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Paradigms for tropical-cyclone intensification

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Abstract:
We review four paradigms of tropical-cyclone intensification that have emerged over the past five decades, discussing the relationship between them and highlighting their positive aspects and limitations. A major focus is on a new paradigm articulated in a series of recent papers by ourselves and colleagues. Unlike the three previous paradigms, all of which assumed axial symmetry, the new one recognizes the importance of rotating deep convection, which possesses local buoyancy relative to the azimuthally-averaged virtual temperature of the warm-cored vortex. This convection comes under increasing rotational control as the vortex intensifies. It exhibits also a degree of randomness that has implications for the predictability of local asymmetric features of the developing vortex. While surface moisture fluxes are required for intensification, the postulated 'evaporation-wind' feedback process that forms the basis of an earlier paradigm is not. The details of the intensification process as well as the structure of the mature vortex are sensitive to the boundary-layer parameterization used in the model.
The spin up of the inner-core winds in the new paradigm occurs within the boundary layer and is associated with the convergence of absolute angular momentum in this layer, where absolute angular momentum is not materially conserved. This spin up is coupled with that of the winds above the boundary layer through boundary-layer dynamics. Balanced and unbalanced contributions to the intensification process are discussed.
An application of the new paradigm is given to help describe and understand a simulated intensification process in a realistic numerical weather prediction model. Copyright © 2011 Royal Meteorological Society

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1 Introduction

The problem of tropical-cyclone intensification continues to challenge both weather forecasters and researchers. Unlike the case of vortices in homogeneous fluids, the tropical-cyclone problem as a whole, and the intensification problem in particular, is difficult because of its convective nature and the interaction of moist convection with the larger scale circulation (e.g., Marks and Shay 1998).

There have been considerable advances in computer technology over the past several decades making it possible to simulate tropical cyclones with high resolution numerical models, with horizontal grid spacing as small as approximately 1 km. Nevertheless, important questions remain about their fluid dynamics and thermodynamics in addition to the obvious practical challenges to successfully forecast the intensity changes of these deadly storms (e.g., Davis *et al.* 2008). Further evaluation of the intensity forecasts produced by high-resolution weather prediction models is certainly necessary in order to develop an appreciation of the strengths and weaknesses of the models in comparison to observations. It is unlikely, however, that significant advances will be made on the intensity-change problem without the concurrent development of a suitable theoretical framework that incorporates the dominant fluid dynamical and thermodynamical mechanisms. Such a framework is necessary to permit a more complete diagnosis of the behaviour of the model forecasts and to identify the essential processes that require further understanding and improved representation. It may be that a paradigm shift is required to break the intensity forecast deadlock.

Although one of the major impediments to intensity forecasts is believed to be associated with the interaction of a tropical cyclone with the vertical shear of the ambient wind (e.g., Riemer *et al.* 2010, Tang and Emanuel 2010 and refs.), it is imperative to have a firm understanding of the physical processes of intensity change for hypothetical storms in environments with no background flow. Such storms are the focus of this paper.

Over the years, several theories have been proposed to explain the intensification of tropical cyclones, each enjoying considerable popularity during their time. The three most established theories are based on axisymmetric considerations and include: Conditional Instability of the Second Kind (CISK); Ooyama's cooperative intensification theory; and Emanuel's air-sea interaction theory (or WISHE[1]). A fourth theory, based on our recent work with colleagues, highlights the intrinsically non-axisymmetric

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[1]The term WISHE, which stands for wind-induced surface heat exchange, was first coined by Yano and Emanuel (1991) to denote the source of fluctuations in subcloud-layer entropy arising from fluctuations in surface wind speed.

nature of the spin-up process and suggests a modified view of the axisymmetric aspects thereof.

In the light of current efforts in many parts of the world to improve forecasts of hurricane intensity, especially cases of rapid intensification near coastal communities and marine assets, we believe it is timely to review the foregoing theories, where possible emphasizing their common features as well as exposing their strengths and weaknesses[2]. An additional motivation for this review is our desire to interpret recent data collected as part of the Tropical Cyclone Structure 2008 (TCS08) field experiment (Elsberry and Harr 2008). While our main focus centres on problems related to the short-range evolution of storms, a more complete understanding of the mechanisms of tropical-cyclone spin-up would seem to be useful also for an assessment of climate-change issues connected with tropical cyclones.

To fix ideas, much of our discussion is focussed on *understanding* various aspects of the prototype problem for tropical-cyclone intensification, which examines the evolution of a prescribed, initially cloud-free, axisymmetric, baroclinic vortex in a quiescent environment over a warm ocean on an f-plane. The effects of an ambient flow, including those of ambient vertical shear, are not considered. It is presumed that the initial vortex has become established and has maximum swirling winds near the ocean surface as a result of some genesis process. This problem has been studied by a large number of researchers. The aim of the paper is not to provide an exhaustive review of all the findings from these studies, but rather to review the various paradigms for tropical-cyclone spin up and to provide an integrated view of the key dynamical and thermodynamical processes involved in the spin up process. The paper is aimed at young scientists who are just entering the field as well as those more established researchers who would like an update on the subject. In particular, our essay presents a more comprehensive view of the intensification process to that contained in the recent World Meteorological Organization sponsored review by Kepert (2010), which until now was the most recent word on this subject.

The paper is structured as follows. First, in section 2 we review some basic dynamical concepts that are required for the subsequent discussion of the various paradigms for tropical-cyclone intensification. These concepts include: the primary force balances in vortices; the thermal wind equation; the meridional (or overturning) circulation as described by balance dynamics; the role of convergence of absolute angular momentum in the spin-up process; and the role of latent heat release and the boundary layer in generating low-level convergence. We decided to include this section for those who are relatively new to the field and it is there that we introduce much of the notation used. Some of our readers will be familiar with much of this material and, since the notation is mostly standard, they may wish to jump straight to section 3. In this and in sections 4 and 5 we discuss the three most established paradigms for intensification. Then, in section 6 we articulate a new paradigm for intensification in which the key dynamical processes are intrinsically non-axisymmetric. Axisymmetric aspects of this paradigm are examined in section 7. Unbalanced aspects of axisymmetric spin up are considered in section 7.1 and balanced aspects are considered in section 7.2. Section 8 examines properties of the asymmetric paradigm. There, we describe tests of the dependence of the spin up process on the postulated WISHE mechanism, on the boundary layer parameterization and on the surface drag coefficient. In section 9, we describe an application of this paradigm to understanding vacillation cycles during the spin up of a model storm. The conclusions are given in section 10 and our view of the road ahead is given in section 11.

[2]Although other reviews have appeared during the past seven years (e.g., Emanuel 2003, Wu and Wang 2004, Houze 2010, Kepert 2010), the review and results presented here are complementary by focusing on the dynamical and thermodynamical aspects of the intensification process.

2 Fundamentals of balanced vortex dynamics and spin up

We commence by reviewing some basic dynamical aspects of tropical cyclone vortices and key processes germane to their spin up. For simplicity, we focus our attention first on *axisymmetric balance* dynamics and discuss then the departures from balance that arise in the frictional boundary layer.

2.1 The primary force balances

Except for small-scale motions, including gravity waves and convection, a scale analysis of the equations of motion for a rotating stratified fluid (Willoughby 1979) shows that the macro motions within a tropical cyclone are in a close state of hydrostatic equilibrium in which the upward-directed vertical pressure gradient force per unit mass is balanced by the gravitational force acting downwards

$$\frac{1}{\rho}\frac{\partial p}{\partial z} = -g, \tag{1}$$

where p is the (total) pressure, ρ is the moist air density, z is the height and g is the acceleration due to gravity. The scale analysis shows also that if the azimuthal mean tangential wind component squared is much larger than the corresponding radial component squared and if frictional forces can be neglected, a state of gradient wind balance prevails in the radial direction wherein the radial pressure gradient is balanced by the sum of the (apparent) Coriolis and centrifugal forces

$$\frac{1}{\rho}\frac{\partial p}{\partial r} = \frac{v^2}{r} + fv, \tag{2}$$

where r is radius from the axis of swirling motion, v is the tangential velocity component, and f is the Coriolis parameter ($2\Omega\sin\phi$, where Ω is the earth's rotation rate and ϕ is the latitude).

Throughout this paper we take f to be constant on the assumption that the latitudinal extent of air motions

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within the vortex circulation is sufficiently small to render corresponding variations of f negligible; this is the so-called f-plane approximation[3].

The validity of gradient wind balance in the lower to middle troposphere in tropical cyclones is supported by aircraft measurements (Willoughby 1990, Bell and Montgomery 2008), but there is some ambiguity from numerical models. In a high-resolution (6 km horizontal grid) simulation of Hurricane Andrew (1992), Zhang *et al.* (2001) showed that the azimuthally-averaged tangential winds above the boundary layer satisfy gradient wind balance to within a relative error of 10%, the main regions of imbalance being in the eyewall and, of course, in the boundary layer (see also, Persing and Montgomery 2003, Appendix A). A similar finding was reported by Smith *et al.* (2009) and Bryan and Rotunno (2009). However, in a simulation of Hurricane Opal (1995) using the Geophysical Fluid Dynamics Laboratory hurricane prediction model, Möller and Shapiro (2002) found unbalanced flow extending far outside the eyewall region in the upper tropospheric outflow layer.

In the next few sections we will focus primarily on the *macro* (non-turbulent) motions within a tropical cyclone vortex and parameterize the convective and subgrid scale motions in terms of macro (or coarse-grained) variables. Then, in terms of the macro variables, the tropical cyclone consists of a horizontal quasi-axisymmetric circulation on which is superposed a thermally-direct transverse (overturning) circulation. These are sometimes referred to as the "primary" and "secondary" circulations, respectively. The former refers to the tangential or swirling flow rotating about the central axis, and the latter to the transverse or "in-up-and-out circulation" (low and middle level inflow, upper-level outflow, respectively). When these two components are combined, a picture emerges in which air parcels spiral inwards, upwards and outwards. The combined spiralling circulation is called energetically "direct" because the rising branch of the secondary circulation near the centre is warmer than the subsiding branch, which occurs at large radial distances (radii of a few hundred kilometres). When warm air rises (or cold air sinks), potential energy is released (Holton 2004, p339). As a tropical cyclone becomes intense, a central cloud-free "eye" forms, or at least a region free of deep cloud. The eye is a region of subsidence and the circulation in it is "indirect", i.e. warm air is sinking (Smith 1980, Shapiro and Willoughby 1982, Schubert *et al.* 2007).

2.2 Thermal wind

Eliminating the pressure in Equations (1) and (2) by cross-differentiation gives the so-called "thermal wind equation":

$$g\frac{\partial \ln \rho}{\partial r} + C\frac{\partial \ln \rho}{\partial z} = -\frac{\partial C}{\partial z}, \tag{3}$$

which relates the radial and vertical density gradients to the vertical derivative of the tangential wind component. Here

$$C = \frac{v^2}{r} + fv \tag{4}$$

denotes the sum of the centrifugal and Coriolis forces per unit mass (see Smith *et al.* 2005). Equation (3) is a linear first-order partial differential equation for $\ln \rho$. The characteristics of the partial differential equation satisfy

$$\frac{dz}{dr} = \frac{C}{g}. \tag{5}$$

and the density variation along a characteristic is governed by the equation

$$\frac{d}{dr}\ln \rho = -\frac{1}{g}\frac{\partial C}{\partial z}. \tag{6}$$

The characteristics coincide with the isobaric surfaces because a small displacement (dr, dz) along an isobaric surface satisfies $(\partial p/\partial r)dr + (\partial p/\partial z)dz = 0$. Then, using the equations for hydrostatic balance $(\partial p/\partial z = -g\rho)$ and gradient wind balance $(\partial p/\partial r = C\rho)$ gives the equation for the characteristics.

The vector pressure gradient per unit mass, $(1/\rho)(\partial p/\partial r, 0, \partial p/\partial z)$ equals $(C, 0, -g)$, which naturally defines the "generalized gravitational vector", $\mathbf{g_e}$, i.e., the isobars are normal to this vector. Given the environmental vertical density profile, $\rho_a(z)$, Equations (5) and (6) can be integrated inwards along the isobars to obtain the balanced axisymmetric density and pressure distributions (Smith 2006). In particular, Equation (5) gives the height of the isobaric surface that has the value $p_a(z)$, say, at radius R.

2.3 The overturning circulation

Where the thermal wind equation is satisfied, it imposes a strong constraint on the evolution of a vortex that is being forced by processes such as diabatic heating or friction. Acting alone, these processes would drive the flow away from thermal wind balance, which the scale-analysis dictates. In order for the vortex to remain in balance, a transverse, or secondary circulation is required to oppose the effects of forcing. The streamfunction of this overturning circulation can be obtained by solving a diagnostic equation, commonly referred to as the Sawyer-Eliassen balance equation, which we derive below. This equation provides a basis for the development of a theory for the evolution of a rapidly-rotating vortex that is undergoing slow[4] forcing by heat and (azimuthal) momentum sources (see section 2.5). It is convenient to define $\chi = 1/\theta$, where θ is the potential temperature. Then, the thermal wind equation (3) becomes

$$g\frac{\partial \chi}{\partial r} + \frac{\partial (\chi C)}{\partial z} = 0\,. \tag{7}$$

[3] The f-plane approximation is defencible when studying the basic physics of tropical-cyclone intensification (Nguyen *et al.* 2008, section 3.2.1), but not, of course, for tropical-cyclone motion (e.g. Chan and Williams 1987, Fiorino and Elsberry 1989).

[4] Slow enough so as not to excite large-amplitude unbalanced inertia-gravity wave motions.

The tangential momentum and thermodynamic equations take the forms

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r} + w\frac{\partial v}{\partial z} + \frac{uv}{r} + fu = F_\lambda\,, \tag{8}$$

and

$$\frac{\partial \chi}{\partial t} + u\frac{\partial \chi}{\partial r} + w\frac{\partial \chi}{\partial z} = -\chi^2\dot{Q}\,, \tag{9}$$

respectively, where u and w are the radial and vertical velocity components, t is the time, F_λ is the tangential component of the azimuthally-averaged force per unit mass (including surface friction), and $\dot{Q} = d\theta/dt$ is the diabatic heating rate for the azimuthally-averaged potential temperature. Neglecting the time derivative of density, which eliminates sound waves from the equations, the continuity equation takes the form

$$\frac{\partial}{\partial r}(\rho r u) + \frac{\partial}{\partial z}(\rho r w) = 0\,. \tag{10}$$

This equation implies the existence of a scalar streamfunction for the overturning circulation, ψ, satisfying

$$u = -\frac{1}{r\rho}\frac{\partial \psi}{\partial z}, \qquad w = \frac{1}{r\rho}\frac{\partial \psi}{\partial r}\,. \tag{11}$$

The Sawyer-Eliassen equation for ψ is obtained by taking $\partial/\partial t$ of Equation (7), eliminating the time derivatives using Equations (8) and (9), and substituting for u and w from (11)[5]. It has the form

$$\frac{\partial}{\partial r}\left[-g\frac{\partial \chi}{\partial z}\frac{1}{\rho r}\frac{\partial \psi}{\partial r} - \frac{\partial}{\partial z}(\chi C)\frac{1}{\rho r}\frac{\partial \psi}{\partial z}\right] +$$
$$\frac{\partial}{\partial z}\left[\left(\xi\chi(\zeta + f) + C\frac{\partial \chi}{\partial r}\right)\frac{1}{\rho r}\frac{\partial \psi}{\partial z} - \frac{\partial}{\partial z}(\chi C)\frac{1}{\rho r}\frac{\partial \psi}{\partial r}\right] =$$
$$g\frac{\partial}{\partial r}\left(\chi^2\dot{Q}\right) + \frac{\partial}{\partial z}\left(C\chi^2\dot{Q}\right) - \frac{\partial}{\partial z}\left(\chi C F_\lambda\right) \tag{12}$$

where $\chi = 1/\theta$, $\xi = 2v/r + f$ is twice the local absolute angular velocity at radius r, and $\zeta = (1/r)(\partial(rv)/\partial r)$ is the vertical component of relative vorticity at radius r. Further algebraic details of the derivation are given in Bui *et al.* (2009). The equation is a linear elliptic partial differential equation for ψ at any instant of time, when the radial and vertical structures of v and θ are known at that time, provided that the discriminant

$$D = -g\frac{\partial \chi}{\partial z}\left(\xi\chi(\zeta + f) + C\frac{\partial \chi}{\partial r}\right) - \left[\frac{\partial}{\partial z}(\chi C)\right]^2 \tag{13}$$

is positive. With a few lines of algebra one can show that $D = g\rho\xi\chi^3 P$, where

$$P = \frac{\chi^2}{\rho}\left[\frac{\partial v}{\partial z}\frac{\partial \chi}{\partial r} - (\zeta + f)\frac{\partial \chi}{\partial z}\right] \tag{14}$$

is the Ertel potential vorticity (Shapiro and Montgomery, 1993).

Since the Sawyer-Eliassen equation (12) is a linear differential equation, the solution for the transverse streamfunction can be obtained by summing the solutions forced individually by the radial and vertical derivative of the diabatic heating rate and the vertical gradient of the azimuthal momentum forcing, respectively. Suitable boundary conditions on ψ are obtained using the relationship between ψ and the velocity components of the transverse circulation, *viz.*, Equation (11). Solutions for a point source of diabatic heating in the tropical cyclone context were presented by Shapiro and Willoughby (1982).

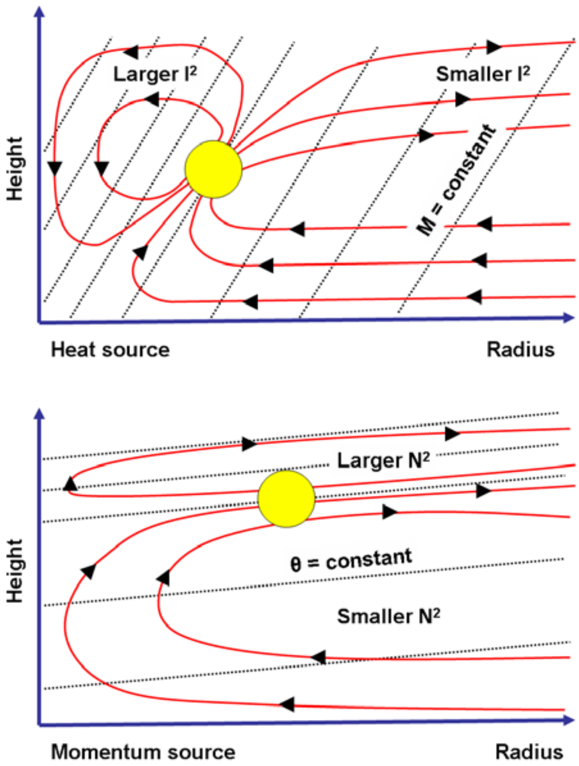

Figure 1. Secondary circulation induced in a balanced vortex by a heat source (upper panel) and a cyclonic momentum source (lower panel) in regions with different magnitudes of inertial stability, I^2 and thermodynamic stability N^2, and baroclinicity S^2. The strong motions through the source follow lines of constant angular momentum for a heat source and of constant potential temperature for a momentum source. Adapted from Willoughby (1995).

Examples of the solution of (12) are shown in Figure 1, which illustrates the secondary circulation induced by point sources of heat and absolute angular momentum in a balanced, tropical-cyclone-like vortex in a partially bounded domain (Willoughby 1995). Willoughby notes that, in the vicinity of the imposed heat source, the secondary circulation is congruent to surfaces of constant absolute angular momentum and is thus primarily vertical. In order to maintain a state of gradient and hydrostatic balance and slow evolution, the flow through the source is directed generally so as to oppose the forcing. Since the vortex is assumed to be stably stratified in the large, the

[5] Smith *et al.* (2005) show that the Sawyer-Eliassen balance equation emerges also from the time derivative of the toroidal vorticity equation when the time rate-of-change of the material derivative of potential toroidal vorticity, $\eta/(r\rho)$, is set to zero. Here $\eta = \partial u/\partial z - \partial w/\partial r$ is the toroidal, or azimuthal, component of relative vorticity.

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induced flow through the heat source causes an adiabatic cooling tendency that tries to oppose the effects of heating. In the vicinity of the imposed momentum source (assumed positive in Figure 1, corresponding to an eddy-induced cyclonic torque), the transverse circulation is congruent to surfaces of constant potential temperature and is primarily outwards. The resulting streamlines in either case form two counter-rotating cells of circulation (or gyres) that extend outside the source. There is a strong flow between these gyres and a weaker return flow on the outside. The flow emerges from the source, spreads outwards through a large volume surrounding it, and converges back into it from below. Thus, compensating subsidence surrounds heat-induced updraughts and compensating inflow lies above and below momentum-induced outflow.

The radial scale of the gyres is controlled by the local Rossby length, NH/I, where $N^2 = (g/\theta)(\partial\theta/\partial z)$, or $-(g/\chi)(\partial\chi/\partial z)$, is a measure of the static stability for vertically displaced air parcels (N being the Brunt-Väisälä frequency), $I^2 = (f+\zeta)\xi$ is a measure of the inertial (centrifugal) stability for horizontally displaced rings of fluid assumed initially in hydrostatic and gradient wind balance, and H is the depth of the overturning layer. The ratio of the horizontal to vertical scales thus scales with N/I. From the foregoing discussion it follows that a heat source located in the middle troposphere induces inflow in the lower troposphere and outflow in the upper troposphere beyond the radius of the source (Figure 1a). At radii inside that of the heat source, a reversed cell of circulation is induced with subsidence along and near the axis. The situation is similar for more realistic distributions of diabatic heating (Bui *et al.* 2009). Similarly, in order to maintain a state of balanced flow a momentum sink associated with surface friction distributed through a frictional boundary layer will induce inflow in the boundary layer and outflow above the layer.

