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On the physics of a new time-dependent theory of tropical cyclone intensification

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Abstract:
Two recent papers (Wang et al. 2021; Li et al. 2024) have developed a new axisymmetric, time-dependent theory of tropical cyclone intensification. Here, we examine the physics of this new theory and point out that intensification in the model has to be the result of an unspecified source of absolute angular momentum. For this reason, we are led to question the physical integrity of the theory. We question also the methodology seeking to tune the unknown parameters introduced in the theory.

KEY WORDS A new tropical cyclone model; classical intensification models; model tuning

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1 Introduction

In two recent papers, Wang et al. (2021) and Li et al. (2024) have developed what they describe as a new time-dependent theory of tropical cyclone intensification. The new theory is based on an extension of that developed by Emanuel (2012), albeit with some notable differences that we detail below. Unfortunately, the authors fall short of providing a conceptual model for the physics of vortex spin up in the new theory. In particular, they do not explain how the model differs from the classical intensification models of Ooyama (1969), Shapiro and Willoughby (1982) and Schubert and Hack (1983), summarized by Smith and Montgomery (2023), Chapter 8. This paper seeks to remove these shortcomings, an endeavour that reveals a number of concerns with the new theory.

2 The new intensification theory in brief

The Emanuel (2012) model, on which the new intensification theory is based, is summarized and appraised by Montgomery and Smith (2019) and in Section 12.3 of Smith and Montgomery (2023). Figure 1 shows the salient features of the assumed flow configuration in radius-height coordinates (r, z). Air is assumed to converge in a shallow frictional boundary layer of constant depth h, acquiring moisture from the surface as it does so. As air parcels ascend out of this layer at inner radii, they are assumed to flow upwards and radially outwards into the upper troposphere, ostensibly[1] conserving their values of saturation specific entropy, s^*, and absolute angular momentum, M, defined as $rv + \frac{1}{2}fr^2$, where r is the radius, v is the tangential velocity component and f is the Coriolis parameter, assumed constant.

The master prognostic equation is one for the moist entropy, s_b, in the boundary layer. It is assumed further that $s^* = s_b$ and that M is continuous at $z = h$. Key assumptions above the boundary layer are that the flow is in hydrostatic and gradient wind balance, implying thermal wind balance, and that the M- and s^*-surfaces are congruent *globally and at all times*. Finally, a closure assumption is made in the upper-tropospheric outflow by introducing a parameterization of the partial derivative of outflow temperature T_o with respect to M, i.e., $\partial T_o/\partial M$, to the derivative of s^* with respect to M, i.e., $\partial s^*/\partial M$. This parameterization is based on the premise that the thermal stratification of the outflow, $\partial T_o/\partial M$, is set by small-scale turbulence that limits the Richardson number to a critical value.

The main outcomes of the theory are an equation for the intensification rate, $\partial V_m/\partial\tau$, of the maximum gradient wind, V_m, given by[2]

$$\frac{\partial V_m}{\partial \tau} = \underbrace{\frac{C_D}{2h}\frac{Ri_c}{r_t^2}M^2}_{\text{turbulent mixing term}} - \frac{C_k}{2h}V_m^2, \tag{1}$$

and, with some further approximations, an analytical solution for V_m. In this equation, τ is the time, C_D

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[1]See Section 3.
[2]See Eq. (A17) of Montgomery et al. (2019).

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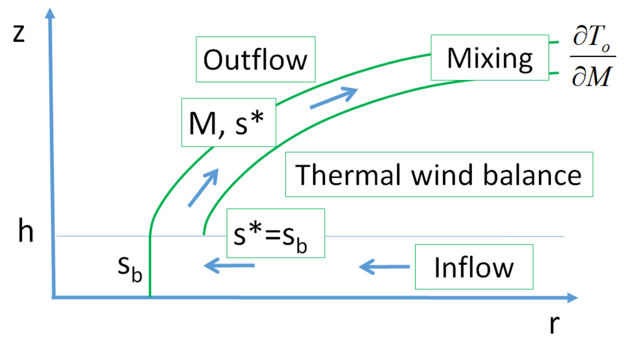

