropy, increase with increasing surface wind speed. Emanuel argues that these opposing effects combine to produce a gestation period for hurricane genesis. There seems little doubt that the inclusion of downdraughts is a necessity in models designed to forecast hurricane intensity change.

In their calculations with the explicit release of latent heat, Emanuel and Rotunno (1987) show that hurricane growth does not require a significant amount of conditional instability, a finding supported by the calculations of DeMaria and Pickle (1988), Emanuel (1989, 1995) and Dengler and Reeder (1997), using various convective parameterization schemes.

What of the future? Do we really understand the interaction between convection and the hurricane circulation well enough to be sure that this is not a factor in our inability

[†] Emanuel (personal communication) holds that while it is not incorrect to argue that the primary energy source for a hurricane is the latent heat release in convective clouds, the argument misses the point. He likens the argument to saying that the primary source of energy for a moving motor car is the torque exerted by the axle. The axle, like convection, is an intermediary in the system, but does not determine the speed of the car, just as convection does not control the rate of vortex intensification.
[*] see e.g. Betts (1978).

to adequately forecast hurricane intensity change? It would be a bold scientist who would answer this question in the affirmative. The studies by Baik *et al.* (1990b), for example, indicate a sensitivity of the time of rapid storm deepening on the choice of the three convective adjustment parameters used in their model. This sensitivity would be significant obstacle to producing timely intensity change forecasts. Similar sensitivity studies using other convection schemes are called for, as are comparisons between different convection schemes used in the same model in the spirit of Baik *et al.* (1992). Even if we believe that we have understood the interaction between convection and the hurricane circulation well enough to formulate a realistic parameterization scheme, it is important to be able to specify the parameters in the scheme with sufficient precision to enable us to correctly forecast the evolution of a hurricane.

There appears to be an upward trend for hurricane researchers to exploit the ever-increasing computer power at their disposal to run mesoscale numerical models with high spatial resolution using existing parameterization schemes or dispensing with such schemes altogether. However, it is well known that the whole concept of "parameterizability" is called into question when the grid scale is reduced to that of only a few active clouds and we reiterate Arakawa's point that even if the clouds can be effectively resolved, one still requires a framework in which to interpret the results from such models. A critical review of cumulus parameterization for mesoscale models is contained in an article by Kuo *et al.* (1997). It is far from clear that the problems of representing convection will go away by increasing the model resolution, or that by doing so our understanding will be enhanced.

Our review has indicated that the simulation of a mature hurricane with some degree of realism is rather insensitive to the detailed treatment of convection, although the details of hurricane evolution do appear to be sensitive to such a treatment. There seems to be a need for the continued development of simpler hurricane models of the type presented by Emanuel (described in section 7b) because they help us to organize our thoughts and are a step towards a deeper understanding of the behaviour of more complex models. A nontrivial task is to remove the axisymmetric limitation of these models so that they can be applied to many of the intrinsically non-axisymmetric aspects of hurricane behaviour, such as hurricane motion and the influence of upper-level troughs on hurricane evolution. Recent attempts to remove this limitation have been made by Zehnder (personal communication) and in my own group in Munich. The results of our findings have been recently submitted for publication (Zhu *et al.*, 2000).

It is important that the modelling studies related to hurricane behaviour and the representation of convection therein continue to be guided by the results of observational studies, especially those arising from field experiments. In turn, the modelling studies have a significant role to play in the construction of testable hypotheses for the design of field experiments.

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10. Glossary of terms

An **aerological diagram** is a thermodynamic diagram in which the state of a sample of dry (moist) air may be characterized by one (two) points and changes in state by one (two) curves (see e.g. Smith, 1997a; Chapter 2).