2.4 Spin up

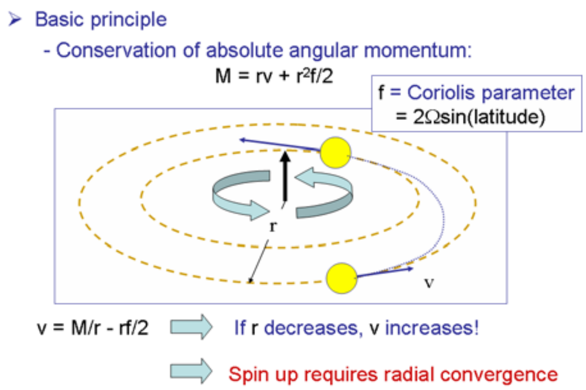

Figure 2. Schematic diagram illustrating the spin up associated with the convergence of absolute angular momentum.

The key element of vortex spin up in an axisymmetric setting can be illustrated from the equation for absolute angular momentum per unit mass

$$M = rv + \frac{1}{2}fr^2 \tag{15}$$

given by

$$\frac{\partial M}{\partial t} + u\frac{\partial M}{\partial r} + w\frac{\partial M}{\partial z} = F \tag{16}$$

where $F = rF_\lambda$ represents the torque per unit mass acting on a fluid parcel in association with frictional or (unresolved) turbulent forces, or those associated with non-axisymmetric eddy processes[6]. This equation is, of course, equivalent to the axisymmetric tangential momentum equation (Equation (8)). If $F = 0$, then M is materially conserved, i.e. $DM/Dt = 0$, where $D/Dt = \frac{\partial}{\partial t} + u\frac{\partial}{\partial r} + w\frac{\partial}{\partial z}$ is the material derivative following fluid particles in the axisymmetric flow. Since M is related to the tangential velocity by the formula:

$$v = \frac{M}{r} - \frac{1}{2}fr, \tag{17}$$

we see that, when M is materially-conserved, both terms in this expression lead to an increase in v as r decreases and to a decrease in v when r increases. Thus *a prerequisite for spin up in an inviscid axisymmetric flow is azimuthal-mean radial inflow*. Conversely, as air parcels move outwards, they spin more slowly. An alternative, but equivalent interpretation for the material acceleration of the mean tangential wind in an axisymmetric inviscid flow follows directly from Newton's second law (see Equation 8) in which the sole force is the generalized Coriolis force[7] associated with the mean radial component of inflow. In regions where frictional forces are appreciable, F is negative definite, and M decreases following air parcels. We will show later that friction plays a crucial dynamical role in the spin up of a tropical cyclone.

2.5 A balance theory for spin up

As intimated in section 2.3, the assumption that the flow is balanced everywhere paves the way for a method to solve an initial-value problem for the slow evolution of an axisymmetric vortex forced by sources of heat ($\dot{Q}$) and tangential momentum (F_λ). Given an initial tangential wind profile $v_i(r,z)$ and some environmental density sounding $\rho_o(z)$, one would proceed using the following basic steps:

- (1) solve Equation (3) for the initial balanced density and pressure fields corresponding to v_i.
- (2) solve the SE-equation (12) for ψ.
- (3) solve for the velocity components u and w of the overturning circulation using Equation (11).

[6] Apart from a factor of 2π, M is equivalent to Kelvin's circulation Γ for a circle of radius r enclosing the center of circulation, i.e., $2\pi M = \Gamma = \int \mathbf{v_{abs}} \cdot \mathbf{dl}$, where $\mathbf{v_{abs}}$ is the absolute velocity and $\mathbf{dl}$ is a differential line segment along the circle. The material conservation of M is equivalent to Kelvin's circulation theorem.

[7] The generalized Coriolis force is $-u(v/r+f)$, where u is the mean radial velocity component.

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- (4) predict the new tangential wind field using Equation (8) at a small time Δt.
- (5) repeat the sequence of steps from item (1).

The method is straightforward to implement. Examples are given by Sundqvist (1970), Schubert and Alworth (1987) and Möller and Smith (1994)). For strong tropical cyclones, the boundary layer and upper-tropospheric outflow region generally develop regions of zero or negative discriminant ($D < 0$). A negative discriminant implies the development of regions supporting symmetric instability and technically speaking the global balance solution breaks down. Nevertheless, it is often possible to advance the balance solution forward in time to gain a basic understanding of the long-time balance flow structure. If, for example, the symmetric instability regions remain localized and do not extend throughout the mean vortex, one may apply a regularization procedure to keep the SE-Equation elliptic and thus invertible (see section 7.2)[8].

2.6 Boundary-layer dynamics and departures from gradient wind balance

We refer to the tropical-cyclone boundary layer as the shallow region of strong inflow adjacent to the ocean surface, which is typically 500 m to 1 km deep and in which the effects of surface friction are important. A scale analysis of the equations of motion indicates that the radial pressure gradient force of the flow above the boundary layer is transmitted approximately unchanged through the boundary layer to the surface (see, e.g. Vogl and Smith 2009). However, beyond some radius outside the radius of maximum gradient wind[9], the centrifugal and Coriolis forces near the ocean surface are reduced because of the frictional retardation of the tangential wind (Figure 3). The resulting imbalance of the radial pressure gradient force and the centrifugal and Coriolis forces implies a radially inward-directed *agradient force*, F_a, that generates an inflow near the surface. The agradient force is defined as the difference between the local radial pressure gradient and the sum of the centrifugal and Coriolis forces per unit mass, i.e. $F_a = -(1/\rho)(\partial p/\partial r) + (v^2/r + fv)$, where the various quantities are as defined earlier. If $F = 0$, the tangential flow is in exact gradient wind balance; if $F < 0$, this flow is *subgradient* and if $F > 0$ it is *supergradient*.

The foregoing considerations naturally motivate a dynamical definition of the boundary layer. Since this layer arises *largely* because of the frictional disruption of gradient wind balance near the surface, we might define the boundary layer as the surface-based layer in which the inward-directed agradient force exceeds a specified threshold value. This dynamical definition is uncontroversial in the outer regions of a hurricane, where there is subsidence into the boundary layer, but it is questionable in the inner core region where boundary-layer air separates from the surface and is being lofted into the eyewall clouds (Smith and Montgomery, 2010).

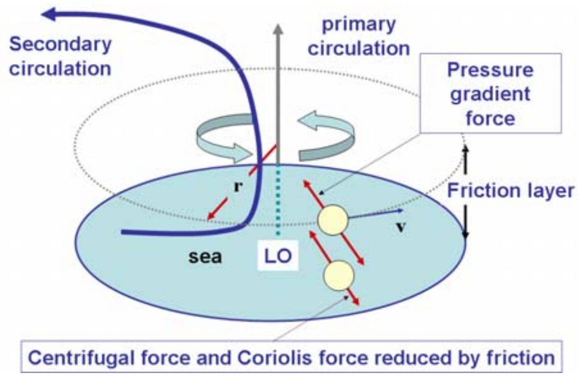

Figure 3. Schematic diagram illustrating the agradient force imbalance in the friction layer of a tropical cyclone and the secondary circulation that it generates.

Because of the pattern of convergence within the boundary layer and the associated vertical velocity at its top, the layer exerts a strong control on the flow above it. In the absence of diabatic heating associated with deep convection, the boundary-layer would induce radial outflow in above it and the vortex would spin down as air parcels move to larger radii while conserving their absolute angular momentum[10]. If the air is stably stratified, the vertical extent of the outflow will be confined. It follows that a requirement for the spin up of a tropical cyclone is that the radial inflow in the lower troposphere induced by the diabatic heating must more than offset the frictionally-induced outflow (e.g. Smith 2000).

Where the boundary layer produces upflow, it plays an additional role by determining the radial distribution of absolute angular momentum, water vapour and turbulent kinetic energy that enter into the vortex above. This latter characteristic is an important feature of the spin up in the three axisymmetric paradigms for tropical-cyclone intensification to be discussed. In particular, the source of moisture that fuels the convection in the eyewall enters the boundary layer from the ocean surface, whereupon the boundary layer exerts a significant control on the preferred areas for deep convection.

As the vortex strengthens, the boundary-layer inflow becomes stronger than the balanced inflow induced

[8]An alternative approach is to formulate the balanced evolution in terms of moist equivalent potential temperature instead of dry potential temperature. A particularly elegant method within this framework is to assume that air parcels rising out of the boundary layer materially conserve their equivalent potential temperature and that the analogous discriminant for the moist SE-Equation is everywhere zero, with an implied zero moist potential vorticity. Such an approach, together with a crude slab boundary layer representation, is employed in a class of time-dependent models that underpin the WISHE paradigm discussed in section 5.1 and the Appendix.

[9]The situation in the inner region is more complex as discussed below.

[10]This mechanism for vortex spin down involving the frictionally-induced secondary circulation is the primary one in a vortex at high Reynolds' number and greatly overshadows the direct effect of the frictional torque on the tangential component of flow in the boundary layer (Greenspan and Howard 1963).

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directly by the diabatic heating and the tangential frictional force as discussed in subsection 2.3. This breakdown of balance dynamics provides a pathway for air parcels to move inwards quickly and we can envisage a scenario in which the boundary layer takes on a new dimension. Clearly, if M decreases less rapidly than the radius following an inward moving air parcel, then it follows from Equation (16) that the tangential wind speed will increase following the air parcel. Alternatively, if rings of air converge quickly enough, i.e. if the generalized Coriolis force exceeds the tangential component of frictional force, the tangential winds can increase with decreasing radius. These considerations raise the possibility that the tangential wind in the boundary layer may ultimately exceed that above the boundary layer in the inner region of the storm. In section 7, this possibility will be shown to be a reality.

3 The CISK-paradigm

In a highly influential paper, Charney and Eliassen (1964) proposed an axisymmetric balance theory for the cooperative interaction between a field of deep cumulus clouds and an incipient, large-scale, cyclonic vortex. A similar theory, but with a different closure for deep convection was proposed independently by Ooyama (1964). These theories highlighted the role of surface friction in supporting the amplification process. They were novel because friction was generally perceived to cause a spin down of an incipient vortex. In their introduction, Charney and Eliassen state: "Friction performs a dual role; it acts to dissipate kinetic energy, but because of the frictional convergence in the moist surface boundary layer, it acts also to supply latent heat energy to the system." This view has prevailed until very recently (see section 7).

The idea of cooperative interaction stems from the closure assumption used by Charney and Eliassen that the rate of latent heat release by deep cumulus convection is proportional to the vertically-integrated convergence of moisture through the depth of the troposphere, which occurs mainly in the boundary layer. Recall from section 2.3 that, in a balanced vortex model where the contribution of friction to the secondary circulation is in initially relatively small, the strength of the azimuthal mean overturning circulation is proportional to the radial gradient of the net diabatic heating rate. In a deep convective regime in which the diabatic heating rate, and therefore its radial gradient, are a maximum in the middle to upper troposphere, this balanced circulation is accompanied by inflow below the heating maximum and outflow above it. At levels where there is inflow, the generalized Coriolis force acting on the inflow accelerates the tangential wind. The increased tangential wind at the top of the boundary layer leads to an increase of the frictional inflow in the boundary and therefore to an increase of moisture convergence in the inflow layer (see Figure 4). The closure that relates the latent heating to the moisture convergence then implies an increase in the heating rate and its radial gradient, thereby completing the cycle.

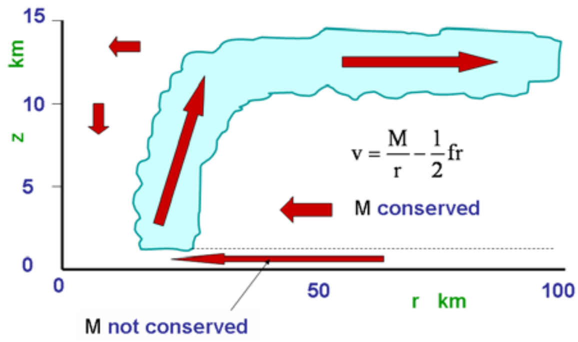

Figure 4. Schematic of the CISK-paradigm and of the cooperative intensification paradigm of tropical-cyclone intensification. The basic tenet is that, in an axisymmetric-mean sense, deep convection in the inner-core region induces inflow in the lower troposphere. Above the frictional boundary layer, the inflowing air conserves its absolute angular momentum and spins faster. Strong convergence of moist air mainly in the boundary layer provides "fuel" to maintain the convection. In the CISK-paradigm, the rate of latent heat release by deep cumulus convection is proportional to the vertically-integrated convergence of moisture through the depth of the troposphere. The bulk of this moisture convergence occurs in the boundary layer. In the cooperative intensification paradigm the representation of latent heat release is more sophisticated.

Charney and Eliassen constructed an axisymmetric, quasigeostrophic, linear model to illustrate this convective-vortex interaction process and found unstable modes at sub-synoptic scales shorter than a few hundred kilometres. In order to distinguish this macro instability from the conventional conditional instability that leads to the initiation of individual cumulus clouds, it was later named Conditional Instability of the Second Kind. A clear picture of the linear dynamics of the intensification process was provided by Fraedrich and McBride (1989), who noted that " ... the CISK feedback is through the spin up brought about by the divergent circulation above the boundary layer", as depicted here in the schematic in Figure 4. Similar ideas had already been suggested much earlier by Ooyama (1969, left column, p18) in the context of a linear instability formulation with a different closure assumption[11].

For many years subsequently, this so-called CISK-theory enjoyed wide appeal by tropical meteorologists. Indeed, the theory became firmly entrenched in the teaching of tropical meteorology and in many notable textbooks (e.g Holton, 1992, section 9.7.2; James, 1994, pp279-281) and the first papers on the topic stimulated much subsequent research. A list of references is given by Fraedrich and McBride (1989).

Despite its historical significance and influence, the CISK theory has attracted much criticism. An important contribution to the debate over CISK is the insightful paper by Ooyama (1982), which articulated the cooperative intensification paradigm for the spin-up process discussed in the next section. Ooyama noted that the CISK

[11] Ooyama's reference to the "lower layer" refers to the lower troposphere *above the boundary layer*, see his Figure 1.

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closure for moist convection is unrealistic in the early stage of development. The reason is that there is a substantial separation of horizontal scales between those of deep convective towers and the local Rossby length for the mean vortex (as defined in section 2.3). Therefore, during this stage, the convection is not under ‘rotational control’ by the parent vortex. Indeed, Ooyama cautioned that the CISK closure and variants thereof were little more than convection in disguise, exhibiting the largest growth rates at the smallest horizontal scales.

Later in the 80’s and 90’s, the CISK theory was critiqued in a number of papers by Emanuel (1986, hereafter E86), Raymond and Emanuel (1993), Emanuel *et al.* (1994), Craig and Gray (1996), Ooyama (1997) and Smith (1997). Raymond and Emanuel *op. cit.* gave an erudite discussion of the issues involved in representing cumulus clouds in numerical models. They recalled that the premise underlying all physical parameterizations is that some aspect of the chaotic microscale process is in statistical equilibrium with the macroscale system. They noted also that the statistical equilibrium assumption implies a particular chain of causality, to wit: “viscous stresses *are caused by* changes in the strain rate; turbulence *is caused by* instability of the macroscale flow; and convection *is caused by* conditional instability. Conditional instability is quantified by the amount of Convective Available Potential Energy (CAPE) in the macroscale system and convection, in turn, consumes this CAPE.” Raymond and Emanuel argued that the CISK closure implies a statistical equilibrium of water substance wherein convection is assumed to consume water (and not directly CAPE) at the rate at which it is supplied by the macroscale system. Indeed, they argued that the closure fundamentally violates causality because convection is not *caused by* the macroscale water supply.

Emanuel (1994) pointed out that the CISK closure calls for the large-scale circulation to replenish the boundary-layer moisture by advecting low-level moisture (and hence CAPE) from the environment. It completely overlooks the central role of surface moisture fluxes in accomplishing the remoistening. Thus, based on CISK theory, cyclone intensification would be just as likely to occur over land as over the sea, contrary to observations.

Another concern is that the heating representation assumed in the CISK theory is not a true “sub-grid scale” parameterization of deep convection in the usual sense, but is simply a representation of moist pseudo-adiabatic ascent (Smith 1997). Furthermore, the assumption that a stronger overturning circulation leads to a greater diabatic heating, while correct, misses the point since the adiabatic cooling following the air parcels increases in step. In other words, the pseudo-equivalent potential temperature, θ_e, of ascending air is materially conserved and is determined by the value of θ_e where the ascending air exits the boundary layer (E86). Thus, the radial gradient of virtual potential temperature at any height in the cloudy air will not change unless there is a corresponding change in the radial gradient of θ_e in the boundary layer.

4 The cooperative-intensification paradigm

Although Charney and Eliassen’s seminal paper continued to flourish for quarter of a century, soon after publishing his 1963 and 1964 papers, Ooyama recognized the limitations of the linear CISK paradigm. This insight led him to develop what he later termed a cooperative intensification theory for tropical cyclones (Ooyama 1982, section 4; Ooyama 1997, section 3.2), but the roots of the theory were already a feature of his simple nonlinear axisymmetric balance model for hurricane intensification (Ooyama 1969). The 1969 study was one of the first successful simulations and consistent diagnostic analyses of tropical-cyclone intensification[12].

The cooperative intensification theory assumes that the broad-scale aspects of a tropical cyclone may be represented by an axisymmetric, balanced vortex in a stably-stratified, moist atmosphere. The basic mechanism was explained by Ooyama (1969, p18) as follows. “If a weak cyclonic vortex is initially given, there will be organized convective activity in the region where the frictionally-induced inflow converges. The differential heating due to the organized convection introduces changes in the pressure field, which generate a slow transverse circulation in the free atmosphere in order to re-establish the balance between the pressure and motion fields. If the equivalent potential temperature of the boundary layer is sufficiently high for the moist convection to be unstable, the transverse circulation in the lower layer will bring in more absolute angular momentum than is lost to the sea by surface friction. Then the resulting increase of cyclonic circulation in the lower layer and the corresponding reduction of the central pressure will cause the boundary-layer inflow to increase; thus, more intense convective activity will follow.”

In Ooyama’s model, the “intensity of the convective activity” is characterized by a parameter η, where $\eta - 1$ is proportional to the difference between the moist static energy in the boundary layer and the saturation moist static energy in upper troposphere. Physically, $\eta - 1$ is a measure of the degree of local conditional instability for deep convection in the vortex. Ooyama noted that, in his model, as long as $\eta > 1$, the positive feedback process between the cyclonic circulation and the organized convection will continue. The feedback process appears to transcend the particular parameterization of deep convection used by Ooyama. Although Ooyama took the cloud-base mass flux to be equal to the convergence of resolved-scale mass flux in the boundary layer, this restriction may be easily relaxed (Zhu *et al.* 2001).

Ooyama’s model contained a simple bulk aerodynamic representation of the surface moisture flux (his Equation (7.4)) in which the flux increases with surface wind speed and with the degree of air-sea moisture disequilibrium. Although Ooyama recognized the need for

[12] In our view, Craig and Gray’s (1996) categorization of the Ooyama (1969) model as being a version of CISK was convincingly refuted by Ooyama (1997).

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such fluxes for intensification, he did not discuss the consequences of their wind-speed dependence. However, he did point out that as the surface pressure decreased sharply with decreasing radius in the inner-core region, this would lead to a concomitant sharp increase in the saturation mixing ratio, q_s^*. The increase in q_s^*, which is approximately inversely proportional to the pressure, may boost the boundary layer moisture provided that the mixing ratio of near-surface air doesn't increase as fast (see the discussion below Equation (7.4) in Ooyama (1969)).

Willoughby (1979) noted that without condensational heating associated with the convection, air converged in the boundary layer would flow outwards just above the boundary layer instead of rising within the clouds and flowing out near the tropopause. In other words, for intensification to occur, the convectively-induced convergence must be large enough to more than offset the frictionally-induced divergent outflow above the boundary layer (Raymond *et al.* 1998, Marin *et al.* 2009, Smith 2000).

The foregoing feedback process differs from Charney and Eliassen's CISK paradigm in that the latter does not explicitly represent the oceanic moisture source and it assumes that the latent heat release within the region of deep convection is proportional to the vertically-integrated radial moisture flux. Unlike the CISK theory, the cooperative intensification theory does not represent a runaway instability, because, as the troposphere warms on account of the latent heat release by convection, the atmosphere becomes progressively less unstable to buoyant deep convection. Moreover, as the boundary-layer moisture increases, the degree of moisture disequilibrium at the sea surface decreases. If the near-surface air were to saturate, the surface moisture flux would be zero, irrespective of the wind speed.

Two aspects of both the CISK theory and Ooyama's cooperative intensification theory that have gained wide acceptance are that:

- the main spin up of the vortex occurs via the convergence of absolute angular momentum above the boundary layer, and
- the boundary layer plays an important role in converging moisture to sustain deep convection, but its dynamical role is to oppose spin up.

Ooyama's theory recognizes also the need for a flux of moisture from the ocean to sustain the intensification process.

