QUARTERLY JOURNAL OF THE ROYAL METEOROLOGICAL SOCIETY
Q. J. R. Meteorol. Soc. **00**: 1–6 (2008)
Published online in Wiley InterScience
(www.interscience.wiley.com) DOI: 10.1002/qj.000

The fundamental role of buoyancy in tropical-cyclone intensification

Roger K. Smith [a] *, and Michael T. Montgomery[b]
[a] *Meteorological Institute, University of Munich, Munich, Germany*
[b] *Dept. of Meteorology, Naval Postgraduate School, Monterey, CA & NOAA's Hurricane Research Division, Miami, FL, USA.*

Abstract:
In this paper we address a central and hitherto unanswered question in tropical meteorology on whether axisymmetric hurricane models correctly represent the essential mechanism of vortex intensification as compared to a three-dimensional model that explicitly simulates local convective structures rather than convective rings. We show that the azimuthally-averaged air density in an idealized, three-dimensional, nonhydrostatic, numerical model for a tropical cyclone is generally *greater* than that which is in thermal wind balance with the azimuthally-averaged tangential wind speed. Density differences between these states are characterized by virtual temperature differences on the order of a few degrees Celsius. We hypothesise that the buoyancy of the vortical hot towers, that have been shown to be the basic coherent structures in the cyclone intensification process, drives a wind field that generally leads the mass field. It follows that there is a fundamental difference between tropical-cyclone development in a three-dimensional non-hydrostatic model and an axisymmetric one, since the intensification of an axisymmetric vortex requires the density to be less than that of the balanced state to drive the necessary in-up-and-out secondary circulation.

KEY WORDS Hurricane; tropical cyclone; typhoon; buoyancy, secondary circulation

Received December 19, 2008; Revised ; Accepted

1 Introduction

In a recent paper, Nguyen *et al.* (2008, henceforth M1) examined tropical-cyclone intensification and predictability in the context of an idealized three-dimensional numerical model on an f-plane. The model has relatively basic physics including a bulk-aerodynamic formulation of the boundary layer and a simple explicit moisture scheme to represent deep convection. In the prototype amplification problem starting with a weak axisymmetric tropical-storm-strength vortex, they showed that the emergent flow becomes highly asymmetric and is dominated by deep convective vortex structures, even though the problem as posed is essentially axisymmetric. Following Hendricks *et al.* (2004) and Montgomery *et al.* (2006) we refer to these structures as "vortical hot towers" (VHTs). These towers have local buoyancy relative to the azimuthally-averaged density field (Montgomery *et al.* 2008) and collectively drive a secondary circulation with convergence in the lower troposphere that is a key feature of the spinup of the mean vortex (Montgomery *et al.* 2006, Smith *et al.* 2008). In addition, they locally enhance the existing rotation of the initial vortex. The foregoing studies indicate that the VHTs are the basic coherent structures in the vortex intensification process. A similar process of evolution occurs even in a minimal tropical-cyclone model (Shin and Smith 2008).

Smith *et al.* (2008) examined the intensification of the azimuthal-mean tangential flow in the two main calculations in M1 in terms of axisymmetric principles. They identified two mechanisms for the spin up of the mean tangential circulation as a result of the inflow produced by the VHTs. The first involves the convergence above the boundary layer in the presence of absolute angular momentum conservation and is a mechanism to spin up the outer circulation. The second involves convergence within the boundary layer with some loss of absolute angular momentum due to friction and is a mechanism to spin up the inner core, typically near or inside the radius of maximum tangential wind speed above the boundary layer.

These ideas differ in important ways from those involved in the currently accepted paradigm for vortex intensification. This paradigm is associated with the so-called WISHE-mechanism[†] as described, for example, by Emanuel *et al.* (1994), Craig and Gray (1996) and in recent reviews by Emanuel (2003, 2004). A central tenet of the WISHE-mechanism holds that cloud buoyancy is not necessary to explain intensification in an axisymmetric framework other than to offset the dissipation of energy. The question is: can one reconcile this paradigm with the findings described above?