Figure 1. Our interpretation of the geometric configuration of the axisymmetric Emanuel (2012) formulation for an intensifying tropical cyclone in cylindrical polar coordinates (r, z). The depth of the layer influenced by friction is denoted by h, which is assumed constant. Other quantities include: the specific entropy in the boundary layer, s_b; the saturation specific entropy above the boundary layer, s^*; and the absolute angular momentum, M. It is assumed that s_b is independent of height. The arrows indicate the secondary circulation with radial inflow in the friction layer and outflow above it. Mixing by shear-stratified turbulence in the outflow layer of the developing tropical cyclone is assumed to determine the thermal stratification of the outflow ($\partial T_o/\partial M$, where T_o is the outflow temperature). (Adapted from Fig. 8 of Emanuel 2000)

and C_k are the surface drag and enthalpy coefficients, respectively, both assumed constant, Ri_c is the critical Richardson number that determines the onset of turbulent mixing in the upper-tropospheric outflow, and r_t is the radius at which the upper-tropospheric mixing begins[3].

It follows from Eq. (1) that an increase of V_m with time requires the first term on the right-hand-side, which contains the effect of the parameterized turbulent mixing, to be sufficiently large. This is because the second term on the right is negative definite. While the reader may be asking at this point how turbulent mixing in the upper-tropospheric outflow leads to vortex intensification, it was shown by Montgomery and Smith (2019) that the mixing itself is irrelevant to the theory.

2.1 The new modifications

The modifications of the Emanuel (2012) theory made by Wang et al. (2021) are significant. Swayed, presumably, by the analysis of Montgomery and Smith (2019), these authors abandoned the mixing formulation in preference to a slight modification of the previous closure introduced by Emanuel (1995), Emanuel (1997) and Emanuel (2000). However, it was the problematic nature[4] of this closure that motivated the development of the upper-level mixing theory of Emanuel (2012).

In the new theory, the time tendency in the equation for boundary-layer moist entropy, $\partial s_b/\partial t$, is omitted and an unknown *ad hoc* parameter, β is introduced into the equation, ostensibly to represent some kind of quasi-equilibrium of the boundary-layer thermodynamics. Finally, recognizing the lack of realism of assuming global congruence of the M and s^* surfaces above the boundary layer during much of the intensification phase (Peng et al. 2018), Wang et al. introduce another quantity A', which is taken to represent the degree to which the M and s^*-surfaces *are not congruent*. With these modifications, they derive an equation analogous to Eq. (1) for the intensification rate of the maximum gradient wind speed, V_m. In this equation (their Eq. (16)), the first term on the right-hand-side involves the quantity A, which is the product of the two unknown quantities A' and β.

Despite the introduction of the parameter A' in the equation for the upper boundary condition, the formulation of this boundary condition, itself, seems to be a retrogressive step in the light of the foregoing remarks of Emanuel and Rotunno (2011) and Emanuel (2012). Moreover, for much of the evolution, it is clear from the solutions of Peng et al. (2018), Smith and Montgomery (2024) and others, that A' has a marked spatial dependence. Therefore, it seems unrealistic to characterize the angle between M and s* surfaces with a parameter that has no spatial dependence.

Notwithstanding the fact that Wang et al. have derived a theory for V_m that is uninfluenced by the flow structure of the vortex outside the M-surface that passes through the location of V_m,[5] it would seem pertinent to enquire about the physics of vortex spin up (i.e., the amplification of V_m) in the new theory. This topic is addressed in the next section.

3 The physics of the new intensification theory

Since the geometrical configuration of the new intensification theory is the same as that in the Emanuel theories in Fig. 1, with radial inflow in a shallow boundary layer and outflow in the free troposphere above, the same question arises as to where V_m is located and how it is amplified. These questions were answered in a recent paper by Smith et al. (2024). There, it was argued that in a balanced warm-cored vortex, V_m must be located at the top of the

[3]The radius r_t is unknown *a priori* and is not determined by the theory: it must be prescribed. In this sense, the theory is not a closed theory for intensification.