5 The WISHE paradigm

In what has become a highly influential paper, E86 developed a steady-state axisymmetric balance model for a mature tropical cyclone that subsequently led to a major paradigm shift in the theory for the intensification of these storms. In order to discuss this new paradigm, it is necessary to review some salient features of the E86 model. Briefly, the hurricane vortex is assumed to be steady and

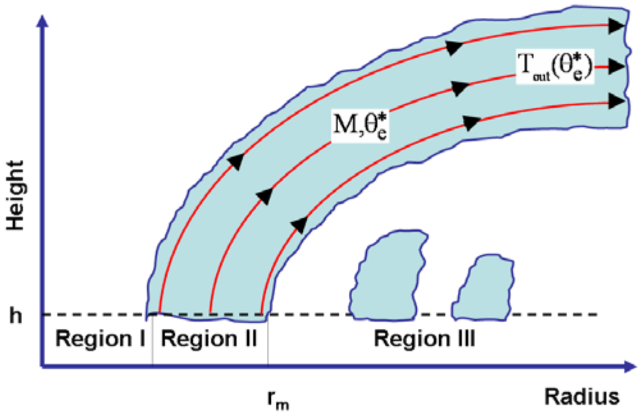

Figure 5. Schematic diagram of Emanuel's 1986 model for a mature steady-state hurricane. The boundary layer is assumed to have constant depth h and is divided into three regions as shown: the eye (Region I), the eyewall (Region II) and outside the eyewall (Region III) where spiral rainbands and shallow convection emanate into the vortex above. The absolute angular momentum per unit mass, M, and equivalent potential temperature, θ_e of an air parcel are conserved after the parcel leaves the boundary layer and ascends in the eyewall cloud. The precise values of these quantities depend on the radius at which the parcel exits the boundary layer. The model assumes that the radius of maximum tangential wind speed, r_m, is located at the outer edge of the eyewall cloud, whereas recent observations (e.g. Marks et al. 2008, Fig. 3) indicate it is closer to the inner edge.

circularly symmetric about its axis of rotation. The boundary layer is taken to have uniform depth, h, and is divided into three regions as shown in Figure 5. Regions I and II encompass the eye and eyewall, respectively, while Region III refers to that beyond the radius, r_m, of maximum tangential wind speed, v_m, at the top of the boundary layer.

E86 takes the outer radius of Region II to be r_m on the basis that precipitation-driven downdraughts may be important outside this radius[13]. The tangential wind field is assumed to be in gradient and hydrostatic balance everywhere, including the boundary layer[14]. Moreover, upon exiting the boundary layer, air parcels are assumed to rise to the upper troposphere conserving their M and saturation moist entropy, s^* (calculated pseudo-adiabatically[15]). Above the boundary layer, the surfaces of moist entropy

[13] Contrary to Emanuel's assumption in this figure, observations show that r_m is located well inside the outer edge of the eyewall (e.g. Marks et al. 2008, Figure 3).

[14] Although the E86 text explicitly assumed gradient wind balance at the top and above the boundary layer (p586: "The model is based on the assumptions that the flow above a well-mixed surface boundary layer is inviscid and thermodynamically reversible, that hydrostatic and gradient wind balance apply ..."), in the slab formulation therein the M and tangential velocity of the slab of air exiting the boundary layer is assumed to equal that of the flow at the top of the boundary layer. Since the swirling wind field at the top of the boundary layer is assumed to be in gradient wind balance (*op cit.*), it is a mathematical consequence that in the region where the air is rising out of the boundary layer the tangential velocity of the slab of air exiting the boundary layer must also be in gradient wind balance.

[15] Although the original formulation and subsequent variants thereof purported to use reversible thermodynamics, it has been recently discovered that the E86 formulation and its variants that make *a priori* predictions for the maximum tangential wind (Emanuel 1995, 1997,

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and absolute angular momentum are assumed to flare outwards with height and become almost horizontal in the upper troposphere. For an appraisal of some aspects of the steady-state model, see Smith *et al.* (2008) and Bryan and Rotunno (2009).

Time-dependent extensions of the E86 model were developed for investigating the intensification process (Emanuel 1989, 1995 and 1997: hereafter E97). These models were formulated in the context of axisymmetric balance dynamics and employ potential radius coordinates. The potential radius, R, is defined in terms of the absolute angular momentum by $\sqrt{2M/f}$ and is the radius to which an air parcel must be displaced, conserving M, so that its tangential wind speed vanishes.

5.1 The role of evaporation-wind feedback

The new paradigm for intensification that emerges from the time-dependent models invokes a positive feedback between the near-surface wind speed and the rate of evaporation of water from the underlying ocean, *which depends on the wind speed* (Emanuel *et al.* 1994, section 5a; Emanuel 2003). Emanuel's time-dependent models have broadly the same ingredients as Ooyama's 1969 model (e.g. gradient wind and hydrostatic balance, wind-speed-dependent surface moisture fluxes), but the closure for moist convection is notably different by assuming undilute pseudo-adiabatic ascent along surfaces of absolute angular momentum rather than upright ascent with entrainment of middle tropospheric air. In the section 5.2 and the supporting critique in Appendix A, we examine carefully the semi-analytical time-dependent E97 model, which has been advanced as the essential explanation of tropical-cyclone intensification (Emanuel 2003). To begin, we discuss the traditional view of the evaporation-wind feedback intensification mechanism known generically as WISHE.

Although the evaporation of water from the underlying ocean has been long recognized as the ultimate energy source for tropical cyclones (Kleinschmidt 1951; Riehl 1954; Malkus and Riehl 1960; Ooyama 1969)[16], Emanuel's contributions with colleagues re-focused attention on the air-sea interaction aspects of the intensification process. Rather than viewing latent heat release in deep convective towers as the 'driving mechanism' for vortex amplification, this body of work showed that certain aspects of these storms could be understood in terms of a simple time-dependent model in which the latent heat release was implicit.

The new dimensions introduced by the WISHE paradigm were the positive feedback process between the wind-speed dependent moisture fluxes and the tangential wind speed of the broad-scale vortex, and the finite-amplitude nature of the instability, whereby the incipient vortex must exceed a certain threshold intensity for amplification to proceed. Emanuel and collaborators emphasized also the non-necessity of CAPE in the storm environment for intensification.

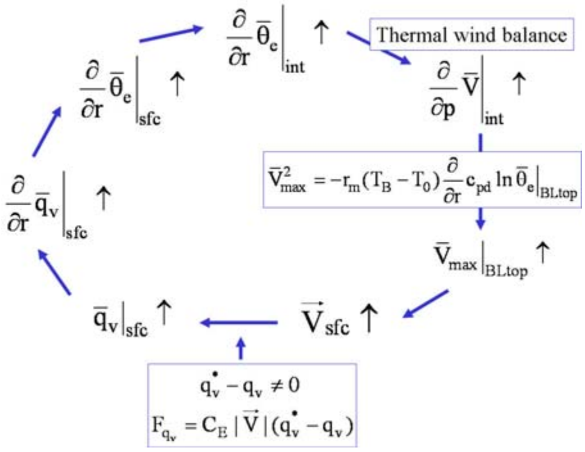

Figure 6. Schematic of the evaporation-wind intensification mechanism known as WISHE as articulated here. The pertinent variables are defined in the text and are otherwise standard. Thermal wind balance refers to the axisymmetric thermal wind equation in pressure coordinates that relates the radial gradient of azimuthal-mean θ_e to the vertical shear of the mean tangential velocity. The discussion assumes that for some reason the near-surface azimuthal mean mixing ratio is increased in the core region. This increase leads to an increase in the corresponding mean radial gradient and also an increase in the mean radial gradient of θ_e throughout the boundary layer. The secondary circulation is imagined to loft this increase in θ_e into the vortex interior thereby warming the core region of the vortex. By the thermal wind relation and its corollary, obtained upon applying a Maxwell thermodynamic relation (presuming reversible thermodynamics, see footnote 15) and then integrating this equation along a surface of constant absolute angular momentum, this increased warmth is invoked to imply an increase in the mean tangential wind at the top of the vortex boundary layer. It is understood here that θ_e values at the boundary layer top correspond to saturated values (Emanuel 1986). Turbulent mixing processes in the boundary layer are assumed to communicate this wind speed increase to the surface, and thereby increase the potential for increasing the sea-to-air water vapour flux F_{q_v}. The saturation mixing ratio must increase approximately in step with the putative increase in near-surface q_v so as to maintain a thermodynamic disequilibrium. If this occurs then the surface mixing ratio will increase, thereby leading to a further increase in the mean tangential wind and so on.

The evaporation-wind feedback intensification mechanism has achieved widespread acceptance in meteorology textbooks and other didactic material (e.g., Rauber *et al.* 2008, Holton 2004, Asnani 2005, Ahrens 2008, COMET course in Tropical Meteorology[17]), tropical weather briefings and the current literature (Lighthill 1998, Smith 2003, Molinari *et al.* 2004, Nong and Emanuel 2004, Montgomery *et al.* 2006, Terwey and Montgomery 2008, Braun *et al.* 2010). Indeed, the last four authors and others have talked about 'igniting the WISHE mechanism' after the vortex (or secondary maximum in the tangential wind) has reached some threshold intensity.

1997, and Bister and Emanuel 1998) tacitly implied pseudo-adiabatic thermodynamics (Bryan and Rotunno, 2009)

[16]Technically speaking, the sun is the ultimate energy source for all atmospheric and oceanic motions.

[17]http://www.meted.ucar.edu/topics_hurricane.php

In his review paper Emanuel (2003) specifically describes the intensification process as follows: "intensification proceeds through a feedback mechanism wherein increasing surface wind speeds produce increasing surface enthalpy flux ..., while the increased heat transfer leads to increasing storm winds." In order to understand his ensuing description of the intensification process, one must return to the E97 paper because two of the key parameters, α and β, in Equation (10) of the 2003 paper are undefined. Appendix A provides a careful examination of the corresponding equation (Eq. (20)) in E97 and the underlying assumptions employed therein. It suffices here to present our own interpretation of the putative WISHE mechanism that was articulated in Montgomery *et al.* (2009).

5.2 The WISHE mechanism in detail

The WISHE process for intensifying the inner-core tangential wind is illustrated schematically in Figure 6. It is based on the idea that, except near the centre, the near-surface wind speed in a tropical cyclone increases with decreasing radius. This increase is typically accompanied by an increase in near-surface specific humidity, which leads to a negative radial gradient of θ_e near the surface and hence throughout the boundary layer by vertical mixing processes. On account of frictional convergence, air parcels in the inner core exit the boundary layer and are assumed to rise upwards and ultimately outwards into the upper troposphere. As they do so, they conserve their θ_e, carrying with them an imprint of the near-surface radial gradient of θ_e into the interior of the vortex. Since the rising air rapidly saturates, the radial gradient of θ_e implies a negative radial gradient of virtual temperature. Thus, in the cloudy region, at least, the vortex is warm cored. The air rising out of the boundary layer is assumed also to conserve its absolute angular momentum. Consequently, as the air parcel moves outwards it spins more slowly about the rotation axis of the storm, which broadly accounts for the observed decrease of the tangential wind speed with height.

Now suppose that for some reason the negative radial gradient of specific humidity increases in the boundary layer. This would increase the negative radial gradient of θ_e, both in the boundary layer and in the cloudy region of the vortex aloft, and thereby warm the vortex core in this region. Assuming that the vortex remains in thermal wind balance during this process, the negative vertical shear of the tangential wind will increase. E86 presents arguments to show that this increase leads to an increase in the maximum tangential wind speed *at the top of the boundary layer* (see, e.g., Montgomery *et al.* 2006, Eq. A1). Assuming that this increase translates to an increase in the surface wind speed, the *surface* moisture flux will increase, thereby increasing further the specific humidity. However, this increase will reduce the thermodynamic disequilibrium in specific humidity between the near-surface air and the ocean surface, unless the saturation specific humidity at the sea surface temperature increases in step. The increase in wind speed could help maintain this disequilibrium if there is an associated reduction in surface pressure. (Recall from section 4 that the saturation specific humidity at the sea surface temperature is approximately inversely proportional to the air pressure). If the specific humidity (and θ_e) in the boundary layer increases further, it will lead to a further increase in tangential wind speed and so on[18].

5.3 WISHE and the cooperative intensification theory

Our discussion of the spin up mechanism in the previous paradigms highlighted the role of convergence of absolute angular momentum in the lower troposphere. The arguments articulated in section 5.2 are silent about the role of the secondary circulation in the vortex-scale amplification process and, because they assume thermal wind balance, the buoyancy of the warm core, or more precisely the system buoyancy, cannot be invoked to 'drive' the secondary circulation. Indeed, cloud updraughts in this idealization have no explicit local buoyancy[19]. In this light, then, it is natural to ask how the spin up actually occurs in the WISHE theory? To answer this question we examine further the time-dependent E97 model referred to in section 5.1.

Recall that the E97 model is founded on the idea that spin-up is controlled by the thermodynamics of the boundary layer and makes the key assumptions that:

(1) The vortex is everywhere in gradient wind balance, even in the boundary layer[20], and
(2) Above the boundary layer, the moist isentropes and M surfaces are congruent and flare out to large radius.

[18] The mechanism as articulated here is quite different from that described by Kepert (2010, p13), who interprets WISHE in the context of a steady state vortex as "The role of the surface enthalpy fluxes in making the expansion of the inflowing boundary layer air isothermal rather than adiabatic ... ." Kepert makes no mention of the necessity of the wind-speed dependence of the fluxes of latent and sensible heat and makes no distinction between dry and moist enthalpy in his discussion.

[19] A related property of the WISHE model is that any latent instability beyond that needed to overcome internal dissipation within cumulus clouds is believed unnecessary and thus irrelevant to the essential dynamics of tropical-cyclone intensification (Rotunno and Emanuel 1987).

[20] In private correspondence, K. Emanuel has expressed strong disagreement with this conclusion stating that "It would be absurd to assume gradient wind balance in the boundary layer ..." . Nevertheless, we hold in our evaluation that the tacit assumption of gradient wind balance in the boundary layer is a property of all three time-dependent models described in E97, Emanuel (1989) and Emanuel (1995). Focusing specifically on the E97 model, the author invokes an axisymmetric invertibility principle in which the "*entire rotational flow* [our emphasis] may be deduced from the radial distribution of θ_e in the boundary layer and the distribution of vorticity at the tropopause" (see first two complete paragraphs on page 1018 of E97). In our view, this invertibility relation would require the assumption of gradient wind balance throughout the vortex flow (including the boundary layer). One page later on p1019, however, the author appears to back off by stating that: "We have assumed gradient and hydrostatic wind balance everywhere above the boundary layer... ." However, since the mathematical model represented by his Equation (20) describes the dynamics of a slab boundary layer, the same scientific reasoning given in footnote 14 applies here. Specifically, because the air at the top of the slab is assumed to be in gradient wind balance with tangential velocity V and because no discontinuities in the tangential velocity between the air rising out of the slab and that at its top are allowed, it is a logical consequence that the slab of air in the boundary layer must be in gradient wind balance.

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In the E86 steady-state model, the flaring out of the M- and θ_e-surfaces implies that there is outflow everywhere above the boundary layer, since both these quantities are materially conserved. The situation in E97 model is less transparent because the model is formulated in potential radius coordinates (see section 5) and even though these surfaces flare outwards in physical space, they will move radially in general as the vortex evolves. If the vortex intensifies, they must move inwards (see section 2.4), consistent with net inflow at low levels above the boundary layer, and if the vortex decreases in intensity, they must move outwards.

The effects of latent heat release in clouds are implicit also in the E97 model, but the negative radial gradient of θ_e in the boundary layer is equivalent to a negative radial gradient of diabatic heating in the interior, which, according to the balance concepts discussed in section 2.3 will lead to an overturning circulation with inflow in the lower troposphere. Thus the spin up above the boundary layer is entirely consistent with that in Ooyama's cooperative intensification theory. Because the tangential wind speed in the slab boundary is assumed to be equal to that at the top of the boundary layer (as in Ooyama's 1969 model), the spin up in the boundary layer is consistent also with Ooyama's 1969 model. Unfortunately, the mathematical elegance of the formulation in E97 is achieved at the cost of making the physical processes of spin up less transparent (at least to us!).

For the foregoing reasons we would argue that the differences between the E97 theory and Ooyama's cooperative intensification theory are perhaps fewer than is widely appreciated. In essence:

- The convective parameterizations are different. E97 assumes pseudoadiabatic ascent, while Ooyama (1969) uses an entraining plume model for deep convection.
- Emanuel gives explicit recognition to coupling between the surface enthalpy fluxes and the surface wind speed (although this coupling was included in Ooyama's 1969 model).
- E97 recognized the importance of convective downdraughts on the boundary layer thermodynamics and included a crude representation of these.

Emanuel would argue that a further difference distinguishing the WISHE paradigm from the others is that it does not require the existence of ambient CAPE, which is crucial in the CISK theory and is present in Ooyama's (1969) model. However, Ooyama (1997, section 3.3) noted that "the initial CAPE had short memory..." and recalled that in his 1969 study he "demonstrated that a cyclone vortex could not develop by the initial CAPE alone...". Further, Dengler and Reeder (1998) have shown that ambient CAPE is not necessary in Ooyama's model or extensions thereof.

One additional feature of the E97 model is its recognition of the tendency of the eyewall to develop a front in M and θ_e. This frontogenesis arises from the convergence of M and θ_e brought about by surface friction. A second feature of the model is a proposed relationship between the lateral mixing of M at the eyewall front and the rate of intensification. These interesting aspects are reviewed briefly in section 5.5.

5.4 Brakes on WISHE

As pointed out by Emanuel *et al.* (1994), the foregoing feedback mechanism is not a runaway process for the present climate and would cease if the boundary layer were to saturate. Moreover, the import of low entropy air into the boundary layer, for example by shallow convection, precipitation-induced downdraughts from deep convection, or vortex-scale subsidence, can significantly offset the moistening of the boundary layer by surface fluxes (e.g., Ooyama 1997). Indeed Emanuel *et al.* (1994) argue that tropical-cyclone amplification requires the middle troposphere to become nearly saturated on the mesoscale so that the downdraughts are weak enough not to negate the feedback process. However, recent evidence from state-of-the-art numerical cloud models suggests that the presence of mid-level dry air is not to strengthen downdraughts, but rather to reduce the strength of updraughts (James and Markowski 2010, G. Kilroy, personnal communication).

Global energetics considerations point also to a brake on the intensification process and a maximum possible intensity. Recall that under normal circumstances the energy dissipation associated with surface friction scales as the cube of the tangential winds, while the energy input via moist entropy fluxes scales with the first power of the wind. Even if the boundary layer remains subsaturated, frictional dissipation will exceed the input of latent heat energy to the vortex from the underlying ocean at some point during the cyclone's intensification. This idea forms the basis for the so-called 'potential intensity theory' of tropical cyclones (E86, Emanuel 1988, 1995; E97; Bister and Emanuel 1998)[21].

[21] At the present time it is thought there is an exception to this general argument when either the sea surface temperature is sufficiently warm or the upper tropospheric temperature is sufficiently cold, or some combination of the two prevails (Emanuel 1988, Emanuel *et al.* 1995). Under such conditions, the vortex is believed to be capable of generating enough latent heat energy via surface moisture fluxes to more than offset the dissipation of energy and a 'runaway' hurricane regime - the so-called 'hypercane regime' - is predicted. However, as these predictions have been formulated only in the context of axisymmetric theory and the corresponding axisymmetric models have not yet been tested carefully for the possible transition to supersonic flow and related shock structure, it remains an open question whether hypercanes are indeed dynamically realizable in a three-dimensional context. Moreover, in a three-dimensional configuration, asymmetric vortex Rossby wave instabilities and their nonlinear lifecycles (e.g., Schubert *et al.* 1999; Montgomery *et al.* 2002; Braun *et al.* 2006), together with their coupling to radiating inertia-buoyancy waves (Plougonven and Zeitlin 2002; Schecter and Montgomery 2004; Schecter 2008 and refs.) and acoustic waves [the coupling to acoustic waves is relevant if the maximum swirling winds attain supersonic speeds] (Broadbent and Moore 1971; Ford 1994a,b; Schecter and Montgomery 2006 and refs.), contribute new pathways to dissipation within the vortex. These pathways are in addition to the usual three-dimensional (Kolmogorov) turbulent dissipation that occurs in the vortex boundary layer and in the convective eyewall at the sub-cumulus cloud scales (e.g., Rotunno *et al.* 2009). These considerations are of course subject to the caveat that our knowledge of the air-sea interaction physics and cloud microphysics

5.5 'Rapid spin up' via interaction with the eye

There are two properties of the E97 model that are new and merit brief discussion. It should be noted that the schematic of the WISHE mechanism in Figure 6 explains physically the sign of the tangential wind tendency in the core region, but it does not provide a means for quantifying the tendency. In the E97 model (Equation (20) and the accompanying discussion on p1020 therein), the spin up of the core tangential wind occurs slowly and fails to achieve its energetically-predicted upper bound (*op. cit.*, Fig. 6a) *unless* there is weak but nonzero radial diffusion of angular momentum across the contracting radius of maximum wind.

In the model, the region of strongly swirling winds is assumed to have moistened the low- to mid-troposphere. Without horizontal diffusion of M across the developing M-front, vortex intensification proceeds slowly until the vorticity becomes infinite at a critical radius in finite time. In this 'pseudo-inviscid' limit wherein lateral mixing of M is neglected, the intensification ceases and the vortex fails to achieve its energetically-predicted upper bound by a wide margin (see Figure 6 of E97).

To extend the solution beyond the time of frontal collapse, Emanuel developed a regularization procedure that assumes non-zero radial diffusion of M throughout the boundary layer, sufficient in magnitude to prevent collapse, and zero radial diffusion of θ_e (i.e. an infinite Prandtl number approximation). As is usual, the radial diffusion of M is a parameterization of asymmetric vortex waves, sub-vortex-scale mesovortices, and turbulent mixing processes near the developing eyewall (e.g., Schubert *et al.* 1999, Montgomery *et al.* 2002, Kossin *et al.* 2002, Braun *et al.* 2006, Rotunno *et al.* 2009). With weak, but nonzero radial diffusion of M into the eye, the model vortex intensifies rapidly to attain its energetic upper bound (E97, p1022-1024 and Figures 7 and 8). This regularized model is consistent with prior work arguing that the tangential winds in the eye are mechanically spun up by eddy processes operating between the eyewall and eye (Malkus 1958, Anthes 1974, Smith 1980, Kurihara and Bender 1982).