*Correspondence to: Roger K. Smith, Meteorological Institute, University of Munich, Theresienstr. 37, 80333 Munich, Germany. E-mail: roger.smith@lmu.de

[†] The term WISHE, which stands for wind-induced surface heat exchange, was first coined by Yano and Emanuel (1991) to denote the source of fluctuations in subcloud-layer entropy arising from fluctuations in surface wind speed. The WISHE-mechanism is articulated in a revised form by Montgomery *et al.* (2008).

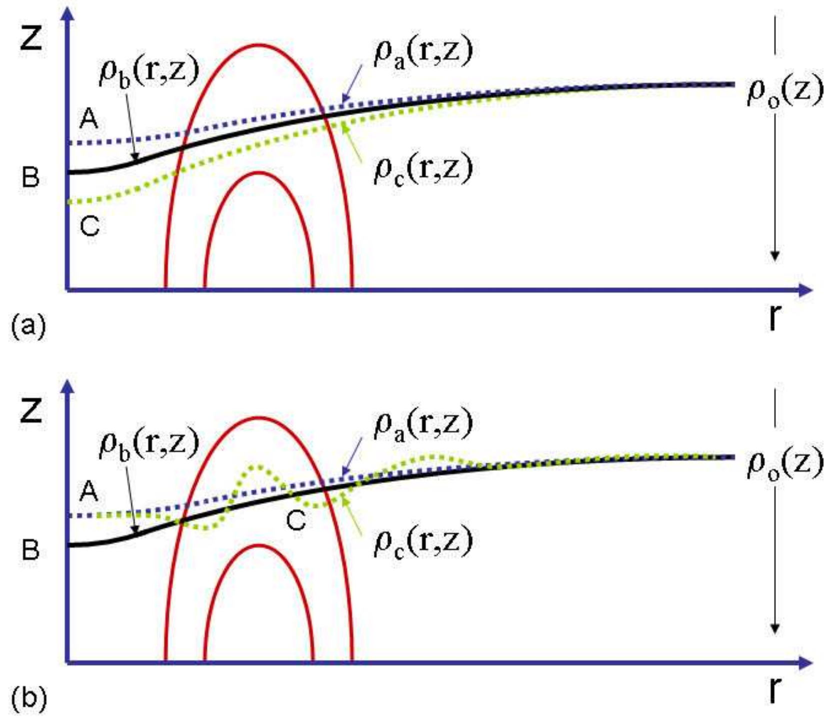

Figure 1. Schematic diagrams illustrating system buoyancy, local buoyancy and unbalanced buoyancy. The red curves in both panels show isotachs of azimuthal-averaged tangential wind speed with speed decreasing with height and the solid curves marked B show an isopleth of constant density, $\rho_b(r, z)$, that is in thermal wind balance with the tangential wind field. The dashed lines represent *azimuthally-averaged density isopleths*, $\rho_a(r, z)$ and $\rho_c(r, z)$, that are in each case different at inner radii than the balanced density. The isopleths marked A in each panel are heavier than the balanced density while the curve C in panel (a) is lighter. However all these densities are less than the density at large radius, $\rho_o(z)$, and therefore posess positive "system buoyancy". Curve A has negative "unbalanced buoyancy" because $\rho_b(r, z) < \rho_a(r, z)$ and curve C has positive "unbalanced buoyancy" because $\rho_c(r, z) < \rho_b(r, z)$. Curve C in panel (b) represents a density isopleth at a given azimuth which deviates from the azimuthally-averaged density. Where it dips below curve A it has positive local buoyancy [$\rho_c(r, z) < \rho_a(r, z)$] and where it lies above curve A [$\rho_c(r, z) > \rho_a(r, z)$] it has negative local buoyancy. Where it dips below curve B [$\rho_c(r, z) < \rho_b(r, z)$], it has positive "unbalanced buoyancy" and where it lies above curve B [$\rho_c(r, z) > \rho_b(r, z)$] it has negative "unbalanced buoyancy".