[4]Emanuel (2012) states: "Recently, Emanuel and Rotunno (2011) demonstrated that in numerically simulated tropical cyclones, the assumption of constant outflow temperature is poor and that, in the simulations, the outflow temperature increases rapidly with angular momentum". In fact, the latter authors state: "The poor solutions that result when constant outflow temperature is assumed, together with the great sensitivity of the solutions to the stratification of the upper troposphere when it is assumed to be positive, motivate a reexamination of the problem".

[5]Note that they admit this fact in their subsequent paper, Li et al. (2024).

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boundary layer. Moreover, V_m can amplify only if the M-surfaces in its neighbourhood at the top of the boundary layer move radially inwards.

Wang et al. (2021) state on page 3858 that the inward movement of the M-surfaces is "driven by surface enthalpy flux under the eyewall", but they do not elaborate on how this driving process works. Note, that the tendency equation for M, implicit in their Eq. (5), does not explicitly involve a term representing the surface enthalpy flux under the eyewall. Therefore, their statement calls for further elaboration, especially since the radial momentum equation is not used in their boundary layer formulation.

If M were materially-conserved at and above the boundary layer top, there would have to be inflow at that level to amplify V_m. However, in the geometry shown in Fig. 1 (and in Fig. 8 of Emanuel 2000), there would appear to be no inflow at this level, or at least a discontinuity in the radial flow. If there is no inflow, for the M-surfaces to move inwards at the location of V_m, there must be a material source of M for V_m to amplify. On the other hand, a discontinuity in the radial flow at the boundary layer top would imply a discontinuity of M there, violating the tacit assumption that v and therefore M are continuous at the top of the boundary layer.

As shown by Smith et al. (2024) in an appendix, the hypothesized global congruence in the Emanuel models does not guarantee either the material conservation of M or of s^*, allowing for material sources of both quantities. To our knowledge, the spatial distribution of these sources and their time evolution have never been calculated and the physical realism of these sources would seem to be highly questionable. The same remarks must be true of the new intensification theory, even when a degree of non-congruence is allowed for by the introduction of the spatially constant parameter A'. Significantly, *nowhere* in formulation of the new Wang et al. model are equations included to ensure the material conservation of M or of s^* at the top of, or above, the boundary layer. This was the reason for introducing footnote 1 regarding the use of the word "ostensibly".

If one accepts the hypothesis outlined by Smith and Montgomery (2024) that the material conservation of M above the frictional boundary layer is the most fundamental principle for understanding the spin up of concentrated vortices, in general, theories which allow for the existence of an unspecified angular momentum source must be considered suspect. It is significant that the classical intensification models referred to in the Introduction are all based on the notion that M is materially conserved above the boundary layer. In these models, deep convection induces radial inflow in a layer above the boundary layer and the inward motion of the M-surfaces in these models is inextricably tied to this radial inflow.

The foregoing situation is not the case in the Emanuel models or their extensions by Wang et al. (2021) and Li et al. (2024) that are the focus of this article. In these models, the inflow is confined to the frictional boundary layer, where M is not materially conserved.

4 The comparison with a full-physics model

It is interesting to ponder the practice of using a full-physics, axisymmetric cloud model to evaluate the new axisymmetric intensification theory as is done by Wang et al. (2021) and Li et al. (2024). Such a comparison presumes that the physics contained in the numerical model and in the model being compared are essentially the same. However, the numerical model does not enforce the same congruence or partial congruence assumption. Moreover, apart from diffusive effects, the numerical model seeks to materially conserve both M and s^* above the frictional boundary layer. In contrast, as noted above, the new intensification theory does not materially conserve M or s^* above the boundary layer. It is therefore unclear what to deduce from the comparison. Although the use of the numerical model to tune the new theory conveys the impression that the theory is sound, this verification procedure seems highly suspect.

5 Conclusions

We have sought to articulate the physics of the new axisymmetric, time-dependent theory for tropical cyclone intensification by Wang et al. (2021) and Li et al. (2024). The analysis raises a number of detailed concerns we have with the formulation of the theory, the most serious being that, as in the Emanuel (2012) model on which it is based, vortex intensification must be the result of an unspecified source of absolute angular momentum. For this reason alone, we are led to question the physical integrity and applicability of the new theory.

Concerns are raised also about the practice of using a full-physics, axisymmetric cloud model to evaluate and validate the new intensification theory.

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