We are uncertain how much weight to place on the proposed role of lateral diffusion in the spin up of the tangential wind field. In view of the results described in section 7 showing that the boundary layer exerts a very strong control on the rate of intensification of the inner-core tangential wind field, it would appear possible that the spin-up rate may be controlled just as much by gradient wind imbalance in the boundary than by radial diffusion of M into the eye. It would appear possible also that the width of the eyewall may be controlled more by the boundary-layer dynamics than the lateral diffusion phenomenology invoked in these time-dependent models (Emanuel 1989, 1995, E97). This is not to say, however, that the eyewall region is not frontogenetic. Rather, the existing articulation of this process may not capture all of the dominant dynamics and our understanding of this process is incomplete.

at such extreme wind speeds is nonexistent. Fortunately, the hypercane regime is not predicted for the present climate or predicted tropical climates of the future barring an unforseen global extinction event (!). We will henceforth confine our attention to the intensification dynamics under current climate conditions.

6 A new asymmetric paradigm

Satellite and radar observations have long suggested that tropical cyclones are highly asymmetric during their intensification phase. Only the most intense storms exhibit a strong degree of axial symmetry in their mature stages and even then, only in their inner-core region. Observations show also that rapidly developing storms are accompanied frequently by 'bursts' of intense convection (e.g., Gentry *et al.* 1970; Black *et al.* 1986; Marks et al. 1992; Molinari *et al.* 1999), which one would surmise possess significant local buoyancy and are accompanied by marked flow asymmetries. These are reasons alone to query the applicability of purely axisymmetric theories to the intensification process and a series of questions immediately arise:

1. How does vortex intensification proceed in three-dimensional models and in reality?
2. Can the evolution of the azimuthally-averaged fields in these models be understood in terms of the axisymmetric paradigms for intensification described in previous sections?
3. If not, can the axisymmetric paradigms be modified to provide such understanding?

These questions are addressed in the discussion below.

There have been many three-dimensional numerical-model studies of vortex amplification in the prototype problem for tropical-cyclone intensification on an f-plane, described in section 1. These studies can be divided into five groups:

- those using hydrostatic models with cumulus parameterization (e.g. Kurihara and Tuleya 1974);
- those using minimal hydrostatic models, with or without cumulus parameterization (e.g., Zhu *et al.* 2001; Zhu and Smith 2002, 2003, Shin and Smith 2008);
- those using hydrostatic models with explicit microphysics (e.g., Wang 2001, 2002a, 2002b);
- those using nonhydrostatic models with highly simplified physics (Nguyen *et al.* 2008 and Montgomery *et al.* 2009); and
- those using nonhydrostatic models with sophisticated representations of physical processes (e.g. Wang 2008, Terwey and Montgomery 2009, Hill and Lackman 2009).

There have been investigations also of the analogous intensification problem on a β-plane, which is a prototype problem for tropical-cyclone motion (e.g., Flatau *et al.* 1994; Dengler and Reeder 1997; Wang and Holland

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1996a, 1996b) [22]. In this scenario, initially symmetric vortices develop asymmetries from the very start of the integration on account of the "β-effect", which is most easily understood in terms of barotropic dynamics. In a barotropic vortex, air parcels conserve their absolute vorticity and, as they move polewards on the eastern side of the vortex, their relative vorticity becomes more anticyclonic. Conversely, air parcels that move equatorwards on the western side become more cyclonic. As time proceeds, this process leads to a vorticity dipole asymmetry that has fine-scale radial structure in the inner-core, where the radial gradient of angular velocity is large, but has a broad coherent structure at large radii where this gradient is small (Chan and Williams 1989, Smith *et al.* 1990, Smith and Ulrich 1993 and references). The asymmetric flow associated with this dipole leads to a westward and poleward motion across the vortex core and is mostly influenced by the coherent vorticity asymmetry at large radii (e.g. Smith *et al.* 1990. p351). In a baroclinic vortex, it is the potential vorticity that is conserved, except where there is diabatic heating associated with convection, but the mechanism of formation of the large-scale vorticity asymmetry is essentially as described above.

While the formation of flow asymmetries is to be expected on a β-plane, it may seem surprising at first sight that appreciable inner-core flow asymmetries emerge also in three-dimensional simulations that start with an axisymmetric vortex on an f-plane. It is the latter aspect that is the main focus of this section.

The development of flow asymmetries on an f-plane was already a feature of the early calculations by Kurihara and Tuleya (1974), but these authors did not consider these asymmetries to be remarkable, even though their problem as posed was, in essence, axisymmetric. Flow asymmetries were found also in simulations using a minimal (three-layer) model for a tropical cyclone that was developed in a series of papers by Zhu *et al.* (2001) and Zhu and Smith (2002, 2003). That model was designed initially to examine the sensitivity of TC intensification to different convective parameterization schemes in the same model and in a configuration that was simple enough to be able to interpret the results. It came as a surprise to those authors that the vortex in their calculations developed flow asymmetries and the cause of these was attributed primarily to the coarse vertical resolution in the vertical and the use of a Lorenz grid for finite differencing in the three layers. In the last paper (Zhu and Smith 2003), it was shown that the amplitude of asymmetries could be reduced by using a Charney-Phillips grid pattern in the vertical.

6.1 The role of deep rotating convection

Subsequent attempts to understand the development and evolution of the flow asymmetries in the prototype intensification problem on an f-plane were described by Nguyen *et al.* (2008) using the non-hydrostatic model, MM5[23] and Shin and Smith (2008) using the minimal model of Zhu *et al*. We describe below the essential features of vortex evolution found in the Nguyen *et al* study. The model was stripped down to what were considered to be the simplest representation of physical processes necessary, including the simplest explicit representation of moist processes that mimicks pseudo-adiabatic moist thermodynamics and a simple bulk formulation of the boundary layer including air-sea exchange processes. The horizontal grid spacing in the main experiments was 5 km and there were 24 levels in the vertical (7 below 850 mb) giving a much higher resolution than Zhu *et al.* 's minimal model, which had a 20 km horizontal grid and only three vertical layers. The calculations were initialized with an axisymmetric warm-cored, cloud-free vortex with a maximum tangential velocity of 15 m s^{-1}.

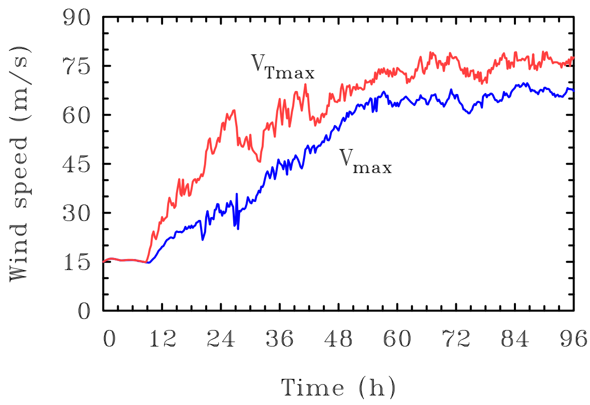

Figure 7. Time-series of maximum azimuthal-mean tangential wind component (V_{max}) and maximum total wind speed (V_{Tmax}) at 900 mb characterizing vortex development in f-plane control experiment in M1.

Figure 7 shows time-series of the maximum total horizontal wind speed, V_{Tmax}, and maximum azimuthally-averaged tangential wind component, V_{max}, at 900 hPa (approximately 1 km high). As in many previous experiments, there is a gestation period during which the vortex slowly decays because of surface friction, but moistens in the boundary layer because of evaporation from the underlying sea surface. During this period, lasting about 9 hours, the vortex remains close to axisymmetric. The imposition of friction from the initial instant leads to inflow in the boundary layer and outflow immediately above it, for the reasons discussed in section 2.6. It is the outflow together with the conservation of absolute angular momentum that accounts for the initial decrease in the tangential wind speed (see section 2.4).

The subsequent evolution is exemplified by Figures 8 and 9, which show the patterns of vertical velocity and the vertical component of relative vorticity at the 850 mb level at selected times. The two left panels show the situation when rapid intensification begins. The inflowing

[22] There have been many more studies of this problem in a barotropic context, but our interest here is focussed on baroclinic models with at least three vertical levels to represent the effects of deep convection.

[23] MM5 refers to the Pennsylvania State University-National Center for Atmospheric Research fifth-generation Mesoscale Model.

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air is moist and as it rises out of the boundary layer and cools, condensation occurs in some grid columns inside the radius of maximum tangential wind speed. Latent heat release in these columns initiates deep convective updraughts with vertical velocities up to 5 m s^{-1} at 850 mb. The updraughts form in an annular region inside the radius of the maximum initial tangential wind speed (i.e. 135 km) and their distribution shows a dominant azimuthal wavenumber-12 pattern around the circulation centre[24] The updraughts rotate cyclonically around the vortex centre and have lifetimes on the order of an hour. As they develop, they tilt and stretch the local vorticity field and an approximate ring-like structure of intense, small-scale, vorticity dipoles emerges (lower left panel of Figure 8). The dipoles are highly asymmetric in strength with strong cyclonic vorticity anomalies and much weaker anticyclonic vorticity anomalies. Early in the intensification phase, the strongest updraughts lie approximately in between the vorticity dipoles whereas later, they are often approximately collocated with the strong cyclonic anomalies. Following Nguyen *et al.* (2008), we refer to the cyclonically-rotating updraughts as "vortical hot towers" (VHTs), a term first coined by Hendricks *et al.* (2004). The development of the updraughts heralds a period lasting about two days during which the vortex intensifies rapidly, with V_{max} increasing at an average intensification rate of about 1 m s^{-1} h^{-1}. During this period, there are large fluctuations in V_{Tmax}, up to 15 m s^{-1}. Eventually, the vortex attains a quasi-steady state, in which V_{max} increases only slightly.

During rapid intensification phase, the number of VHTs decreases from twelve at 9 h to no more than three by the end of the period of rapid intensification, but their structure remains spatially irregular and during some periods, the upward motion occupies a contiguous region around the vortex centre. The cyclonic vorticity anomalies associated with the VHTs grow horizontally in scale due to merger and axisymmetrization with neighbouring cyclonic vorticity anomalies. This upscale growth occurs in tandem with the convergence of like-sign vorticity into the circulation of the VHTs. They move slowly inwards also and become segregated from the anticyclonic vorticity anomalies, which move slowly outwards relative to them. As the anticyclonic anomalies move outwards they decrease in amplitude and undergo axisymmetrization by the parent vortex. We summarize the elements of the segregation process in section 6.2.

The right panels of Figure 8 show the situation at 24 hours. At this time there are five strong updraughts, each possessing greatly enhanced cyclonic vorticity (lower right panel), so that the initial monopole structure of the vortex is completely dwarfed by the local vorticity of the VHTs.

Comparing Figures 8 and 9 shows that the VHTs move inwards over time. Spiral inertia-gravity waves propagating outwards occur throughout the vortex evolution and these are especially prominent in animations of the vertical velocity fields. The pattern of strong updraughts changes continuously with time, but is mainly monopolar, dipolar, or tripolar and always asymmetric. By the end of the rapid intensification period, no more than three VHTs are active around the circulation centre.

The simulated vortex exhibits many realistic features of a mature tropical cyclone (e.g., Willoughby 1995, Kossin *et al.* 2002, Corbosiero *et al.* 2006, Marks *et al.* 2008) with spiral bands of convection surrounding an approximately symmetric eyewall and a central eye that is free of deep convection (Figure 9a,b). In the mature phase, the evolution of the vortex core is characterized by an approximately axisymmetric circulation superimposed on which are:

- small, but finite-amplitude vortex Rossby waves that propagate azimuthally, radially and vertically on the mean potential vorticity gradient of the system scale vortex (Shapiro and Montgomery 1993, Guinn and Schubert 1993, Montgomery and Kallenbach 1997, Möller and Montgomery 2000, Chen and Yau 2001, Wang 2002a,b, Chen *et al.* 2003, McWilliams *et al.* 2003, Martinez 2008, Martinez *et al.* 2011)[25];
- barotropic-baroclinic shear instabilities (Schubert *et al.* 1999, Nolan and Montgomery 2001, Nguyen *et al.* 2011);
- eyewall mesovortices (Schubert *et al.* 1999, Montgomery *et al.* 2002, Braun *et al.* 2006); and
- trochoidal "wobbling" motion of the inner-core region (Nolan and Montgomery 2000, Nolan *et al.* 2001).

Of course, these processes are not adiabatic and inviscid as they are coupled to the boundary layer and convection.

6.2 The vorticity segregation process

The vorticity segregation process occurs in the presence of persistent lower tropospheric inflow and upper-tropospheric outflow. Therefore, it cannot be explained solely by the mean advection associated with the diabatically-forced axisymmetric secondary circulation. To gain further understanding of the process, it is first useful to recall that vortex axisymmetrization involves the ingestion of like-sign vorticity anomalies accompanied by the formation of like-sign vorticity filaments surrounding the parent vortex as well as the expulsion of opposite-signed vorticity anomalies (e.g., Montgomery and Enagonio 1998, Figures 9, 11, 12; Reasor and Montgomery 2001, Figure 18 and accompanying discussions. During the merger phase, the amplitude of the cyclonic vorticity anomalies is more than an order of magnitude larger than the vorticity of the parent vortex. Consequently the cyclonic anomalies are not easily axisymmetrized by the parent vortex.

[24]It turns out that the number of updraughts that form initially increases as the horizontal resolution increases, but the subsequent evolutionary picture is largely similar.

[25]The restoring mechanism for vortex Rossby waves and shear instabilities related thereto is associated with the radial and vertical gradient of dry potential vorticity of the system-scale vortex.

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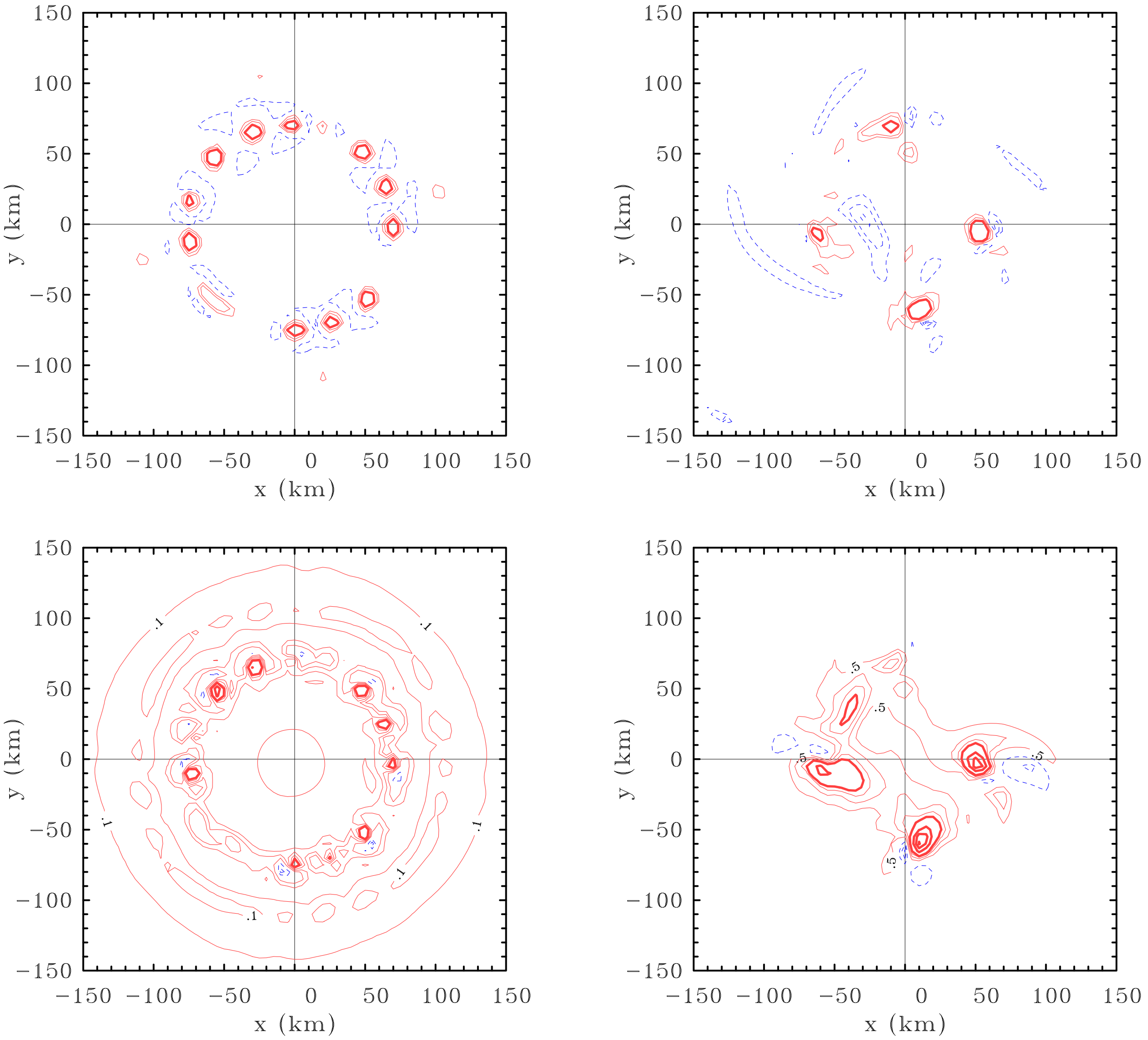

Figure 8. Vertical velocity fields (upper panels) and fields of the vertical component of relative vorticity (lower panels) at 850 hPa at 9.75 hours (left panels) and 24 hours (right panels) in the control experiment in M1. Contour interval for vertical velocity: thick curves 2.0 m s^{-1} and thin solid curves 0.5 m s^{-1} with highest value 1.0 m s^{-1}, thin dashed curves for negative values with interval 0.25 m s^{-1}. Contour interval for relative vorticity: at 9.75 hours, thick curves 5.0×10^{-3} s^{-1} and thin curves 1.0×10^{-4} s^{-1}; at 24 hours, thick curves 2.0×10^{-2} s^{-1} and thin curves 5.0×10^{-3} s^{-1}. Positive values are solid curves in red and negative values are dashed curves in blue. The zero contour is not plotted.

From vortex motion theory applied to a vortex monopole on a beta plane, intense cyclonic vorticity anomalies generally "move up the ambient vorticity gradient" and vice versa for the anticyclonic vorticity anomalies (McWilliams and Flierl 1979, Smith and Ulrich 1990, Schecter and Dubin 1999). In other words, intense cyclonic anomalies immersed in the negative radial vorticity gradient of the parent vortex will tend to move towards the centre and the anticyclonic anomalies will tend to move outwards relative to the low-level inflow. As the parent vortex evolves, the azimuthally-averaged relative vorticity distribution progressively takes on a ring-like structure as convection tries to organize and form an eyewall. Immediately outside of this vorticity ring, the azimuthally-averaged relative vorticity still possesses a negative radial gradient (see lower panels of Figure 9). Therefore, the segregation mechanism continues to operate outside of this ring. The divergent flow above the inflow layer weakens the anticyclonic anomalies leaving them susceptible to axisymmetrization. Thus, during the period of intensification the eddy dynamics associated with the VHTs make a direct contribution to the system-scale spin up (Montgomery *et al.* 2006), resulting in a mesoscale, inner-core region rich in cyclonic relative vorticity.

In a recently published study, Wissmeier and Smith (2011) have shown that significant amplification of ambient low-level vorticity occurs, even for a background rotation rate typical of the undisturbed tropical atmosphere, and even for clouds of only moderate vertical extent. Moreover, it is not necessarily the deepest clouds that produce the maximum amplification, but rather those that

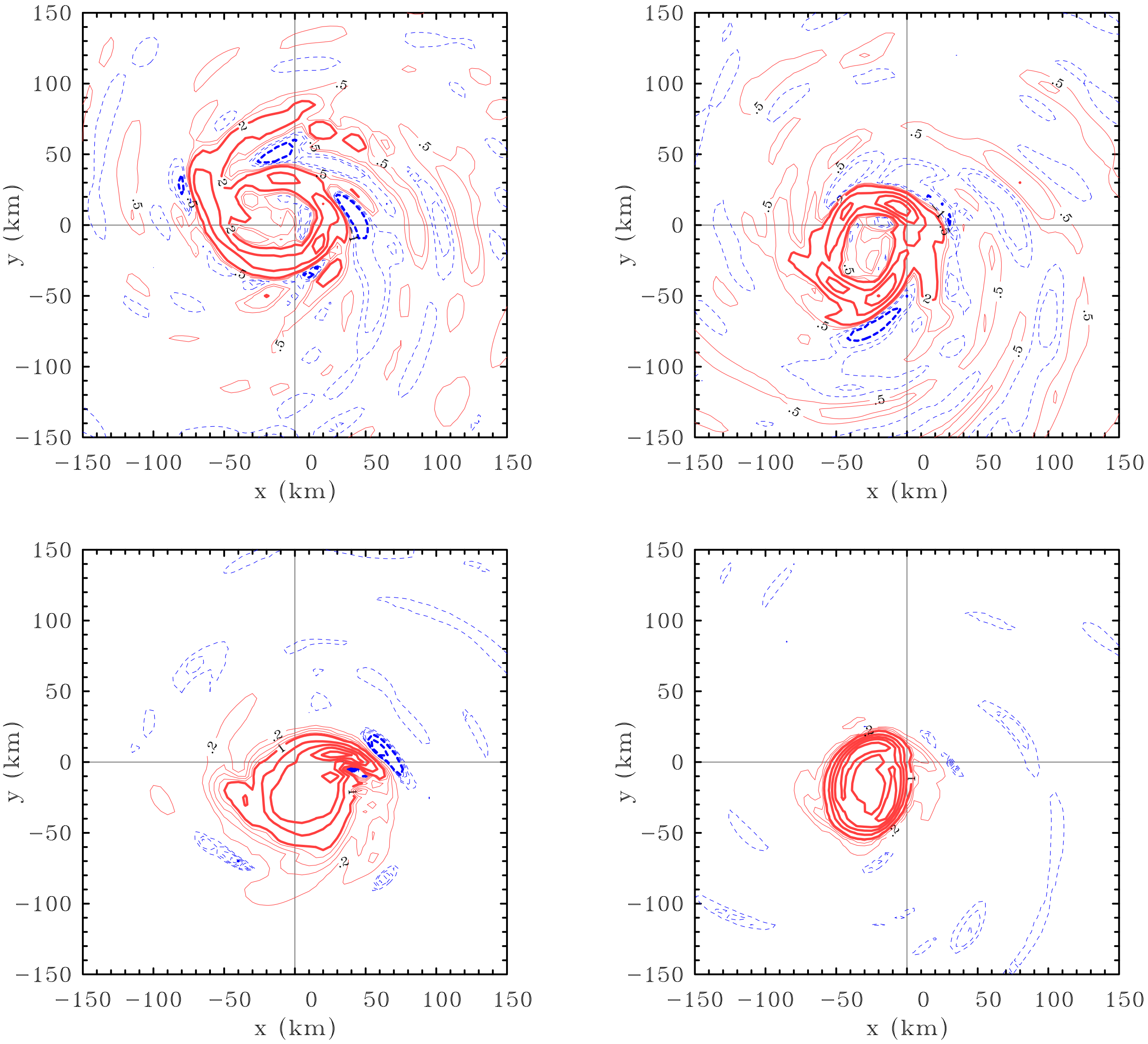

Figure 9. Vertical velocity fields (upper panels) and fields of the vertical component of relative vorticity (lower panels) at 850 hPa at 48 h (left panels) and 96 h (right panels) in the control experiment in M1. Contour interval for vertical velocity: thick curves 4.0 m s^{-1} with lowest absolute value 2.0 m s^{-1} and thin curves 0.5 m s^{-1} with highest absolute value 1.0 m s^{-1}. Contour interval for relative vorticity: thick curves 1.0×10^{-2} s^{-1} and thin curves 1.0×10^{-3} s^{-1}. Positive values are solid curves in red and negative values are dashed curves in blue. The zero contour is not plotted.

have the largest low-level buoyancy. These findings suggest that all non shallow convection away from the equator plays a role in amplifying ambient vorticity, even though the associated tangential winds are small, no more than a few metres per second.