To explore the foregoing question it is necessary to review the concept of buoyancy as it relates to a rapidly-rotating fluid. The concept is discussed by Smith *et al.* (2005). First it should be remembered that the buoyancy force has both a radial and vertical component, but for tropical cyclones, the radial component is small compared to the vertical component. Secondly, the vertical component of the buoyancy, b, is not a unique force, but depends on the choice of reference density, ρ_{ref}, used to define it. Thus $b = -g(\rho - \rho_{ref})/\rho$, where ρ is the density of an air parcel and g is the acceleration due to gravity. Smith *et al.* distinguish between *system buoyancy* and *local buoyancy*. System buoyancy is defined relative to an *ambient* reference density, $\rho_o(z)$, that depends only on the height z above the surface (see Fig. 1). In contrast, *local buoyancy* is defined relative to a reference density that varies with radius and height (and possibly also in time), say $\rho_a(r, z)$ (Curves A in Fig. 1). Smith *et al.* took this reference density to be in thermal-wind balance with the azimuthal-mean tangential wind speed, say $\rho_b(r, z)$. Then a balanced vortex has only system buoyancy and no local buoyancy. Here we define the local buoyancy relative to $\rho_a(r, z)$ and reserve the term *unbalanced buoyancy* for the buoyance relative to the reference density $\rho_b(r, z)$ (Curve B in Fig. 1a). This distinction allows us to consider the situation when $\rho_a(r, z)$ is *not* in thermal wind balance, itself. Thus, when $\rho_b(r, z) < \rho_a(r, z)$, the azimuthally-averaged density field has negative unbalanced buoyancy (Curves A in Fig. 1) and when $\rho_a(r, z) < \rho_b(r, z)$ it has positive unbalanced buoyancy (Curve C in Fig. 1a). Panel (b) of Fig. 1 illustrates the case where the density isopleth at some azimuth (Curve C) has both local buoyancy, represented by the deviation of this curve from the azimuthally-averaged density (Curve A), as well as unbalanced buoyancy, represented by the deviation of this curve from the balanced density (Curve B). Montgomery *et al.* (2008) showed that the VHTs in the calculations in M1 possess significant local buoyancy relative to the azimuthally-averaged density field. Then two questions naturally arise:

- Is this buoyancy dynamically important in the intensification process, or does it play a passive role as suggested by Emanuel (1986, 1989, 1994, 1995, 1999, 2003, 2004)?
- Does the azimuthally-averaged density field differ from that of the density field in thermal wind balance with the azimuthally-averaged tangential wind field, i.e. does it have unbalanced buoyancy?

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An answer to the first question was given by Montgomery *et al.* (2008). Specifically, it was shown that non-trivial and modest local buoyancy contributed to the generation of accelerating updrafts and significant local vortex tube stretching - enhancing the cyclonic vorticity of the parent initial vortex by more than an order of magnitude in the early period of intensification. In particular, the spin up of several of these VHTs has a collective impact on the low-level spin up of the system-scale azimuthal-mean vortex as documented in Smith *et al.* (2008). Thus, the VHT dynamics provide an asymmetric tropical-cyclone intensification pathway, the 'VHT-pathway', that is distinct from the widely accepted 'WISHE-pathway'.

At least in a non-hydrostatic axisymmetric vortex model, one might argue on the basis of elementary fluid dynamical principles that vortex amplification can proceed only if the unbalanced buoyancy is slightly positive to enable it to drive a secondary circulation with low-level convergence. In this case, the inner-core virtual temperature must be slightly larger than that required by thermal-wind balance. If the unbalanced buoyancy were negative, it would drive a secondary circulation with low-level outflow. Of course, in a model that assumes thermal wind balance such as those of Emanuel, the latent heat release implicit in cloud updrafts is assumed to go instantaneously into elevating the system buoyancy and there can be no local buoyancy. The question remains, however, is this assumption supported by the azimuthally-averaged fields in the three-dimensional calculations in M1? This brings us back to the second question itemized above, i.e. is the azimuthally-averaged density field in the three-dimensional calculation the same or slightly less than the corresponding density field that is in thermal wind balance with the azimuthally-averaged tangential wind field? The present paper examines this important question. As it turns out the answer is no: it is significantly larger. In other words, the azimuthally-averaged density field has negative unbalanced buoyancy. This finding raises fundamental questions concerning the applicability of axisymmetric models for studying or predicting tropical-cyclone intensification.

