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On: "A balanced model of a hurricane vortex coupled to a boundary layer" by Robert J. Beare and Mike J. P. Cullen

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Abstract:
In this comment we pose some questions about the physics embodied in a recent axisymmetric balance model formulation that seeks to remedy a deficiency of the traditional axisymmetric Eliassen balance vortex model for hurricane-like vortices.

KEY WORDS An axisymmetric balance model for tropical cyclones

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1 Comment

In their paper, Beare and Cullen (2023) have sought to extend the gradient wind balance approach of Bui et al. (2009) as a basis for "understanding hurricane dynamics and synoptic-scale vortices in middle latitudes". They claim to have "improved the balance theory of hurricane dynamics (Bui et al. 2009) by including an explicit boundary layer balance". They summarize their theory as being "analogous to the Cartesian balance theory of Beare and Cullen (2013), but now extended to cylindrical polars including the centrifugal terms" and improving on Bui et al. (2009) "by introducing extra positive definiteness to the circulation equation and thus solvability in the boundary layer". In their Conclusions, Beare and Cullen concede that "unbalanced motions are still present in real cases (Smith and Montgomery 2008) ... ", but state that their balanced perspective "provides an invaluable starting point to *understanding the flow* (our emphasis)". With these facts in mind, we have a number of questions about the physics embodied in their hurricane solution. These are enumerated below.

(1) It is unclear to us whether this theory is for an evolving vortex or one that is in a steady state. Certainly, no metrics of time evolution are shown.
(2) If the vortex is evolving, is it spinning up or spinning down? From Figs. 4a and 5a showing the radial velocity and streamfunction for the overturning circulation, it is clear that there is mostly outflow above the boundary layer top (defined to be 1.5 km for radii at and beyond the radius of maximum tangential velocity) - see point 4 also. This would imply that the vortex would be spinning down almost everywhere, especially at the top of the boundary layer, unless there is an implicit source of cyclonic absolute angular momentum in the mathematical formulation (Smith et al. 2024). If there is no substantive source of cyclonic angular momentum in the formulation and the vortex is spinning down, in what way does this formulation improve the balance theory of hurricane dynamics by Bui et al. (2009)? After all, the main objective of that paper was *to better understand vortex spin up in realistic tropical cyclones using the traditional axisymmetric Eliassen balance vortex model*.
(3) If the vortex is in a steady state, is the overturning circulation along absolute angular momentum surfaces above the boundary layer? Moreover, what is the source of cyclonic angular momentum to maintain this steady state (e.g., Smith et al. 2014)?
(4) Given that the maximum diabatic heating rate, shown in Fig. 3a, occurs in the middle troposphere, the traditional Eliassen balance model would predict inflow below the heating rate maximum provided that the boundary layer inflow is not overly intense. However, in the present balance formulation, such a deep inflow structure does not occur. Although there is evidence of a small region of inflow aloft (Fig. 5a) just below the diabatic heating maximum (Fig. 3a), the theory predicts mostly outflow above the boundary layer. In particular, the outflowing air depicted by the streamfunction field near the radius and height of the tangential wind maximum (Fig 1a) suggests that the maximum tangential wind would spin down. Is it the case that the diabatic heating rate is too weak to ventilate mass at the rate it is converging in the boundary layer (cf. Smith and Wang 2018)?

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(5) Figure 2b shows a radius height distribution of the potential temperature anomaly calculated for the imposed hurricane-like vortex as given by Eq. (10). The deduced temperature anomaly that solves their modified thermal wind balance equation is given by Eq. (20). This temperature anomaly has a warm maximum of about 10 K located at the ocean surface near the center of circulation and the anomaly declines with height. However, observations of real hurricanes are well known to have a maximum temperature anomaly and potential temperature anomaly in the upper troposphere near the axis (e.g., Smith and Montgomery 2023, their Fig. 1.9b). Does the unrealistic temperature structure in the model have implications for its integrity?

(6) It is well known that inflow in a vortex boundary layer is a result of an inward agradient force brought about by the frictional disruption of approximate gradient wind balance above the boundary layer (e.g., Smith and Montgomery 2023, Chapter 6): the boundary layer is *not* in gradient wind balance. What, then, is the physical justification for seeking a more sophisticated balance formulation for the boundary layer in a hurricane vortex? Does the improved "solvability in the boundary layer" of the Beare and Cullen model come at the expense of the physical realism of the boundary layer formulation? What is the physical mechanism for driving inflow in this model in the boundary layer?

(7) What is the physical significance of having a diabatic heating rate of 12.5 K per hour *in* the boundary out to large radial distances?

(8) What are the implications of the no slip boundary condition on providing a realistic drag formulation at the sea surface? The no-slip boundary condition could have important implications for the ability for the boundary layer air to be ventilated by the prescribed diabatic heating rate (point 4).

We look forward to Beare and Cullen's answers to the foregoing questions, which would be helpful in appraising their new model formulation.

References

Beare, H. H., and M. Cullen, 2023: A balanced model of a hurricane vortex coupled to a boundary layer. *Quart. Journ. Roy. Meteor. Soc.*, **149**, 3290—-3300.

Bui, H. H., R. K. Smith, M. T. Montgomery, and J. Peng, 2009: Balanced and unbalanced aspects of tropical-cyclone intensification. *Quart. Journ. Roy. Meteor. Soc.*, **135**, 1715–1731.

Smith, R. K., and M. T. Montgomery, 2008: Balanced depth-averaged boundary layers used in hurricane models. *Quart. Journ. Roy. Meteor. Soc.*, **134**, 1385–1395.

Smith, R. K., and M. T. Montgomery, 2023: *Tropical cyclones: Observations and basic processes*. Elsevier, London, 411pp.

Smith, R. K., M. T. Montgomery, and J. Persing, 2014: On steady state tropical cyclones. *Quart. J. Roy. Meteor. Soc.*, **140**, 2638–2146.

Smith, R. K., M. T. Montgomery, and S. Wang, 2024: Can one reconcile the classical theories and the WISHE-theories of tropical cyclone intensification? *Tropical Cyclone Research and Review*, **14**, 105–118.

Smith, R. K., and S. Wang, 2018: Axisymmetric balance dynamics of tropical cyclone intensification: Diabatic heating versus surface friction. *Quart. Journ. Roy. Meteor. Soc.*, **144**, 2350–2357.