6.3 Observational evidence for VHTs

The relatively recent discovery of VHTs in three-dimensional numerical-model simulations of tropical cyclogenesis and tropical-cyclone intensification has motivated efforts to document such structures in observations. Two early studies were those of Reasor *et al.* (2005), who used airborne Doppler radar data to show that VHTs were present in the genesis phase of Hurricane Dolly (1996), and Sippel *et al.* (2006), who found evidence for VHTs during the development of Tropical Storm Alison (2001). It was not until very recently that Houze *et al.* (2009) presented the first detailed observational evidence of VHTs in a depression that was intensifying and which subsequently became Hurricane Ophelia (2005). The updraught that they documented was 10 km wide and had vertical velocities reaching 10-25 m s^{-1} in its upper portion, the radar echo of which reached to a height of 17 km. The updraught was contiguous with an extensive stratiform region on the order of 200 km in extent. Maximum values of vertical vorticity averaged over the convective region during different fly-bys were on the order of 5-10 $\times 10^{-4}$ s^{-1} (see Houze *et al.* Figure 20).

Bell and Montgomery (2010) analysed airborne Doppler radar observations from the recent Tropical Cyclone Structure 2008 field campaign in the western

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North Pacific and found the presence of deep, buoyant and vortical convective plumes within a vertically-sheared, westward-moving pre-depression disturbance that later developed into Typhoon Hagupit. Raymond *et al.* (2010) carried out a similar analysis of data from the same field experiment, in their case for different stages during the intensification of Typhoon Nuri and provided further evidence for the existence of VHT-like structures.

6.4 Relationship to other rotating convection problems?

From the above results it is evident that the asymmetric paradigm is distinguished from the others by the presence of local, deep vortical convective cells that grow in the rotation-rich environment of the incipient storm and contribute to the vortex-scale circulation. A natural question arises as to whether similar phenomena are encountered in other areas of fluid dynamics. Laboratory and high resolution numerical simulations of high Rayleigh number rotating convection demonstrate that the convective regime is one of vertical plume structures that control principally the vertical heat transfer (e.g. Siggia 1994; Julien *et al.* 1996).

The phenomenology of rotating Rayleigh convection appears most similar to that of dust devils in the earth's atmosphere (e.g., Sinclair 1973) and those observed on Mars in recent years[26]. However, unlike the TC intensification problem the warm vortical structures found in the rotating Rayleigh problem do not aggregate to form a larger-scale vortex structure, but remain largely isolated from one another. At present we lack a general explanation for this significant difference in convective phenomenology. Further examination of this interesting topic would be worthwhile.

6.5 Predictability issues

The association of the inner-core flow asymmetries with the development of VHTs led to the realization that the asymmetries, like the deep convection itself, would have a limited "predictability". The reason is that the pattern of convection is strongly influenced by the low-level moisture field (Nguyen *et al.* 2008, Shin and Smith 2008). Further, observations have shown that low-level moisture has significant variability on small space scales and is not well sampled (Weckwerth 2000), especially in tropical-depression and tropical-cyclone environments.

The sensitivity of the flow asymmetries to low-level moisture in an intensifying model cyclone is demonstrated in Nguyen *et al.* (2008) in an ensemble of ten experiments in which a small (± 0.5 g kg^{-1}) random moisture perturbation is added at every horizontal grid point in the innermost domain of the model below 900 mb. As shown in Figure 10, the evolution in the local intensity as characterized by VT_{max} at 900 hPa is similar in all ensemble members, but there is a non-negligible spread, the maximum difference at any given time in the whole simulation being as high as 20 m s^{-1}.

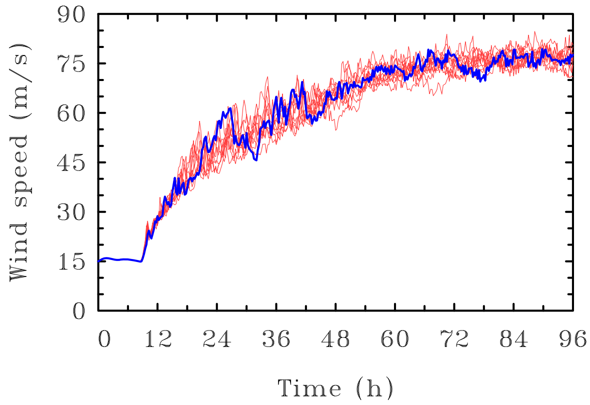

Figure 10. Time-series of maximum total wind speed at 900 hPa in the control experiment in Nguyen *et al.* (2008) (blue) and in the 10 ensemble experiments therein (thin, red).

The pattern of evolution of the flow asymmetries is significantly different between ensemble members also. These differences are exemplified by the relative vorticity fields of the control experiment and two randomly-chosen realizations at 24 hours (compare the two panels in Figure 11 with Figure 8b). In general, inspection of the relative vorticity and vertical velocity fields of each ensemble member shows similar characteristics to those in the control experiment, the field being non-axisymmetric and still dominated by locally intense cyclonic updraughts. However, the detailed pattern of these updraughts is significantly different between the ensemble members.

The foregoing results are supported by the calculations of Shin and Smith (2008), who performed similar ensemble calculations using the minimal three-layer hydrostatic model of Zhu and Smith (2003). Their calculations had twice the horizontal grid spacing used in Nguyen *et al.* (2008), and a much coarser vertical grid spacing. In fact, the calculations indicate a much larger variance between the ensembles than those in Nguyen *et al.*, suggesting that the predictability limits may be resolution dependent also. Since the flow on the convective scales exhibits a degree of randomness, *the convective-scale asymmetries are intrinsically chaotic and unpredictable. Only the asymmetric features that survive in an ensemble average of many realizations can be regarded as robust.*

6.6 Inclusion of warm-rain physics

The simple representation of condensation in the Nguyen *et al.* (2008) calculations neglects one potentially important process in tropical-cyclone intensification, namely the effects of evaporatively-cooled downdraughts. When these are included through a representation of warm rain processes (i.e. condensation-coalescence without ice), the vortex intensification is delayed and the vortex intensifies more slowly than in the control experiment (see, e.g., Figure 12). The intensity after four days is considerably lower

[26] http://science.nasa.gov/science-news/science-at-nasa/2005/14jul_dustdevils/

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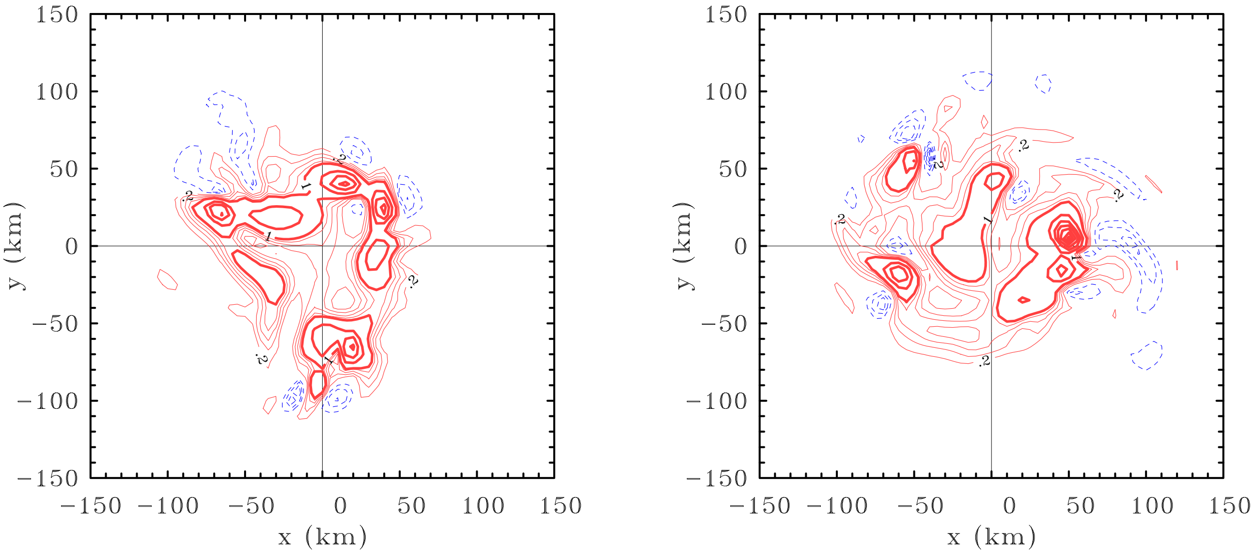

Figure 11. Relative vorticity fields of 2 representative realizations from the f-plane ensemble at 24 h in Nguyen *et al.* (2008). Contour interval is 1.0×10^{-2} s^{-1} (thick curves) and 2.0×10^{-3} s^{-1} (thin curves). Positive values are solid curves in red and negative values are dashed curves in blue. The zero contour is not plotted.

also. Nguyen *et al.* (2008) attributed this lower intensity to a reduction in the convective instability that results from downdraughts associated with the rain process and to the reduced buoyancy in clouds on account of water loading.

6.7 Vortex intensification on the β-plane

In the previous section we showed that, on an f-plane, the flow asymmetries are highly sensitive to the initial moisture distribution. Due in part to the practical connection of hurricane track to storm-scale asymmetries, it is of interest to enquire whether a similar result is true when there is a mechanism to force the asymmetries, such as occurs on a β-plane, or when a vortex is exposed to ambient vertical wind shear. To investigate the former question, Nguyen *et al.* (2008) repeated the foregoing experiments on a β-plane. Although the vortex moves north-northwest as a result of the β effect, the main characteristics in the vortex core region are unchanged and the intensification period is again dominated by VHTs.

The asymmetric forcing implied by the presence of β may be expected to produce an effect similar to a moisture perturbation on the initial pattern of convective cells. In order to determine whether there is a significant difference between the f- and β-plane experiments, *one must compare the average of the ensemble means: it is insufficient to compare just two deterministic experiments*.

Figure 13 shows the ensemble average of the total wind speed and the azimuthally-mean tangential wind speed at 900 hPa as functions of time for the f-plane and β-plane ensemble experiments. The corresponding curves lie close to each other with one higher for some periods and lower for other periods. However, the difference between the two never exceeds 3 m s^{-1}. In particular, the differences lie well inside the intervals of the largest difference between any two members of either the f-plane or β-plane ensemble experiments (indicated by the vertical bars). The spread found in the β-plane set is typically larger (no greater than 5 m s^{-1}) than that in the f-plane case. From these results, Nguyen *et al.* concluded that the β-effect has essentially no impact on the maximum intensity of the vortex during the course of the simulations.

The foregoing result concerning the influence of β on the intensification rate and final intensity differs profoundly from those of Kwok and Chan (2005), Wu and Braun (2004) and Peng *et al.* (1999). These studies found that β caused significant weakening of the final intensity, but they were limited by much coarser horizontal resolution and by the use of parameterized convection.

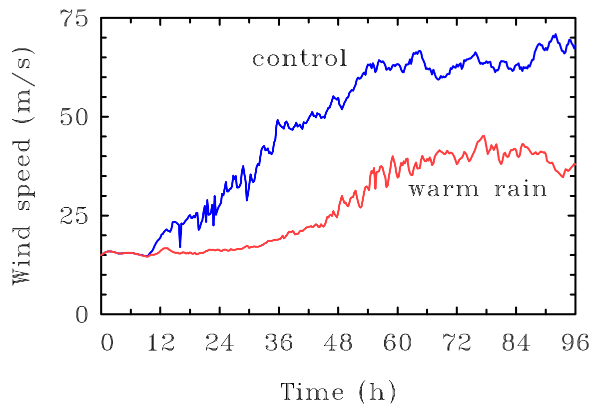

Figure 12. Time-series of azimuthal-mean maximum tangential wind speed at 900 hPa in the control experiment in Nguyen *et al.* (2008) and in the corresponding experiment with a representation of warm rain processes.

6.8 Summary of the new paradigm

The foregoing simulations, both on an f-plane and on a β-plane, show that the VHTs dominate the intensification

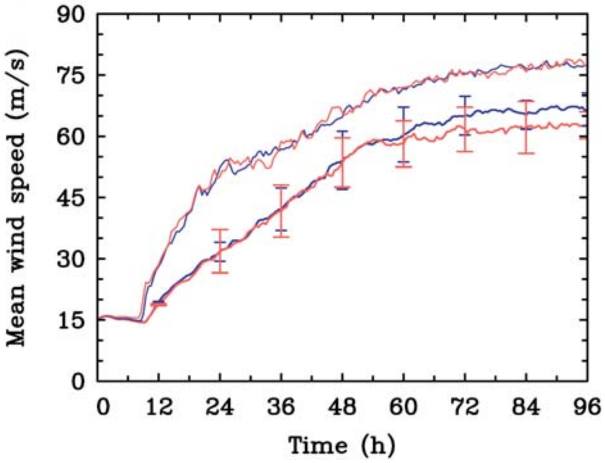

Figure 13. Ensemble average time series of the azimuthally-averaged tangential wind speed (two lower curves) and the maximum local wind speed (two upper curves) in the f-plane (blue line) and β-plane experiments (red line) in Nguyen *et al.* (2008). The corresponding vertical bars mark the maximum differences among ensemble members.

period at early times and that the progressive segregation, merger and axisymmetrization of the VHTs together with the low-level convergence they generate is fundamental to the tropical-cyclone intensification process. The emerging flow asymmetries are not merely a manifestation of the numerical representation of the equations[27], but rather a reflection of a key physical process, namely moist convective instability. The results of Nguyen *et al.* support those of Montgomery *et al.* (2006) in an idealized study of the transformation of a relatively-weak, axisymmetric, middle-level, cold-cored vortex on an f-plane to a warm-cored vortex of tropical-cyclone strength and by those of near cloud-resolving numerical simulations of tropical storm Diana (1984) (Davis and Bosart 2002; Hendricks et al. 2004), which drew attention to the important role of convectively-amplified vorticity in the spin-up process. All of these studies indicate that VHTs are the basic coherent structures of the tropical-cyclone intensification process.

The simulations in Nguyen *et al.* (2008) show that axisymmetrization is never complete and that there is always a prominent low azimuthal-wave-number asymmetry (often with wave number 1 or 2) in the inner-core relative vorticity. Therefore the question arises: *to what extent are the axisymmetric paradigms discussed in sections 4 and 5 relevant to explaining the spin-up in the three-dimensional model?* We address this question in the following section.

[27]It is true that the initial convective updraughts in the control experiment in Nguyen *et al.* (2008) form more or less within an annular envelope inside the radius of maximum tangential winds and that their precise location, where grid-scale saturation occurs first in this annulus, is determined by local asymmetries associated with the representation of an initially symmetric flow on a square grid. However, the updraughts rapidly forget their initial configuration and become randomly distributed.

7 An axisymmetric view of the new paradigm

The new paradigm for tropical-cyclone spin up discussed in the previous section highlights the collective role of rotating deep convection in amplifying the storm circulation. These convective structures are intrinsically asymmetric, but it remains possible that the cooperative intensification theory, the WISHE theory, or a modification of these may still provide a useful integrated view of the intensification process in an azimuthally-averaged sense. With this possibility in mind, Smith *et al.* (2009) examined the azimuthally-averaged wind fields in the control experiment of Nguyen *et al.* (2008) and found that they could be interpreted in terms of a version of the cooperative intensification theory with an important modification. This modification, discussed in this section, recognizes that as intensification proceeds, the spin up becomes progressively focussed in the boundary layer.

Until recently, the perception seems to have been that surface friction plays only an inhibiting role in vortex intensification. For example, Raymond *et al.* (1998, 2007) assume that the boundary layer is generally responsible for spin-down. Specifically, Raymond *et al.* (2007) note that " ... cyclone development occurs when the tendency of convergence to enhance the low-level circulation of a system defeats the tendency of surface friction to spin the system down", while Marin *et al.* 2008 state that: "The primary balance governing the circulation in the planetary boundary layer is between the convergence of environmental vorticity, which tends to spin up the storm, and surface friction, which tends to spin it down".

A similar idea is presented in Kepert's (2010) review on the role of the boundary-layer circulation: "The boundary-layer circulation can only *spin down* (our emphasis) the storm, since inwards advection of (absolute, our insertion) angular momentum by the frictional inflow is countered by the surface frictional torque and by outwards advection in the return branch of the gyre". We find this statement misleading. Indeed, long ago, Anthes (1971) discussed " ... the paradox of the dual role of surface friction, with increased friction yielding more intense circulation ... ". This idea of a dual role is different from that envisaged by Charney and Eliassen (1964) in that it focuses on the dynamical role of the boundary layer as opposed to its thermodynamical role in supplying moisture to feed the inner-core convection. In retrospect, Anthes' statement points to the possibility that the spin up of the inner-core winds might actually occur within the boundary layer. This possibility is further supported by the ubiquitous tendency in numerical models of the vortex boundary layer for supergradient winds to develop in the layer (e.g. Zhang *et al.* 2001, Kepert and Wang 2001, Nguyen *et al.* 2002, Smith and Vogl 2008).

There was already some evidence that spin up occurs in the boundary layer in unpublished calculations performed by our late colleague, Wolfgang Ulrich. Using a simple axisymmetric tropical-cyclone model and performing back trajectory calculations, he found that in all calculations examined, the ring of air associated with the maximum tangential wind speed invariably emanated from the

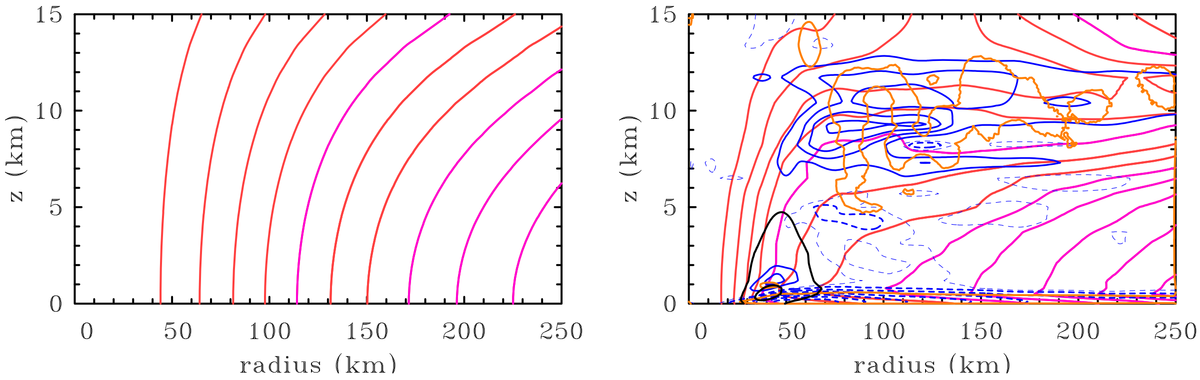

Figure 14. Absolute angular momentum surfaces at (a) the initial time, and (b) at 48 hours (red and pink contours). Contour interval 4×10^5 m^2 s^{-1}. Contour with value 20×10^5 m^2 s^{-1} and those with values $\geq 32 \times 10^5$ m^2 s^{-1} in pink. Shown also in (b) are contours of radial wind component in blue, with contour interval 2 m s^{-1} (the thin dashed contour shows the 1 m s^{-1} inflow), and the 40 and 50 m s^{-1} isotachs of the tangential wind component (solid black contours). The orange contours delineate the the zero value of the discriminant D in Equation (13) and encircle regions in which the azimuthally-averaged flow satisfies the necessary condition for symmetric instability.

boundary layer at some large radius from the storm axis. Numerical simulations of Hurricane Andrew (1992) by Zhang *et al.* (2001) indicated that spin up occurred in the boundary layer of this storm.