The paper is organized as follows. First we review briefly in Section 2 the Nguyen *et al.* model and outline the method for calculating the balanced temperature field of the vortex. The results, their interpretation and their implications are presented in Section 3 and the conclusions in Section 4.

2 Model setup

The numerical experiments are carried out using a modified version of the Pennsylvania State University-National Center for Atmospheric Research fifth-generation Mesoscale Model (MM5; version 3.6) (Dudhia 1993; Grell *et al.* 1995), which is suitable for the study of idealized problems. The model is configured with three domains: one with a coarse mesh of 45-km resolution and two, two-way nested domains of 15 and 5 km resolution, respectively. The domains are square and are 5400 km, 1800 km, 600 km on each side. There are 24 σ-levels in the vertical, 7 of which are below 850 mb. The calculations are performed on an f-plane centred at 20^0N. To keep the experiments as simple as possible, the main physics options chosen are the bulk-aerodynamic boundary-layer scheme and either the simplest explicit moisture scheme that mimics pseudo-adiabatic ascent, or a warm-rain scheme. These schemes are applied in all domains. The sea surface temperature is set to a constant 27^oC. For simplicity, radiative cooling is neglected. The initial vortex is axisymmetric with a maximum tangential wind speed of 15 m s^{-1} at the surface at a radius of 135 km. The strength of the tangential wind decreases sinusoidally with height, vanishing at the top model level (50 mb). The temperature field is initialized to be in gradient wind balance with the wind field using the method described by Smith (2006). The far-field temperature and humidity are based on Jordan's Caribbean sounding (Jordan 1958).

The vortex centre is defined as the centroid of relative vorticity at 900 mb over a circular region of 200 km radius from a "first-guess" centre, which is determined by the minimum of the total wind speed at 900 mb. Azimuthal averages are computed relative to this centre. Two of Nguyen *et al.* 's calculations are examined: Experiment 1, the control calculation which assumes a pseudo-adiabatic representation of latent heat release and Experiment 2, the corresponding calculation with a representation of warm rain processes.

To answer the questions posed in Section 1 requires the balanced temperature field of the vortex to be determined. We do this using the method described by Smith (2006). This method assumes the tangential wind field is given as a function of radius and height at a particular time and it integrates a general form of the axisymmetric thermal wind equation from a selected grid point to some large radius where the vertical profile of pressure, density (or virtual temperature) are known. The thermal wind equation may be written in the form:

$$\frac{\partial}{\partial r}\ln\rho + \frac{C}{g}\frac{\partial}{\partial z}\ln\rho = -\frac{1}{g}\frac{\partial C}{\partial z}, \qquad (1)$$

where r is the radius, z is the height, ρ is the density, $C = v^2/r + fv$ is the sum of the centrifugal and Coriolis forces per unit mass, v is the azimuthally-averaged tangential wind speed, f is the Coriolis parameter and g is the acceleration due to gravity. This is a linear, first-order, partial differential equation for $\ln\rho$ and may be solved by the method of characteristics. The characteristics satisfy the ordinary differential equation

$$\frac{dz}{dr} = \frac{C}{g}, \qquad (2)$$

whereupon $\ln\rho$ satisfies the ordinary differential equation

$$\frac{d}{dr}\ln\rho = -\frac{1}{g}\frac{\partial C}{\partial z}. \qquad (3)$$

It turns out that the characteristics are simply the isobars and the integration involves the simultaneous solution of