Buoyancy is defined as the net upward force on an air parcel on account of its density ρ being less than some reference density, ρ_o, normally taken to be the density of the far environment at the same height as the air parcel: specifically, the buoyancy force per unit mass, $b = -g(\rho - \rho_o)/\rho$, where g is the acceleration due to gravity. It is important to remember that buoyancy is not the sole force acting in the vertical, nor is it unique as it depends on the choice of reference density. There is also a perturbation pressure gradient force $-(1/\rho)(\partial p'/\partial z)$, where z is the height and the perturbation pressure p' is defined relative to the reference pressure, $p_o(z)$, which satisfies $dp_o/dz = -g\rho_o$ (see Smith, 1980, p1228). It is important also to note that buoyancy involves a comparison of densities (or virtual/density temperature) at the same height with the assumption that the air parcel has the same pressure as its environment.

The Coriolis parameter f is twice the local vertical component of the Earth's angular velocity.

Divergence/Convergence (or more strictly **horizontal** divergence/convergence) refer to situations where fluid flows locally away from a point or towards it. Continuity of mass requires that there be a compensating flow away from or towards the point in a vertical plane.

An **Ekman layer** is a region of flow adjacent to a boundary normal to the axis of rotation in a rapidly-rotating fluid where viscous effects are important. An important characteristic of such a layer is its constant depth (assuming a constant fluid viscosity); another is the linear relationship between the normal flow out of or into the layer and the normal component of relative vorticity.

Gradient wind balance refers to a state of fluid motion in which the local horizontal pressure gradient acting on a fluid parcel is balanced by the centrifugal and Coriolis forces acting on the parcel.

The **level of neutral buoyancy** is the level at which the pseudo-adiabat through the lifting condensation level becomes cooler than the environment.

The **level of non-divergence** is the height at which the horizontal divergence is zero.

The **lifting condensation level** is the height at which a sample of moist unsaturated air becomes saturated as it ascends adiabatically (i.e. without heat exchange with its environment).

The **potential temperature** is the temperature that a parcel of dry air would have if brought adiabatically to a standard pressure of 1000 mb (or hPa).

A **pseudo-adiabat** is a line on an aerological diagram that characterizes the state of a saturated air parcel as it ascends adiabatically in the atmosphere on the assumption that all condensate falls out of the parcel as soon as it is formed. The latent heat of condensation is assumed to heat the air parcel in the process.

Potential radius is the radius to which an air parcel must be moved (conserving absolute angular momentum) in order to change its tangential velocity component to zero. It is proportional to the square root of the absolute angular momentum per unit mass about the storm centre.

Quasi-geostrophic refers to a fluid flow that is dominated by background rotation to the extent that the acceleration of fluid parcels is an order of magnitude smaller than the Coriolis acceleration.

The **relative vorticity** is equal to twice the local angular velocity of fluid parcels.

The **Rossby deformation radius** is a natural length scale for adjustment in a rotating stratified fluid. For an axisymmetric disturbance with depth scale H, it is defined as NH/I, where N is the buoyancy frequency associated with the stable stratification and I is the inertial stability parameter defined by $I^2 = (f + \zeta)(f + 2v/r)$, where ζ is the vertical component of relative vorticity, f is the Coriolis parameter and v is the tangential velocity component at radius r (see e.g. Shapiro and Willoughby, 1982, Frank, 1983). For weak disturbances, $I \approx f$.

The **vertical p-velocity**, ω, is defined as Dp/Dt, where D/Dt is the material derivative. For hydrostatic motion it is approximately equal to $-g\rho w$, where w is the vertical velocity. It follows that $-\omega/g$ has the dimensions of a vertical mass flux. The water vapour mixing ratio of an air sample is defined as the mass of water vapour in an air parcel expressed per unit mass of dry air.

The **Brunt-Väisälä frequency**, or buoyancy frequency, is the frequency of oscillation of a hypothetical air parcel that is displaced vertically in a stably-stratified atmosphere (i.e. one in which the potential temperature, or more strictly the virtual potential temperature, increases with height).

The **troposphere** is region of the atmosphere in which most weather systems are confined. Its depth varies from a maximum of around 16 km in the deep tropics to about 8 km in the polar regions.