7.1 Two spin-up mechanisms

Smith *et al.* 's (2009) analysis indicated the existence of two mechanisms for the spin up of the azimuthal-mean tangential circulation of a tropical cyclone, both involving the radial convergence of M (defined in section 2.4). The first mechanism is associated with the radial convergence of M *above the boundary layer* induced by the inner-core convection as discussed in section 4. It explains why the vortex expands in size and may be interpreted in terms of balance dynamics (see section 7.2).

The second mechanism is associated with the radial convergence of M *within the boundary layer* and becomes progressively important in the inner-core region as the vortex intensifies. Although M is *not* materially conserved in the boundary layer, the largest wind speeds anywhere in the vortex can be achieved in the boundary layer. This happens if the radial inflow is sufficiently large to bring the air parcels to small radii with a minimal loss of M. Stated in another way, the reduction of M in the formula for v is more than offset by the reduction in r[28]. This mechanism is coupled to the first through boundary-layer dynamics because the radial pressure gradient of the boundary layer is slaved to that of the interior flow as discussed in section 2.6[29]

The spin up of the inner-core boundary layer requires the radial pressure gradient to increase with time, which, in turn, requires spin up of the tangential wind at the top of the boundary layer by the first mechanism. The boundary layer spin up mechanism explains why the maximum azimuthally-averaged tangential wind speeds in the model calculations of Smith *et al.* (2009) and in those of other authors (e.g. Braun and Tao 2000, Zhang *et al.* 2001) are located near the top of the boundary layer.

The two mechanisms may be illustrated succinctly by the evolution of the azimuthally-averaged M-surfaces from the control experiment in Nguyen *et al.* (2008) shown here in Figure 14. The left panel depicts the M-surfaces at the initial time of model integration. Certain M surfaces are coloured blue to highlight their inward movement with time. We note that the initial vortex is centrifugally stable so that the M-surfaces increase monotonically outwards. The right panel shows the M-surfaces at 48 hours integration time. Shown also are the radial wind component, the zero contour of the discriminant D defined in Equation (13), and two contours of the tangential wind component. The boundary layer as defined here is the layer of strong radial wind associated with the effect of surface friction. This layer is about 1 km deep.

The inflow velocities above the boundary layer are typically less than 1 m s^{-1} through the lower and middle troposphere. Nevertheless, this weak inflow is persistent, carrying the M surfaces inwards and accounting for the amplification of the tangential winds above the boundary layer. Across the boundary layer, the M surfaces have a large inward slope with height reflecting the removal of M near the surface by friction. Notably, the largest inward displacement of the M surfaces occurs near the top of the boundary layer where frictional stresses are small. This pattern of M surfaces explains why the largest tangential wind speed at a given radius occurs near the top of the boundary layer. Note that the bulge of the innermost blue contour of M is approximately where the tangential wind speed maximum occurs and that this contour has moved radially inwards about 60 km by 48 h.

[28] As discussed in section 2.4, an alternative, but equivalent interpretation for the material acceleration of v follows directly from Newton's second law in which the sole force is the generalized Coriolis force (see footnote 6).

If there is inflow, this force is positive for a cyclonic vortex and this term will contribute to the material acceleration of v. If rings of air can converge quickly enough (i.e. if $|u|$ is sufficiently large), the generalized Coriolis force can exceed the tangential component of frictional force and the tangential winds of air parcels will increase with decreasing radius in the boundary layer. It is precisely for this reason that supergradient winds can arise in the boundary layer.

[29] In Smith *et al.* (2009) these spin up processes were erroneously stated to be independent.

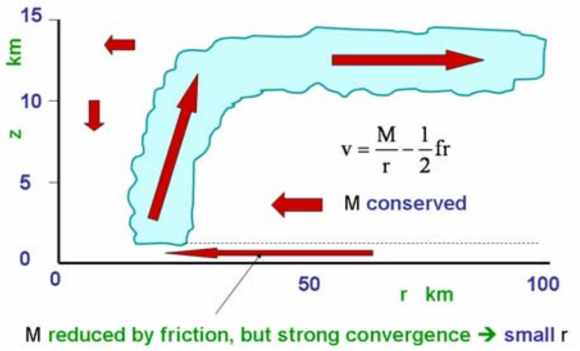

Figure 15. Schematic of the revised view of tropical-cyclone intensification.

There are two regions of strong outflow. The first is just above the inflow layer in the inner-core region, where the tangential velocity is a maximum and typically supergradient. The second is in the upper troposphere and marks the outflow layer produced by the inner-core convection. As air parcels move outwards M is approximately conserved so that the M surfaces are close to horizontal. However, beyond a radius of about 100 km, the M surfaces slope downwards, whereupon the radial gradient of M is negative. Consequently, regions develop in which the D becomes negative. There is a region also in the boundary layer where the D is negative on account of the strong vertical shear of the tangential wind (last term in Equation 13).

The foregoing features are found in all of the numerical simulations we have conducted and lead to a revised view of spin up that includes the cooperative intensification mechanism, but emphasizes the important *dynamical role* of the boundary layer. A schematic of the revised view is illustrated in Figure 15, which is a modification of Figure 4.

The revised axisymmetric view stimulates a number of important questions:

- To what extent can the spin up processes be captured in a balance dynamics framework?
- How important is the WISHE paradigm to vortex spin up in three dimensions?
- How sensitive are the spin up and inner-core flow structure to the parameterization of the boundary layer in a tropical cyclone forecast model?
- How sensitive is spin up to the assumed surface exchange coefficients?

The first question is addressed in the following subsection and the others in section 8.

7.2 An axisymmetric balance view of spin up

The applicability of the axisymmetric balance theory, summarized in section 2.5, to the revised view of tropical-cyclone intensification encapsulated in Figure 15, was investigated in Bui *et al.* (2009). That study built upon those of Hendricks *et al.* (2004) and Montgomery *et al.* (2006), who used a form of the SE-equation to investigate the extent to which vortex evolution in a three-dimensional, cloud-resolving, numerical model of hurricane genesis could be interpreted in terms of balance dynamics. The idea was to diagnose the diabatic heating and azimuthal momentum sources from azimuthal averages of the model output at selected times and to solve the SE-equation for the balanced streamfunction with these as forcing functions (see section 2.3). Then one can compare the azimuthal-mean radial and vertical velocity fields from the numerical model with those derived from the balanced streamfunction. Hendricks *et al.* and Montgomery *et al.* found good agreement between the two measures of the azimuthal-mean secondary circulation and concluded that the vortex evolution proceeded in a broad sense as a balanced response to the azimuthal-mean forcing by the VHTs.

Bui *et al.* (2009) applied the same methodology to the Nguyen *et al.* (2008) calculations on an f-plane, described in section 6.1. A specific aim was to determine the separate contributions of diabatic heating and boundary-layer friction to producing convergence of absolute angular momentum above and within the boundary layer, which, as discussed in section 7.1, are the two intrinsic mechanisms of spin-up in an axisymmetric framework. The study investigated also the extent to which a balance approach is useful in understanding vortex evolution in the Nguyen *et al.* calculations.

The development of regions of symmetric instability in the upper-tropospheric outflow layer and the frictional inflow layer in the Nguyen *et al.* calculations, generally requires the regularization of the SE-equation prior to its solution. A consequence of symmetric instability is the appearance of a shallow layer (or layers) of inflow or reduced outflow in the upper troposphere, both in the three-dimensional model and in the non-regularized balanced solutions, which were examined also by Bui *et al.* (2009). Using this regularization procedure it was found that, during the intensification process, the balance calculation captures a major fraction of the azimuthally-averaged secondary circulation in two main calculations, except in the boundary layer where the gradient wind balance assumption breaks down (see Bui *et al.* Figures 5 and 6). Moreover, the diabatic forcing associated with the inner-core convection negates the divergence above the boundary layer that would be induced by friction alone, producing radial inflow both within the boundary layer and in the lower troposphere (Bui *et al.* 2009, Figure 4).

An important limitation of the balanced solution is that it considerably underestimates both the strength of the inflow and the intensification rate in the boundary layer, a result that reflects its inability to capture the important inertial effects of the boundary layer in the inner core region (Bui *et al.* 2009, Figures 5, 6 & 9). Specifically, these effects include the development of supergradient winds in the boundary layer, the ensuing rapid radial deceleration of the inflow and the eruption of the boundary layer into the interior. Some consequences

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of these important inertial effects are discussed in sections 8.2 and 8.3.

8 Properties of the asymmetric paradigm

8.1 Is WISHE essential?

To explore the relationship of the asymmetric intensification process to that of the evaporation-wind feedback mechanism discussed in section 5, Nguyen *et al.* (2008) conducted a preliminary investigation of the intensification of the vortex when the dependence of wind speed in the bulk aerodynamic formulae for latent- and sensible-heat fluxes was capped at 10 m s^{-1}. This wind-speed cap gave sea-to-air water vapour fluxes that never exceeded 130 W m^{-2}, a value comparable to observed trade-wind values in the summer-hemisphere.

A more in depth study of the role of surface fluxes was carried out by Montgomery *et al.* (2009) and the principal results are summarized in their Figure 9. When the wind-speed dependence of the surface heat fluxes is capped at 10 m s^{-1}, the maximum horizontal wind speed is a little less than in the uncapped experiments, but the characteristics of the vortex evolution are qualitatively similar, in both the pseudo-adiabatic experiments and when warm rain processes are included. In contrast, when the latent and sensible heat fluxes are suppressed altogether, the system-scale vortex does not intensify, even though there is some transient hot-tower convection and local wind-speed enhancement. Despite the non-zero CAPE present in the initial sounding, no system-scale amplification of the wind occurs. This finding rules out the possibility that vortex intensification with capped fluxes is an artefact of using an initially unstable moist atmosphere (cf. Rotunno and Emanuel, 1987). A minimum value of approximately 3 m s^{-1} was found necessary for the wind-speed cap at which the vortex can still intensify to hurricane strength in the pseudo-adiabatic configuration discussed above.

When warm rain processes are included, the vortex still intensifies with a wind-speed cap of 5 m s^{-1}. These experiments suggest that large latent heat fluxes in the core region are not necessary for cyclone intensification to hurricane strength and indicate that the evaporation-wind feedback process discussed in section 5 is not essential.

A word of caution is needed if one tries to interpret the intensification process discussed in section 6 in terms of the schematic of Figure 6. At first sight it might appear that a capped wind-speed in the sea-to-air water vapour flux no longer gives the needed boost in the inner-core boundary layer mixing ratio and concomitant elevation of θ_e required for an air parcel to ascend the warmed troposphere created by prior convective events. The pitfall with this idea is that *it is not necessary to have the water vapour flux increase with wind speed to generate an increase in boundary layer moisture and hence an increase in boundary layer* θ_e. The boundary layer θ_e will continue to rise as long as the near-surface air remains unsaturated at the sea surface temperature. While this is the case, the boundary layer θ_e will continue to increase towards the saturation value at the corresponding sea surface temperature. In some sense, this discussion echoes what was said in section 4 concerning the increase in saturation mixing ratio with decreasing surface pressure. However, the above results add further clarification by showing that the wind-speed dependence of the surface moisture flux is not essential for sustained intensification, even though it may generally augment the intensification process[30].

8.2 Dependence of spin up on the boundary-layer parameterization

A recent assessment of the state-of-the-art Advanced Hurricane Weather Research and Forecasting model (WRF; Skamarock *et al.* 2005) by Davis *et al.* (2007) examined, *inter alia*, the sensitivity of predictions of Hurricane Katrina (2005) to the model resolution and the formulation of surface momentum exchange. Interestingly, it did not consider that there might be a sensitivity also to the boundary-layer parameterization used in the model. This omission reflects our experience in talking with model developers that the choice of boundary-layer parameterization is low on the list of priorities when designing models for the prediction of tropical cyclones: in several instances the modeller had to go away and check exactly which scheme was in use! In view of this situation, and in the light of the theoretical results described above indicating the important role of boundary-layer dynamics and thermodynamics on tropical-cyclone spin-up, we review briefly some work that has attempted to address this issue. Two important questions that arise are: how sensitive are intensity predictions to the choice of boundary-layer parameterization scheme and which scheme is optimum for intensity prediction. These questions have been addressed in the context of case studies (Braun and Tao 2000, Nolan *et al.* 2009a,b) and idealized simulations (Hill and Lackmann 2009, Smith and Thomsen 2010), but the latter question remains to be answered.

Smith and Thomsen (2010) investigated tropical-cyclone intensification in an idealized configuration using five different boundary-layer parameterization schemes available in the MM5 model, but with all the schemes having the *same* surface exchange coefficients. Values of these coefficients were guided by results from the coupled boundary-layer air-sea transfer experiment[31] (CBLAST) to facilitate a proper comparison of the schemes. Like Braun and Tao, they found a significant sensitivity of vortex evolution, final intensity, inner-core low-level wind structure, and spatial distribution of eddy diffusivity, K,

[30] These considerations transcend the issue of axisymmetric versus three-dimensional intensification. While it has been shown that tropical cyclones can intensify without evaporation-wind feedback in an axisymmetric configuration (Montgomery *et al.* 2009, their section 5.3), our focus here is on the three-dimensional problem, which we believe to be the proper benchmark.

[31] See Black *et al.* (2007), Drennan *et al.* (2007), French *et al.* (2007), Zhang *et al.* (2008).

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to the particular scheme used. In particular, the less diffusive schemes have shallower boundary layers and more intense radial inflow, consistent with some early calculations of Anthes (1971) using a constant-K formulation. The stronger inflow occurs because approximately the same surface stress is distributed across a shallower layer leading to a larger inward agradient force in the outer region of the vortex. In turn, the stronger inflow leads to stronger supergradient flow in the inner core.

The sensitivity to K is problematic because the only observational estimates for this quantity that we are aware of are those analysed recently from flight-level wind measurements at an altitude of about 500 m in Hurricanes Allen (1980) and Hugo (1989) by Zhang *et al.* (2011a). In Hugo, maximum K-values were about 110 m^2 s^{-1} beneath the eyewall, where the near-surface wind speeds were about 60 m s^{-1}, and in Allen they were up to 74 m^2 s^{-1}, where wind speeds were about 72 m s^{-1}. Based on these estimates. one would be tempted to judge that the MRF and Gayno-Seaman schemes studied by Braun and Tao and Thomsen and Smith are much too diffusive as they have maximum values of K on the order of 600 m^2 s^{-1} and 250 m^2 s^{-1} respectively. On the other hand, the other schemes have broadly realistic diffusivities.

From the latter results alone, it would be premature to draw firm conclusions from a comparison with only two observational estimates. However, in recent work, Zhang *et al.* (2011b) have analyzed the boundary layer structure of a large number of hurricanes using the Global Positioning System dropwindsondes released from National Oceanic and Atmospheric Administration and United States Air Force reconnaissance aircraft in the Atlantic basin. This work is complemented by individual case studies of Hurricanes and Typhoons examining the boundary-layer wind structure of the inner-core region (Kepert 2006a,b, Montgomery *et al.* 2006, Bell and Montgomery 2008, Schwendike and Kepert 2008, Sanger 2011). These results show that the inner-core boundary-layer depth is between 500 m and 1 km[32]. If the observed boundary layer depth is taken to be an indication of the level of diffusivity, these results imply that boundary layer schemes with deep inflow layers, such as the MRF scheme, are too diffusive and fail to capture the important inertial dynamical processes discussed in section 7.1.

The studies by Braun and Tao and Thomsen and Smith have elevated awareness of an important problem in the design of deterministic forecast models for hurricane intensity, namely which boundary layer scheme is most appropriate? They provide estimates also of forecast uncertainty that follow from the uncertainty in not knowing the optimum boundary-layer scheme to use.

8.3 Dependence of spin up on the drag coefficient

An interesting result emerging from the E86, steady-state, tropical-cyclone model discussed in section 5 is that the square of the maximum swirling wind is proportional to the ratio of the surface exchange coefficients for enthalpy[33], C_K, and momentum, C_D (E86, Emanuel 1995):

$$|V_{max}|^2 = \frac{C_K}{C_D}\left(\frac{T_s - T_o}{T_s}\right)(k_o^* - k), \qquad (18)$$

where T_s is the sea surface temperature, T_o is an entropy-weighted average of the temperature over the outflow layer of the storm, k_o^* is the saturation moist enthalpy at the sea surface temperature and k is the subcloud-layer moist enthalpy, which is assumed to be approximately well mixed. In this expression the dissipative heating effect as formulated by Bister and Emanuel (1998), which replaces the sea-surface temperature with the outflow temperature in the denominator, has not been included.

A particularly noteworthy property of the equation for $|V_{max}|^2$ is that it predicts a monotonic decrease in V_{max} as the drag coefficient increases. Indeed, Emanuel's time-dependent axisymmetric balance model confirms a nearly linear decrease in V_{max} as the drag coefficient increases (see Figure 1 of Emanuel 1995). Another finding from the Emanuel (1995) study was that the development of intense hurricanes (category 3 and higher on the Saffir-Simpson scale) was curbed when the ratio[34] C_K/C_D fell below a value of 0.75, thereby implying a threshold value of C_K/C_D below which incipient tropical depressions will not intensify to major storms[35].

In a complimentary study using the axisymmetric, numerical model of Rotunno and Emanuel (1987), Craig and Gray (1996) found that the rate of intensification increases with increasing values of the transfer coefficients for heat and moisture. They found further that the intensification rate is relatively insensitive to changes in the drag coefficient and noted that “frictional convergence is of secondary importance (for intensification), but may represent a sink of energy that decreases the growth rate.” A particular result of Craig and Gray's study was the finding that the largest intensification rate occurred with no surface friction at all (see page 3537 of their paper).

All of the above results appear to be in conflict with the result of Anthes (1971, p264), who found that an increase in the surface drag coefficient could lead to an increase in intensity rather than a decrease. The reason is related to the behaviour when the (vertical) eddy-diffusivity is reduced, as discussed in section 8.2. An increase in drag could result in an increase in the frictionally-induced inflow and hence the generalized Coriolis force associated with this inflow. This inflow acts to accelerate the tangential flow (section 2.4). This conflict and its resolution are summarized briefly here.

[32] These studies show also that the maximum mean tangential velocity occurs within the boundary layer, supporting the revised conceptual model presented in section 7.1.

[33] The total heat transfer is equivalent to the transfer of moist enthalpy if the transfer coefficients for sensible and latent heat are the same (E86, Emanuel 1995). This equality will be assumed herein.

[34] Emanuel allowed both C_D and C_K to increase with wind speed at the same rate so that their ratio was independent of wind speed

[35] These results assume that dissipation heating is absent. To stay in accord with the set up of the Emanuel (1995) study, the model results summarized in this section do not have dissipative heating switched on.

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The sensitivity of tropical-cyclone models to the surface drag coefficient was examined by Montgomery *et al.* (2010) using a modified version of the three-dimensional model described in section 6. The modifications involved using an approximately moist-neutral environmental sounding and constant surface exchange coefficients. In contrast to the results from the axisymmetric models reviewed above, it was found that the intensification rate and intensity of the simulated tropical cyclones increase with increasing surface drag coefficient until a certain threshold value is attained and then decrease. In particular, tropical-depression strength vortices intensify to major hurricane intensity for values of C_K/C_D as small as 0.1. This value is significantly smaller than the critical threshold value of about 0.75 for major hurricane development predicted by Emanuel (1995) using an axisymmetric model (without dissipative heating). These findings are supported by the calculations cited by Wang and Wu (2004, p269), and those presented in Cram *et al.* (2007) and Hill and Lackman (2009, p762); none of these studies suggest a threshold in C_K/C_D for major hurricane development.

The foregoing behaviour has been verified by Smith and Thomsen (2010) when a wind-speed dependent drag coefficient is included. However, the behaviour is again at odds with the predictions of Emanuel's (*balance*) theory for the maximum intensity of a tropical cyclone, which, as discussed above, predicts a monotonic decrease in intensity with the drag coefficient. The behaviour is at odds also with a recent axisymmetric numerical study of the maximum intensity by Bryan and Rotunno (2009). One reason for this discrepancy may be because the vertical grid spacing of 250 m used in the majority of their calculations that varied the surface drag coefficient is inadequate; another reason may be because the vertical mixing length employed for representing sub-grid scale processes does not yield a physically realistic value for the vertical eddy diffusivity (see section 8.2). Both are necessary to properly represent the structure of the boundary layer and the corresponding unbalanced component of the boundary-layer dynamics as discussed in section 7.1. Indeed, Smith and Thomsen (2010) showed that the foregoing dependence on drag coefficient depends on the boundary-layer scheme used and occurs for the less diffusive schemes. As mentioned in the previous section, the observations of Zhang *et al.* (2011a,b) suggest that the less diffusive schemes most closely reproduce the salient characteristics of the observed hurricane boundary layer structure.

The foregoing findings further highlight the need to adequately represent the boundary layer in order to capture its influence on vortex intensification.

9 Interaction between rotating deep convection, vortex Rossby waves and the mean vortex

As another illustration of the phenomenology associated with the new paradigm, recent work using the Australian Bureau of Meteorology's operational model has shown that, during the intensification stage of a Hurricane Katrina simulation there occurred vacillations between a more symmetric phase and a more asymmetric phase (Nguyen *et al.* 2011). The symmetric phase is characterized by a ring-like structure in the low-level potential vorticity (PV), θ_e and vertical velocity, and is accompanied by a rapid acceleration of the maximum azimuthal-mean tangential wind. In contrast, during the asymmetric phase the low-level PV and θ_e fields exhibit a monopole-like structure, with the corresponding maximum at the vortex centre. Although the maximum in these latter two quantities is at the center of circulation, the asymmetric phase is punctuated by localized deep rotating updrafts with corresponding signatures in vertical velocity, PV and θ_e. Figure 16 gives a representative picture of the vertical velocity structure during the symmetric and asymmetric phases of the simulated intensification process. The asymmetric phase is accompanied by relatively low intensification rates of the azimuthal-mean tangential wind. These two phases are very similar to the two observed regimes reported by Kossin and Eastin (2001), the symmetric phase to their Regime 1 and the asymmetric phase to their Regime 2.