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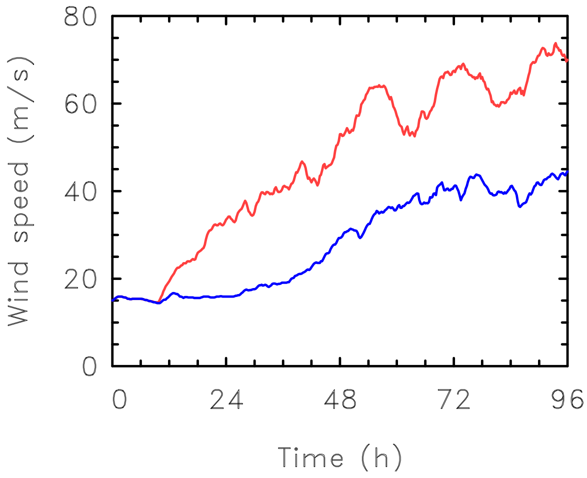

Figure 2. Time-series of azimuthal-mean maximum tangential wind component at a height of 500 m in Experiment 1 (red curve) and Experiment 2 (blue curve).

(2) for the height of the isobars and (3) for $\ln\rho$ along the isobars as functions of r. Application of the method requires an interpolation of the azimuthally-averaged tangential wind so that it may be calculated at an arbitrary radius and height at any given time. Here we use a biratational spline approximation to accomplish the interpolation.

3 Results

The evolution of the vortex intensity in the two MM5 calculations is characterized by the maximum azimuthally-averaged tangential wind speed at 500 m shown in Figure 2. After a short gestation period lasting about 10 h, the two vortices rapidly intensify. The intensification rate is markedly slower in Experiment 2 because the warm-rain process includes the effects of precipitation-driven downdrafts, which lower the equivalent potential temperature in the boundary layer.

Figure 3 shows radius-height cross-sections of the azimuthally-averaged tangential wind speed and virtual potential temperature[‡] at 24 h intervals to 96 h in Experiment 1. It shows also the differences in the virtual potential temperature between the full solution and that calculated assuming thermal wind balance as described in section 2. At all times the vortex is warm-cored in terms of virtual potential temperature contrasts at a fixed height as one expects for a tangential wind field that decreases with height: in other words the system buoyancy is positive (Smith *et al.* 2008). The same is true also of the balanced solution, except at low levels where, as a result of friction, the tangential wind increases sharply with height.

[‡]The virtual potential temperature, θ_v is defined by $\theta_v = T_v(p_o/p)^{R_d/c_{pd}}$, where $T_v = p/(\rho R_d)$ is the virtual temperature, p is the pressure, $p_o = 1000$ mb, ρ is the density, R_d is the specific gas constant for dry air, and c_{pd} is the specific heat of dry air at constant pressure.

However, it is clear from the figure that the azimuthally-averaged virtual potential temperatures in the MM5 calculation are a few degrees less than those that would be in thermal wind balance with the azimuthally-averaged tangential wind field, i.e. the mean vortex has negative unbalanced buoyancy. The same is true in Experiment 2 as exemplified by Figure 4, which shows similar radius-height cross-sections for this experiment to those in Figure 3.

The foregoing results point to a fundamental difference between the axisymmetric paradigm for tropical-cyclone intensification (the 'WISHE pathway') and the asymmetric paradigm, which involves the VHTS (the 'VHT pathway'). The behaviour in the three-dimensional calculations may be explained by recalling the generalized Rossby adjustment problem wherein a stably stratified circular vortex initially in gradient and hydrostatic balance is subject to an unbalanced perturbation. For horizontal perturbation scales that are much less than the local Rossby radius of deformation, the mass field will tend to adjust to the wind field. Here, the VHTs represent the unbalanced perturbations to the mean vortex and the local Rossby radius of deformation for the mean vortex is on the order of 150 km (e.g., Shapiro and Montgomery 1993). Now, remembering that the adjustment time for the mean vortex is typically on the order of several local inertial periods $(2\pi/\sqrt(f+2\Omega)(f+d(rv)/rdr) \approx \pi/\bar{\Omega} \approx 2$ h, where $\bar{\Omega}$ is the azimuthally averaged angular velocity of the system vortex), we should not be surprised to find $\bar{\theta}$ systematically lagging behind $\bar{\theta}_b$.