While Kossin and Eastin (2001) propose barotropic instability as an explanation for the transition from Regime 1 to Regime 2, the analysis of Nguyen *et al.* suggests that convective instability couples with the barotropic modes and significantly augments the growth rates. This hybrid instability is an efficient pathway to converting eddy diabatic heating to eddy potential energy. Moreover, a number of processes in the inner core of the tropical cyclone appear to play roles during the vacillation.

1. During the symmetric-to-asymmetric transition, the release of barotropic-convective instability promotes asymmetries within the eyewall in the form of VHTs. These VHTs effectively stir vorticity and θ_e between the eye and the eyewall, bringing the vortex to an asymmetric state with a monopole structure. Consistent with the vorticity arguments of Schubert *et al.* (1999), which highlight the spin up of the eye during these mixing episodes, the minimum pressure falls most rapidly during the asymmetric phase.
2. The asymmetric-to-symmetric transition is accompanied by a weakening of the VHTs as the convective instability is exhausted and the local vertical wind shear increases. The weakened VHTs become stretched bands of moderate convection and move outward as VRWs, which become more axisymmetrized in a strongly sheared mean flow. During this time, convection develops beyond the radius of maximum tangential wind speed in a region of reduced strain and high convective instability.

It has yet to be determined whether these vacillation cycles are a common feature of intensification in other models. The new paradigm provides a framework for such an investigation.

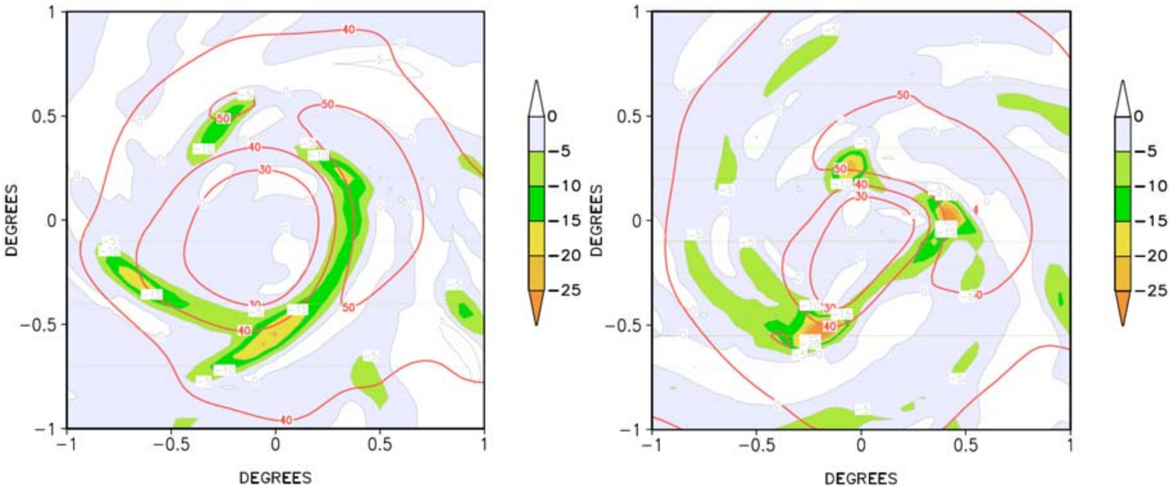

Figure 16. Simulated vortex structure at 850 hPa during symmetric phase 'S1' (left figure) and asymmetric phase 'A2' (right figure) from the recent study by Nguyen et al. (2011). The figures depict vertical velocity in pressure coordinates [$\omega = Dp/Dt$; units of Pa s^{-1}] (updraughts are shaded) and horizontal wind magnitude (contours).

10 Conclusions

In his recent review of tropical-cyclone dynamics, Kepert (2010) states that "the fundamental view of tropical cyclone intensification as a cooperative process between the primary and secondary circulations has now stood for four decades, and continues to underpin our understanding of tropical cyclones." Here we have provided what we think is a broader perspective of the basic intensification process including a new, asymmetric paradigm, not discussed by Kepert.

In the early days of hurricane research, the intensification problem was viewed and modelled as an axisymmetric phenomenon and, over the years, three main paradigms for intensification emerged: the CISK paradigm; the cooperative intensification paradigm; and the WISHE paradigm. We have analysed these paradigms in detail and have sought to articulate the relationship between them. We noted that the process of spin up is similar in all of them, involving the convectively-induced inflow in the lower troposphere, which converges absolute angular momentum to accelerate the tangential wind component. This process is less transparent in the WISHE paradigm, which is usually framed using potential radius coordinates. The main processes distinguishing the various paradigms are the methods for parameterizing deep convection and, in the case of the CISK paradigm, the lack of attention given to surface moisture fluxes in maintaining the boundary-layer moisture and the reliance on ambient CAPE.

The new paradigm is distinguished from those above by the presence of rotating deep convection that grows in the rotation-rich environment of the incipient storm and contributes to the vortex-scale circulation. The processes involved are intrinsically asymmetric and the details thereof possess a stochastic component, which reflects the convective nature of the inner-core region. In this paradigm, it is the progressive segregation, merger and axisymmetrization of these convective towers and the lower-tropospheric convergence they generate that are fundamental to building the system-scale vortex core, but axisymmetrization is never complete. There is always a prominent low azimuthal-wave-number asymmetry (often for azimuthal-wave-number 1 or 2) in the inner-core relative vorticity and other fields. In the mature phase, the evolution of the vortex core is characterized by an approximately axisymmetric circulation superimposed on which are primarily small, but finite-amplitude vortex Rossby waves, eyewall mesovortices, and their coupling to the boundary layer and convection.

The axisymmetric (or 'mean field') view that results after azimuthally averaging the three-dimensional fields provides a new perspective into the axisymmetric dynamics of the spin up process. When starting from a prototypical vortex less than tropical-storm strength, there are two mechanisms for spinning up the mean tangential circulation. The first involves convergence of absolute angular momentum above the boundary layer where this quantity is approximately materially conserved. This mechanism acts to spin up the bulk circulation and is captured in a balance theory wherein the convergence can be thought of as being "driven" by the radial gradient of diabatic heating associated with deep inner-core convection. In particular, it acts to broaden the outer circulation at radii where the boundary-layer flow is subgradient.

The second mechanism involves the convergence of absolute angular momentum within the boundary layer, where this quantity is not materially conserved, but where air parcels are displaced much further radially inwards than air parcels above the boundary layer. Ultimately, this mechanism becomes the dominant one for spinning up the inner-core winds of the intensifying storm and it is accompanied by the development of supergradient wind speeds in the boundary layer. The mechanism is coupled to the first through boundary-layer dynamics because the radial pressure gradient of the boundary layer is slaved to that of the interior flow. The spin up of the inner-core boundary layer requires the radial pressure gradient to

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increase with time, which, in turn, requires spin up of the tangential wind at the top of the boundary layer by the first mechanism. The second spin-up mechanism cannot be represented by balance theory.

In the new paradigm, the underlying fluxes of latent heat are necessary to support the intensification of the vortex. However, the wind-evaporation feedback mechanism that has become the accepted paradigm for tropical-cyclone intensification is not essential, nor is it the dominant intensification mechanism in the idealized numerical experiments that underpin the paradigm.

It is hoped that this new way of thinking about the intensification process will guide the research and modelling community in the development of an improved understanding of observed intensification events.

11 The road ahead

The ability to numerically simulate a hurricane in near real-time with high horizontal resolution using complex microphysical parameterizations can lead to a false sense of understanding without commensurate advances in understanding of the underlying fluid dynamics and thermodynamics that support the intensification process. The situation is elegantly summed up by Ian James who, in reference to the Held-Hou model for the Hadley circulation, writes: “This is not to say that using such [simple] models is folly. Indeed the aim of any scientific modelling is to separate crucial from incidental mechanisms. Comprehensive complexity is no virtue in modelling, but rather an admission of failure” (James 1994, p93). We endorse this view wholeheartedly in the context of tropical-cyclone modelling.

We believe that a consistent dynamical framework of tropical-cyclone intensification is essential to diagnose and offer guidance for improving the three-dimensional hurricane forecast models being employed in the USA and several other countries affected by these deadly storms. To this end, we have provided a review and synthesis of intensification theory for a weak, but finite amplitude, cyclonic vortex in a quiescent tropical atmosphere. However, there is still much work to be done. As intimated in the Introduction, there is a need to investigate the effects of an environmental flow, including one with vertical shear, on the dynamics of the intensification process.

There is a need also to document further the lifecycle of VHTs and the way they interact with one another in tropical-depression environments through analyses of existing field data and those from future field experiments. We see a role also for idealized numerical simulations to complement these observational studies.

There is a need to investigate the limits of predictability of intensity and intensity change arising from the stochastic nature of the VHTs, which has implications for the utility of assimilating Doppler radar data as a basis for deterministic intensity forecasts.

The relative insensitivity of the intensification rate and intensity for drag coefficients typical of high wind-speeds over the ocean calls for a reassessment of the need for a complicated wave component of coupled ocean-wave-atmospheric models to accurately forecast tropical cyclone intensity.

Finally, a strategy is required to answer the question: what is the “optimum” boundary-layer parameterization for use in numerical models used to forecast tropical-cyclone intensity? The current inability to determine “the optimum scheme” has implications for the predictability of tropical-cyclone intensification using current models.

Appendix

E97 proposed an idealized axisymmetric, time-dependent model that would appear to include the WISHE intensification mechanism. A similar formulation has been used by Gray and Craig (1998) and Frisius (2006). In light of the significance of this model in apparently underpinning the WISHE paradigm for intensification as discussed in the main text, it is of interest to examine it more thoroughly here. An expression for the time rate-of-change of tangential wind at the radius of maximum tangential wind derived by E97 (his Eq. (20)) equates the tangential wind tendency to the sum of three terms, two of which are always negative definite. The third term is positive only if the radial gradient of an ‘ad hoc’ parameter is negative; vortex intensification requires the radial gradient of this ad-hoc parameter to be sufficiently negative to offset the other two terms. The ad-hoc parameter is introduced to “crudely represent the effects of convective and large-scale downdraughts, which import low θ_e air into the subcloud layer” (E97, p1019, below Eq. (16)). Under such circumstances, the theory predicts *an intensification rate that decreases monotonically with increasing drag coefficient* (cf. section 8.3). One weakness of the theory is the lack of a rigorous basis for the ad-hoc parameter. A second weakness is the tacit assumption of gradient wind balance in the boundary layer[36] as discussed in footnote 16. Since neither of these assumptions can be defended or derived from first principles, we do not regard this model as the archetype for tropical-cyclone intensification.

In addition to the foregoing problems, we have a further issue with this model. On page 1019 of E97, Emanuel draws attention to the “... crucial presence of downdraughts by reducing the entropy tendency there (outside the radius of maximum tangential wind, our insertion) by a factor β.” However, the WISHE mechanism articulated in section 5.2 did not require downdraughts and moreover we showed in section 6.1 that vortex intensification proceeds optimally in the pseudo-adiabatic case in which downdraughts are absent altogether!

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[36]The scale analysis of the steady-state boundary layer equations performed by Smith and Montgomery (2008) is readily generalized to the intensification problem and shows that the gradient wind balance approximation cannot be justified for an intensifying tropical cyclone.

and a first draft was completed in June of the same year during a visit to Taiwan National University in Taipei. We are grateful to Prof. Phan Van Tan of the VNU and Prof. Chun-Chieh Wu of the NTU for providing us with a stimulating environment for our work. We are grateful also for perceptive reviews of the original manuscript by Kerry Emanuel, Sarah Jones and Dave Raymond.

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13 References

Aberson SD Montgomery MT Bell MM Black ML. 2006 Hurricane Isabel (2003): New insights into the physics of intense storms. Part II - Extreme localized wind. *Bull. Amer. Meteorol. Soc.*, **87,** 1349-1357.

Ahrens CD. 2008 *Meteorology today: An introduction to weather, climate and the environment*. Thomson Brooks/Cole, Stamford, Connecticut, 624pp.

Anthes RA. 1971 Iterative solutions to the steady-state axisymmetric boundary-layer equations under an intense pressure gradient. *Mon. Wea. Rev.*, **99,** 261-268.

Anthes RA. 1974 The dynamics and energetics of mature tropical cyclones. *Rev. Geophys. Phys.*, **12**, 495-522.

Asnani GC. 2005 *Tropical Meteorology, revised edition*. Pune, India.

Bell MM Montgomery MT. 2008 Observed structure, evolution, and potential intensity of Category 5 Hurricane Isabel (2003) from 12 to 14 September. *Mon. Wea. Rev.*, **65**, 2025-2046.

Bell MM Montgomery MT. 2010 Sheared deep vortical convection in pre-depression Hagupit during TCS08. *Geo. Res. Letters*, **37**, L06802.

Bister M Emanuel KA. 1998 Dissipative heating and hurricane intensity. *Meteor. Atmos. Phys.*, **65,** 233-240.

Black PG Marks FD Black RA. 1986 Supercell structure in tropical cyclones. Preprints, Joint Sessions 23d Conf. on Radar Meteorology and Conf. on Cloud Physics, Vol. 3, Snowmass, CO, *Amer. Meteor. Soc.*, JP255-JP259.

Braun SA Tao W-K. 2000 Sensitivity of high-resolution simulations of Hurricane Bob (1991) to planetary boundary layer parameterizations. *Mon. Wea. Rev.*, **128**, 3941-3961.

Braun SA Montgomery MT Pu Z. 2006 High-resolution simulation of Hurricane Bonnie (1998). Part I: The organization of eyewall vertical motion. *J. Atmos. Sci.*, **63**, 19-42.

Braun SA Montgomery MT Mallen KJ Reasor PD. 2010 Simulation and interpretation of the genesis of Tropical Storm Gert (2005) as part of the NASA Tropical Cloud Systems and Processes Experiment. *J. Atmos. Sci.*, **67**, 999-1025.

Broadbent EG Moore DW. 1979 Acoustic destabilization of vortices. *Philos. Trans. Roy. Soc. London A*, **290**, 353-371.

Bryan GH Rotunno R. 2009 The influence of near-surface, high-entropy air in hurricane eyes on maximum hurricane intensity. *J. Atmos. Sci.*, **66**, 148-158

Bui HH Smith RK Montgomery MT Peng J. 2009 Balanced and unbalanced aspects of tropical-cyclone intensification. *Q. J. R. Meteorol. Soc.*, **135,** 1715-1731.

Chan JCL Williams RT 1987 Analytical and numerical studies of the beta-effect in tropical cyclone motion. Part I: Zero mean flow. *J. Atmos. Sci.*, **44**, 1257-1265.

Cram TA Persing J Montgomery MT Braun SA. 2007 A Lagrangian trajectory view on transport and mixing processes between the eye, eyewall, and environment using a high resolution simulation of Hurricane Bonnie (1998). *J. Atmos. Sci.*, **64,** 1835-1856.

Charney JG Eliassen A. 1964 On the growth of the hurricane depression. *J. Atmos. Sci.*, **21**, 68-75.

Chen Y Yau MK. 2001 Spiral bands in a simulated hurricane. Part I: vortex Rossby wave verification. *J. Atmos. Sci.*, **58**, 2128-2145.

Chen Y Brunet G Yau MK. 2003 Spiral Bands in a Simulated Hurricane. Part II: Wave Activity Diagnostics. *J. Atmos. Sci.*, **60** , 1240-1256.

Corbosiero KL Molinari J Aiyyer AR Black ML. 2006 The Structure and Evolution of Hurricane Elena (1985). Part II: Convective Asymmetries and Evidence for Vortex Rossby Waves. *Mon. Wea. Rev.*, **134**, 3073–3091.

Craig GC. Gray SL. 1996 CISK or WISHE as a mechanism for tropical cyclone intensification. *J. Atmos. Sci.*, **53**, 3528-3540.

Davis CA Bosart LF. 2002 Numerical simulations of the genesis of Hurricane Diana (1984). Part II: Sensitivity of track and intensity prediction. *Mon. Wea. Rev.*, **130**, 1100-1124.

Davis CA Wang W Chen SS Chen Y Corbosiero K DeMaria M Dudhia J Holland G Klemp J Michalakes J Reeves H Rotunno R Snyder C Xiao Q. 2008 Prediction of landfalling hurricanes with the advanced hurricane WRF model. *Mon. Wea. Rev.*, **136**, 1990–2005.

Dengler K Reeder MJ. 1997 The effects of convection and baroclinicity on the motion of tropical-cyclone-like vortices. *Q. J. R. Meteorol. Soc.*, **123**, 699-727.

Elsberry R Harr P. 2008 Tropical cyclone structure (TCS08) Field experiment scientific basis, observational platforms, and strategy. Asia-Pacific Journal of Atmospheric Sciences, **44**, 1-23.

Emanuel KA. 1986 An air-sea interaction theory for tropical cyclones. Part I: Steady state maintenance. *J. Atmos. Sci.*, **43**, 585-604.

Emanuel KA. 1988 The maximum intensity of hurricanes. *J. Atmos. Sci.*, **45,** 1143-1155.

Emanuel KA. 1989 The finite amplitude nature of tropical cyclogenesis. *J. Atmos. Sci.*, **46**, 3431-3456.

Emanuel KA Raymond DJ. (Eds.) 1993 *The representation of cumulus convection in numerical models*. American Meteorological Society, Meteorological Monograph, No., **46,** 246pp.

Emanuel KA. 1994 *Atmospheric convection.* Oxford University Press, 580pp.

Emanuel KA Neelin JD and Bretherton CS. 1994 On large-scale circulations of convecting atmospheres. *Q. J. R. Meteorol. Soc.*, **120,** 1111-1143.

Emanuel KA. 1995 Sensitivity of tropical cyclones to surface exchange coefficients and a revised steady-state model incorporating eye dynamics. *J. Atmos. Sci.*, **52**, 3969-3976.

Emanuel KA Speer K Rotunno R Srivastava R Molina M. 1995 Hypercanes: A possible link in global extinction scenarios. *J. Geophys. Res.*, **100**, 13 755-765.

Emanuel, KA. 1997 Some aspects of hurricane inner-core dynamics and energetics. *J. Atmos. Sci.*, **54**, 1014-1026.

Emanuel KA. 2003 Tropical Cyclones. *Annu. Rev. Earth Planet. Sci.*, **31**, 75-104.

Emanuel KA. 2004 Tropical Cyclone Energetics and Structure. In *Atmospheric Turbulence and Mesoscale Meteorology*, E. Fedorovich, R. Rotunno and B. Stevens, editors, Cambridge University Press, pp280.

Fiorino M Elsberry R. 1989 Some aspects of vortex structure related to tropical-cyclone motion. *J. Atmos. Sci.*, **46**, 975-990.

Flatau M Schubert WH Stevens DE. 1994 The role of baroclinic processes in tropical cyclone motion: The influence of vertical tilt. *J. Atmos. Sci.*, **51**, 2589-2601. 1994.

Ford R. 1994a The instability of an axisymmetric vortex with monotonic potential vorticity in rotating shallow water. *J. Fluid Mech.*, **280**, 303-334.

Ford R. 1994b The response of a rotating ellipse of uniform potential vorticity to gravity wave radiation. *Phys. Fluids*, **6**, 3694-3704.

Frisius T. 2002 Surface-flux-induced tropical cyclogenesis within an axisymmetric atmospheric balanced model. *Q. J. R. Meteorol. Soc.*, **132**, 26032623.

Fraedrich K and McBride JL. 1989 Large-scale convective instability revisited. *J. Atmos. Sci.*, **52**, 1914-1923.

Gentry RC Fujita TT Sheets RC. 1970 Aircraft, spacecraft, satellite and radar observations of Hurricane Gladys, 1968. *J. Appl. Meteor.*, **9**, 837850.

Greenspan HP Howard LN. 1963 On a time-dependent motion of a rotating fluid. *J. Fluid Mech.*, **17**, 385-404.

Gray SL Craig GC. 1998 A simple theoretical model for the intensification of tropical cyclones and polar lows. *Q. J. R. Meteorol. Soc.*, **124**, 919-947.

Guinn T Schubert WH. 1993 Hurricane spiral bands. *J. Atmos. Sci.*, **50**, 3380 - 3403.

Hendricks EA Montgomery MT Davis CA. 2004 On the role of "vortical" hot towers in formation of tropical cyclone Diana (1984). *J. Atmos. Sci.*, **61**, 1209-1232.

Hill KA Lackmann GM. 2009 Analysis of idealized tropical cyclone simulations using the Weather Research and Forecasting model: sensitivity to turbulence parameterization and grid spacing. *Mon. Wea. Rev.*, **137**, 745-765.

Holton JR. 1992 *An introduction to dynamic meteorology* Academic Press, London, 507pp.

Holton JR. 2004 *An introduction to dynamic meteorology* Academic Press, London, 535pp.

Houze RA Chen SS Lee W-C Smull BF Bell MM. 2007 Hurricane intensity and eyewall replacement. *Science*, **315**, 1235-1239.

Houze, RA Lee W-C Bell MM. 2009 Convective contribution to the genesis of Hurricane Ophelia (2005). *Mon. Wea. Rev.*, **137**, 2778-2800.

Houze, RA Jr. 2010 Clouds in Tropical Cyclones. *Mon. Wea. Rev.*, **138**, 293-344.

James IN. 1994 *Introduction to circulating atmospheres*, Cambridge University Press, London, 422pp.

James RP Markowski PM. 2010 A numerical investigation of the effects of dry air aloft on deep convection. *Mon. Wea. Rev.*, **138**, 140-161.

Julien K Legg S McWilliams J Werne J. 1996 Rapidly rotating turbulent Rayleigh-Benard convection *J. Fluid Mech.*, **322**, pgs. 243 - 273

Kepert JD. 2001 The dynamics of boundary layer jets within the tropical cyclone core. Part I: Linear Theory. *J. Atmos. Sci.*, **58,** 2469-2484.

Kepert JD. 2006a Observed boundary-layer wind structure and balance in the hurricane core. Part I. Hurricane Georges. *J. Atmos. Sci.*, **63,** 2169-2193

Kepert JD. 2006b Observed boundary-layer wind structure and balance in the hurricane core. Part II. Hurricane Mitch. *J. Atmos. Sci.*, **63,** 2194-2211

Kepert JD. 2010 Tropical cyclone structure and dynamics. In *Global perspectives on Tropical cyclones: From science to mitigation*. (Ed. Chan and Kepert) World Scientific Series on Asia-Pacific Weather and Climate - Vol. 4 448pp

Kepert JD Wang Y. 2001 The dynamics of boundary layer jets within the tropical cyclone core. Part II: Nonlinear enhancement. *J. Atmos. Sci.*, **58,** 2485-2501.

Kleinschmidt K, 1951: Grundlagen einer Theorie der tropischen Zyklonen. *Arch. Meteor. Geophys. Bioklim.*, **A4**, 53-72.

Kossin J Eastin M. 2001 Two distinct regimes in the kinematic and thermodynamic structure of the hurricane eye and eyewall. *J. Atmos. Sci.*, **58** 1079–1090.

Kossin J McNoldy BD Schubert WH. 2002 Vortical swirls in hurricane eye clouds. *Mon. Wea. Rev.*, **130**, 3144-3149.

Kurihara Y Tuleya RE. 1974 Structure of a tropical cyclone developed in a three-dimensional numerical simulation model. *J. Atmos. Sci.*, **31,** 893-919.

Kurihara Y Bender M. 1982 Structure and analysis of the eye of a numerically simulated tropical cyclone. *J. Meteor. Soc. Japan*, **60**, 381-395.

Kwok HY Chan JCL. 2005 The influence of uniform flow on tropical cyclone intensity change. *J. Atmos. Sci.*, **62**, 3193-3212.

Lighthill J. 1998 Fluid mechanics of tropical cyclones. *Theoret. Comput. Fluid. Dyn.*, **10,** 321.

Lorosolo S Zhang JA Marks FD Gamache J. 2009 Estimation and mapping of hurricane turbulent energy using airborne Doppler measurements. Submitted to *Mon. Wea. Rev.*,

Malkus JS. 1958 On the structure and maintenance of the mature hurricane eye. *J. Meteor.*, **15**, 337-349.

Malkus JS Riehl H. 1960 On the dynamics and energy transformations in steady-state hurricanes. *Tellus*, **12**, 1-20.

McWilliams JC Flierl R. 1979 On the evolution of isolated nonlinear vortices. *J. Phys. Oceanogr.*, **9**, 1155-1182.

McWilliams JC Graves P Montgomery MT. 2003 A formal theory for vortex Rossby waves and vortex evolution. *Geophys. Astro. Fluid Dynamics*, **97**, 275-309.

Möller JD Smith RK. 1994 The development of potential vorticity in a hurricane-like vortex. *Q. J. R. Meteorol. Soc.*, **120,** 1255-1265.

Möller JD Montgomery MT. 2000 Tropical cyclone evolution via potential vorticity anomalies in a three-dimensional balance model. *J. Atmos. Sci.*, **57**, 3366-3387.

Möller JD Shapiro LJ. 2002 Balanced contributions to the intensification of Hurricane Opal as diagnosed from a GFDL model forecast. *Mon. Wea. Rev.*, **130,** 1866-1881.

Marks FD Shay LK. 1998: Landfalling tropical cyclones: Forecast problems and associated research opportunities. *Bull. Amer. Meteor. Soc.*, **79**, 305-323.

Marks FD Black PG Montgomery MT Burpee RW. 2008 Structure of the Eye and Eyewall of Hurricane Hugo (1989). *Mon. Wea. Rev.*, **136**, 1237-1259.

Marin JC Raymond DJ Raga GB. 2009 Intensification of tropical cyclones in the GFS model. *Atmos. Chem. Phys. Discuss.*, **8**, 17803-17839.

Martinez, YH. 2008 Diagnostic study of hurricane asymmetries using empirical normal modes. Ph.D. thesis, McGill University, 165 pp.

Martinez YH Brunet G Yau MK Wang X. 2011 On the Dynamics of Concentric Eyewall Genesis: SpaceTime Empirical Normal Modes Diagnosis. *J. Atmos. Sci.*, **68**, 458-476.

Molinari J Skubis S Vollaro D. 1995 External influences on hurricane intensity. Part III: Potential vorticity structure. *J. Atmos. Sci.*, , **52**, 3593-3606.

Montgomery MT Kallenbach RJ. 1997 A theory for vortex Rossby waves and its application to spiral bands and intensity changes in hurricanes. *Q. J. R. Meteorol. Soc.*, **123**, 435-465.

Montgomery MT Enagonio J. 1998 Tropical cyclogenesis via convectively forced vortex Rossby waves in a three-dimensional quasi-geostrophic model. *J. Atmos. Sci.*, **55**, 3176-3207.

Montgomery MT Vladimirov VA Denissenko PV. 2002 An experimental study on hurricane mesovortices. *J. Fluid. Mech.*, **471**, 1-32.

Montgomery MT Nicholls ME Cram TA Saunders AB. 2006 A vortical hot tower route to tropical cyclogenesis. *J. Atmos. Sci.*, **63**, 355-386.

Montgomery MT Nguyen SV Smith RK Persing J. 2009 Do tropical cyclones intensify by WISHE? *Q. J. R. Meteorol. Soc.*, **135,** 1697-1714.

Montgomery MT Smith RK Nguyen SV. 2010: Sensitivity of tropical cyclone models to the surface exchange coefficients *Q. J. R. Meteorol. Soc.*, **136**, 19451953.

Nguyen CM Smith RK Zhu H Ulrich W. 2002 A minimal axisymmetric hurricane model. *Q. J. R. Meteorol. Soc.*, **128**, 2641-2661.

Nguyen SV Smith RK Montgomery MT. 2008 Tropical-cyclone intensification and predictability in three dimensions. *Q. J. R. Meteorol. Soc.*, **134**, 563-582.

Nguyen CM Reeder MJ Davidson NE Smith RK Montgomery MT. 2011 Inner-core vacillation cycles during the intensification of Hurricane Katrina *Q. J. R. Meteorol. Soc.*, **137**, in press.

Nolan DS Montgomery MT. 2000 The algebraic growth of wavenumber one disturbances in hurricane-like vortices, *J. Atmos. Sci.*, **57**, 3514-3538.

Nolan DS Montgomery MT Grasso LD. 2001 The wavenumber-one instability and trochoidal motion of hurricane-like vortices. *J. Atmos. Sci.*, **58**, 3243-3270.

Nolan DS Zhang JA Stern DP. 2009a Evaluation of planetary boundary layer parameterizations in tropical cyclones by comparison of in-situ observations and high-resolution simulations of Hurricane Isabel (2003). Part I: Initialization, maximum winds, and the outer core boundary layer. *Mon. Wea. Rev.*, **137**, 3651-3674.

Nolan DS Zhang JA Stern DP. 2009b Evaluation of planetary boundary layer parameterizations in tropical cyclones by comparison of in-situ observations and high-resolution simulations of Hurricane Isabel (2003). Part II: Inner core boundary layer and eyewall structure. *Mon. Wea. Rev.*, **137**, 3675-3698.

Nong S Emanuel KA. 2004 A numerical study of the genesis of concentric eyewalls in hurricanes. *Q. J. R. Meteorol. Soc.*, **129,** 3323-3338.

Ooyama KV. 1964 A dynamical model for the study of tropical cyclone development. *Geophys. Int.*, **4,** 187-198.

Ooyama KV. 1969 Numerical simulation of the life cycle of tropical cyclones. *J. Atmos. Sci.*, **26**, 3-40.

Ooyama KV. 1982 Conceptual evolution of the theory and modeling of the tropical cyclone. *J. Meteor. Soc. Japan*, **60,** 369-380.

Ooyama, KV. 1997 Footnotes to 'Conceptual Evolution'. Extended Abstracts, 22nd Conference on Hurricanes and Tropical Meteorology, *American Meteorological Society, Boston*, 13-18.

Peng MS Jeng B-F Williams RT. 1999 A numerical study on tropical cyclone intensification. Part I: Beta effect and mean flow effect. *J. Atmos. Sci.*, **56**, 1404-1423.

Persing J Montgomery MT. 2003 Hurricane superintensity. *J. Atmos. Sci.*, **60**, 2349-2371.

Plougonven R Zeitlin V. 2002 Internal gravity wave emission from a pancake vortex: An example of wave vortex interaction in strongly stratified flows. *Phys. Fluids*, **14**, 1259-1268.

Powell MD Uhlhorn EW Kepert JD. 2009 Estimating maximum surface winds from hurricane reconnaissance measurements. *Wea. Forecasting*, **24**, 868-883.

Rauber R M Walsh J E Charlevoix DJ *Severe and Hazardous Weather. An Introduction to High Impact Meteorology*. Third Edition. Kendall/Hunt Publishing Company, 642 pp.

Raymond DJ Emanuel KA. 1993 The Kuo cumulus parameterization. In *The representation of cumulus convection in numerical models*. Meteorological Moonograph No. 46, American Meteorological Society, Boston, Mass. USA, pp145-147.

Raymond DJ López-Carrillo C Cavazos LL. 1998 Case studies of developing east Pacific easterly waves. *Q. J. R. Meteorol. Soc.*, **124**, 2005-2034.

Raymond DJ López Carillo C. 2010 The vorticity budget of Typhoon Nuri (2008). Atmos. Chem. Phys. Discuss., **10**, 16589-16635.

Reasor PD Montgomery MT. 2001 Three-dimensional alignment and co-rotation of weak TC-like vortices via linear vortex Rossby waves. *J. Atmos. Sci.*, **58**, 2306-2330.

Schwendike J Kepert JD. 2008 The boundary layer winds in Hurricane Danielle (1998) and Isabel (2003). *Mon. Wea. Rev.*, **136**, 3168-3192.

Reasor PD Montgomery MT Bosart L. 2005 Mesoscale observations of the genesis of hurricane Dolly (1996). *J. Atmos. Sci.*, **62**, 3151-3171.

Riehl H. 1954 *Tropical Meteorology*. McGraw-Hill, 392pp.

Riemer M Montgomery MT Nicholls ME 2010 A new paradigm for intensity modification of tropical cyclones: Thermodynamic impact of vertical wind shear on the inflow layer, *Atmos. Chem. and Phys.*, **10**, 3163-3188.

Rotunno R Emanuel KA. 1987 An air-sea interaction theory for tropical cyclones. Part II: Evolutionary study using a nonhydrostatic axisymmetric numerical model. *J. Atmos. Sci.*, **44**, 542-561.

Rotunno R Chen W Wang W Davis C Dudhia J Holland GH. 2009 Large-eddy simulation of an idealized tropical cyclone. *Bull. Amer. Meteor. Soc.*, bf 90, 1783-1788.

Sanger N. 2011 An Observational Study of Tropical Cyclone Spinup in Supertyphoon Jangmi and Hurricane Georges. Ph.D. Dissertation. U.S. Naval Postgraduate School. Monterey, California, 120 pp.

Schecter DA Dubin DHE. 1999 Vortex motion driven by a background vorticity gradient. *Phys. Rev. Letters*, **83**, 2191-2194.

Schecter DA Montgomery MT. 2003 On the symmetrization rate of an intense geophysical vortex. *Dyn. Atmos. Oceans*, **37**, 55-88.

Schecter DA Montgomery MT. 2004 Damping and pumping of a vortex Rossby wave in a monotonic cyclone: Critical layer stirring versus inertiabuoyancy wave emission. *Phys. Fluids*, **16** , 1334-1348.

Schecter DA Montgomery MT. 2006 Conditions that inhibit the spontaneous radiation of spiral inertia-gravity waves from an intense mesoscale cyclone. *J. Atmos. Sci.*, **63**, 435-456.

Schecter DA Montgomery MT. 2007 Waves in a cloudy vortex. *J. Atmos. Sci.*, **64**, 314 - 337.

Schecter DA. 2008 The spontaneous imbalance of an atmospheric vortex at high Rossby number. *J. Atmos. Sci.*, **65**, 2498-2521.

Schubert WH Alworth BT. 1987 Evolution of potential vorticity in tropical cyclones. *Q. J. R. Meteorol. Soc.*, **113**, 147-162.

Schubert WH Rozoff CM Vigh JL McNoldy BD Kossin JP. 2007 On the distribution of subsidence in the hurricane eye. *Q. J. R. Meteorol. Soc.*, **133**, 1-20.

Schubert WH Montgomery, MT Taft RK Guinn TA Fulton SR Kossin JP Edwards JP. 1999 Polygonal eyewalls, asymmetric eye contraction, and potential vorticity mixing in hurricanes. *J. Atmos. Sci.*, **56**, 1197-1223.

Schwendike J Kepert JD. 2008 The boundary layer winds in Hurricane Danielle (1998) and Isabel (2003). *Mon. Wea. Rev.*, **136**, 3168-3192.

Shapiro LJ Willoughby H. 1982 The response of balanced hurricanes to local sources of heat and momentum. *J. Atmos. Sci.*, **39**, 378-394.

Shapiro LJ. 1983 The asymmetric boundary layer flow under a translating hurricane. *J. Atmos. Sci.*, **40**, 1984-1998.

Shapiro LJ Montgomery MT. 1993 A three-dimensional balance theory for rapidly-rotating vortices. *J. Atmos. Sci.*, **50**, 3322-3335.

Shin S Smith RK. 2008 Tropical-cyclone intensification and predictability in a minimal three-dimensional model. **134**, 1661-1671.

Siggia ED. 1993 High Rayleigh number convection. *Ann. Rev. Fluid Mech.*, **26—**, 137-168.

Sinclair, PC. 1973 The lower structure of dust devils. *J. Atmos. Sci.*, **30**, 1599-1619.

Sippel JA Nielsen-Gammon JW Allen SE. 2006: The multiple vortex nature of tropical cyclogenesis. *Mon. Wea. Rev.*, **134**, 1796-1814.

Skamarock WC Klemp JB Dudhia J Gill DO Barker DM Wang W Powers JG. 2005: A description of the Advanced Research WRF: Version 2. NCAR Tech. Note 468 1 STR, 88pp.

Smith RK. 1980 Tropical cyclone eye dynamics *J. Atmos. Sci.*, **37**, 1227-1232.

Smith RK Ulrich W. 1990 An analytical theory of tropical cyclone motion using a barotropic model. *J. Atmos. Sci.*, **47**, 1973-1986.

Smith RK Ulrich W Dietachmayer G. 1990 A numerical study of tropical cyclone motion using a barotropic model. Part I. The role of vortex asymmetries. *Q. J. R. Meteorol. Soc.*, **116**, 337-362.

Smith RK Ulrich W. 1993 Vortex motion in relation to the absolute vorticity gradient of the environment. *Q. J. R. Meteorol. Soc.*, **119**, 207-215.

Smith RK. 1997 On the theory of CISK. *Q. J. R. Meteorol. Soc.*, **123,** 407-418.

Smith RK. 2000 The role of cumulus convection in hurricanes and its representation in hurricane models. *Rev. Geophys.*, **38,** 465-489.

Smith RK Montgomery MT Zhu H. 2005 Buoyancy in tropical cyclones and other rapidly rotating vortices. *Dyn. Atmos. Oceans*, **40**, 189-208.

Smith RK. 2006 Accurate determination of a balanced axisymmetric vortex. *Tellus*, **58A**, 98-103.

Smith RK Vogl S. 2008 A simple model of the hurricane boundary layer revisited. *Q. J. R. Meteorol. Soc.*, **134**, 337-351.

Smith RK Montgomery MT and Vogl S. 2008 A critique of Emanuel's hurricane model and potential intensity theory. *Q. J. R. Meteorol. Soc.*, **134**, 551-561.

Smith RK Montgomery MT. 2008 Balanced boundary layers in hurricane models. *Q. J. R. Meteorol. Soc.*, **134**, 1385-1395.

Smith RK Montgomery MT Nguyen SV. 2009 Tropical cyclone spin up revisited. *Q. J. R. Meteorol. Soc.*, **135,** 1321-1335.

Smith RK Montgomery MT. 2010 Hurricane boundary-layer theory. *Q. J. R. Meteorol. Soc.*, **136**, 1665-1670.

Smith RK Thomsen GL. 2010 Dependence of tropical-cyclone intensification on the boundary-layer representation in a numerical model. *Q. J. R. Meteorol. Soc.*, **136**, 1671-1685.

Smith RK Montgomery MT Thomsen GL. 2010 Sensitivity of tropical cyclone models to the surface drag coefficient in different boundary-layer schemes. *Q. J. R. Meteorol. Soc.*, (submitted).

Sundqvist H. 1970 Numerical simulation of the development of tropical cyclones with a ten-level model. Part I. *Tellus*, **22**, 359-389.

Tang B Emanuel KA. 2010 Mid-level ventilation's constraint on tropical-cyclone intensity. *J. Atmos. Sci.*, **67**, 1817-1830.

Terwey WD Montgomery MT. 2008 Secondary eyewall formation in two idealized, full-physics modeled hurricanes. *J. Geophys. Res.*, **113**, D12112.

Vogl S Smith RK. 2009 A simple model of the hurricane boundary layer revisited. *Q. J. R. Meteorol. Soc.*, **135**, 337-351.

Wang Y Holland GJ. 1996a The beta drift of baroclinic vortices. Part I: Adiabatic vortices. *J. Atmos. Sci.*, **53**, 411-427.

Wang Y Holland GJ. 1996b The beta drift of baroclinic vortices. Part II: Diabatic vortices. *J. Atmos. Sci.*, **53**, 3737-3756.

Wang Y. 2001 An explicit simulation of tropical cyclones with a triply nested movable mesh primitive equation model: TCM3, Part I: Model description and control experiment. *Mon. Wea. Rev.*, **129**, 1370-1394.

Wang Y. 2002a Vortex Rossby waves in a numerical simulated tropical cyclone. Part I: Overall structure, potential vorticity, and kinetic energy budgets. *J. Atmos. Sci.*, **59**, 1213-1238.

Wang Y. 2002b Vortex Rossby waves in a numerical simulated tropical cyclone. Part II: The role in tropical cyclone structure and intensity changes. *J. Atmos. Sci.*, **59**, 1239-1262.

Wang Y Wu C-C. 2004 Current understanding of tropical cyclone structure and intensity changes - a review. *Meteorol. Atmos. Phys.*, **87**, 257-278.

Wang, Y. 2008 Rapid filamentation zone in a numerically simulated tropical cyclone model. *J. Atmos. Sci.*, **65**, 1158-1181.

Weatherford CL Gray WM. 1988 Typhoon structure as revealed by aircraft reconnaissance. Part I. Data analysis and climatology. *Mon. Wea. Rev.*, **116**, 1032-1043.

Weckwerth TM. 2000 The effect of small-scale moisture variability on thunderstorm initiation. *Mon. Wea. Rev.*, **128**, 4017-4030.

Willoughby HE. 1979 Forced secondary circulations in hurricanes. *J. Geophys. Res.*, **84**, 3173-3183.

Willoughby HE. 1988 The dynamics of the tropical cyclone core. *Aust. Met. Mag.*, **36**, 183-191.

Willoughby HE. 1990 Gradient balance in tropical cyclones. *J. Atmos. Sci.*, **47**, 465-489.

Willoughby HE. 1995 Mature structure and evolution. In *Global Perspectives on Tropical Cyclones. pp21-62. WMO/TD-No 693 (Ed. R. L. Elsberry), World Meteorological Organization, Geneva*, 289pp.

Wissmeier U Smith RK. 2011: Tropical-cyclone convection: the effects of ambient vertical vorticity. *Q. J. R. Meteorol. Soc.*, **137**, (in press).

Wu L Braun SA. 2004 Effects of environmentally induced asymmetries on hurricane intensity: a numerical study. *J. Atmos. Sci.*, **61**, 3065-3081.

Yano J-I Emanuel KA. 1991 An improved model of the equatorial troposphere and its coupling with the stratosphere. *J. Atmos. Sci.*, **48**, 377-389.

Zhang D-L Liu Y Yau MK. 2001 A multi-scale numerical study of Hurricane Andrew (1992). Part IV: Unbalanced flows. *Mon. Wea. Rev.*, **129**, 92-107.

Zhang J Drennan WM Black PB French JR. 2009 Turbulence structure of the hurricane boundary layer between the outer rainbands. *J. Atmos. Sci.*, **66**, 2455-2467.

Zhang JA Marks FD Montgomery MT and Lorsolo S. 2011a An Estimation of Turbulent Characteristics in the Low-Level Region of Intense Hurricanes Allen (1980) and Hugo (1989). *Mon. Wea. Rev.*, in press.

Zhang JA Rogers RF Nolan DS and Marks FD. 2011b On the characteristic height scales of the hurricane boundary layer. *Mon. Wea. Rev.*, in press.

Zhu H Smith RK Ulrich W. 2001 A minimal three-dimensional tropical cyclone model. *J. Atmos. Sci.*, **58**, 1924-1944.

Zhu H Smith RK. 2002 The importance of three physical processes in a three-dimensional tropical cyclone model. *J. Atmos. Sci.*, **59**, 1825-1840.

Zhu H Smith RK. 2003 Effects of vertical differencing in a minimal hurricane model. *Q. J. R. Meteorol. Soc.*, **129**, 1051-1069.