4 Conclusions

We have shown that there is a fundamental difference between the axisymmetric paradigm for tropical-cyclone intensification and the asymmetric paradigm, in which the vortical hot towers are key elements of the evolution. Indeed, these are the only coherent structures that have positive local buoyancy. The azimuthally-averaged field has negative unbalanced buoyancy, which we defined here as buoyancy relative to the reference density of the balanced state. Therefore it cannot support an intensifying circulation by itself. As a result, we conclude that this circulation must be driven by the local buoyancy in the VHTs. We believe that this important result questions the integrity of axisymmetric models for representing or predicting tropical-cyclone intensity change. To assess the quantitative importance of these findings requires a more systematic comparison between two and three dimensional models. We aim to report on such a study in due course.

5 Acknowledgements

We are grateful to our colleague, Sang Nguyen, who kindly provided the MM5 data for this study. This basic research on tropical cyclones was supported in part by the Office of Naval Research Grant N001408WR20129, by National Science Foundation Grants ATM-0649944,

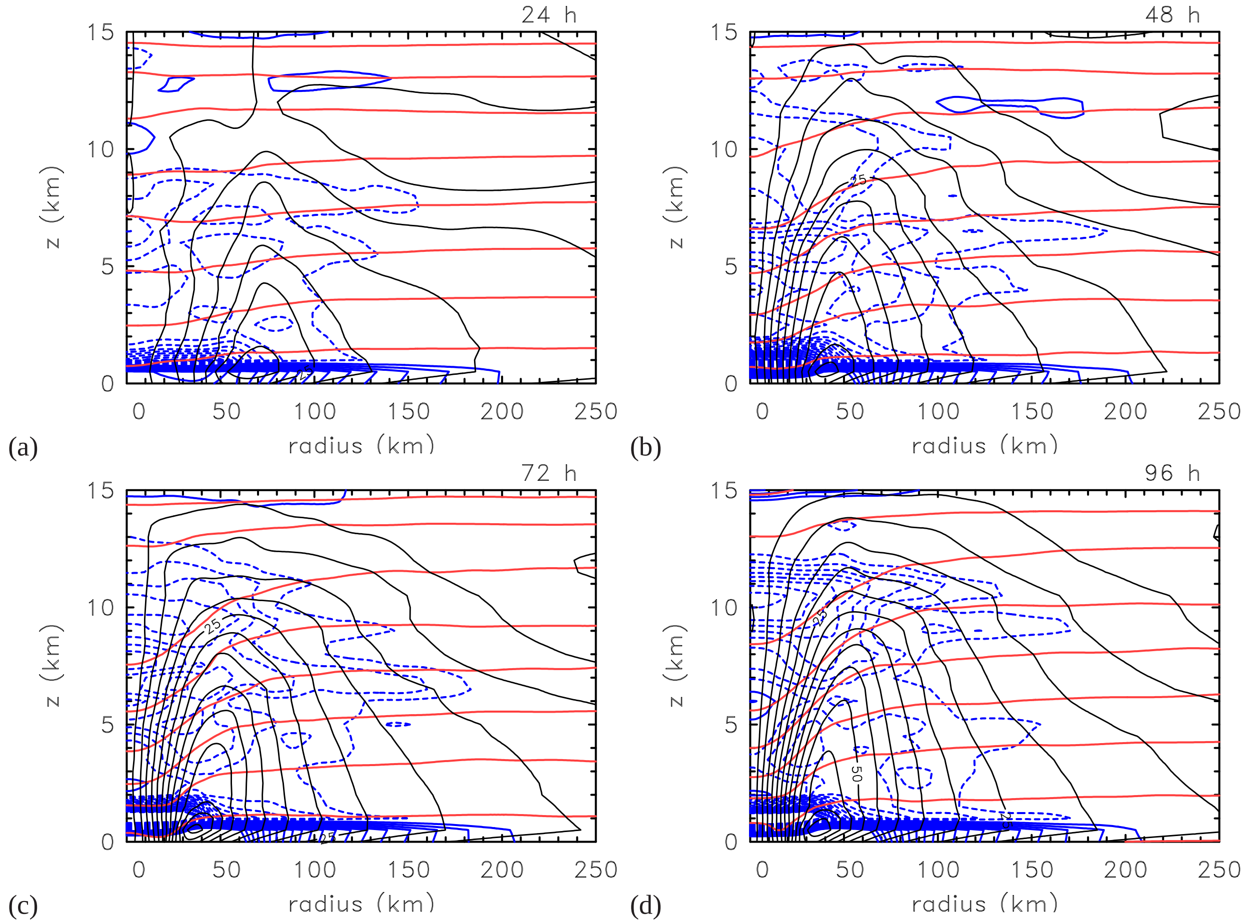

Figure 3. Radius-height cross-sections of the azimuthally-averaged tangential wind speed isotachs (black lines, contour interval 5 m s^{-1}) and potential temperature isopleths (red lines, contour interval 10 K) at (a) 24 h, (b) 48 h, (c) 72 h, (d) 96 h in Experiment 1 together with isopleths of the difference in potential temperature between the full solution and the balanced solution (blue contours, contour interval 1 K, dashed curves indicate negative values, the zero contour is not plotted).

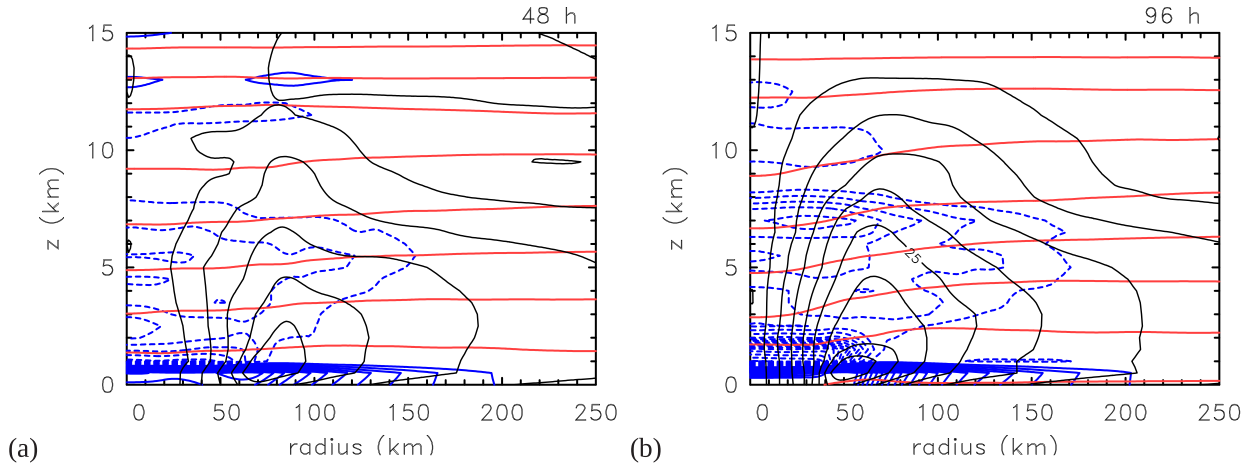

Figure 4. Legend as for Figure 3, except for Experiment 2 at (a) 48 h, and (b) 96 h.

ATM-0649946, ATM-0715426, NOAA grant 2007-AOML-E8R2H and by a grant from the German Research Council (DFG). The work was initiated during a visit by both authors to the U.S. Hurricane Research Division of NOAA/AOML. We thank HRD for their generous hospitality and for creating a stimulating environment for pursuing hurricane research.

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