TROPICAL CYCLONE RESEARCH REPORT
TCRR **0**: 1–3 (2020)
Meteorological Institute
Ludwig Maximilians University of Munich

Comments on: How much does the upward advection of supergradient component of boundary-layer wind contribute to tropical cyclone intensification and maximum intensity? by Yuanlong Li, Yuqing Wang and Yanluan Lin

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KEY WORDS Tropical cyclones

Date: June 19, 2020; Revised ; Accepted

1 Introduction

In a recent paper, Li et al. (2020) carried out an extensive ensemble of axisymmetric numerical simulations to examine the "the importance of supergradient winds in TC (tropical cyclone, our insertion) intensification", claiming that this topic "is still under debate". In their Introduction they state: "One view is that the spinup of the eyewall occurs by the upward advection of high tangential momentum associated with supergradient winds from the boundary layer. The other view argues that the upward advection of supergradient winds by eyewall updrafts results in an outward agradient force, leading to the formation of a shallow outflow layer immediately above the inflow boundary layer". One might ask why these are considered to be "separate views"? One could argue that they are part of the same picture, irrespective of the degree to which the ascending air is supergradient. If the air that exits the boundary layer is supergradient, it must surely move outwards. What other force would make the air move inwards against the positive agradient force (which includes, of course, the radial pressure gradient force)?

Their paper seems to be motivated by arguments presented by Schmidt and Smith (2016) and Montgomery and Smith (2017), who did indeed argue that the spinup of the eyewall occurs by the upward advection of high tangential momentum associated with supergradient winds from the boundary layer, a result that Li et al. seem to regard as debatable. However, much of their paper is based on a misinterpretation of these arguments that leads them to carry out an ensemble of numerical model simulations that we suggest are physically unrealistic. As a preliminary to appraising their simulations, it may be helpful to review the arguments presented by Schmidt and Smith (2016) and Montgomery and Smith (2017).

In a cylindrical coordinate system (r, λ, z), with r the radius, λ the azimuth and z the height, the tendency equation for the tangential velocity component v in an axisymmetric vortex may be written as:

$$\frac{\partial v}{\partial t} = -(\zeta + f)u - w\frac{\partial v}{\partial z} + F_\lambda, \tag{1}$$

where u and w are the radial and vertical velocity components, respectively, t is the time and F_λ represents the frictional and/or sub grid scale diffusion of tangential momentum. This equation simply expresses the azimuthal component of Newton's second law of motion[1]. Assuming that, above the frictional boundary layer, F_λ can be neglected, the only way that v can increase locally in a cyclonic vortex $(\zeta + f > 0)$ when the radial flow is outwards $u > 0$ is if the vertical advection of tangential momentum $-w\partial v/\partial z$ is positive and exceeds the radial flux of absolute vorticity, $(\zeta + f)u$ in magnitude. This result seems so basic, it is hard to imagine why Li et al. would consider it to be "still

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[1] The equation can be written alternatively as one for the absolute angular momentum, $M = rv + \frac{1}{2}fr^2$, i.e.

$$\frac{\partial v}{\partial t} = -\frac{u}{r}\frac{\partial M}{\partial z} - \frac{w}{r}\frac{\partial M}{\partial z} + rF_\lambda. \tag{2}$$

Writing the right-hand-side in vector form shows that for v to increase locally, there must be a component of flow across the M surfaces towards low M so that the first two terms on the right-hand-side are positive and outweigh the friction term rF_λ, which for an axisymmetric cyclonic (Northern Hemisphere) flow must be negative.

under debate”. Moreover, it is hard to imagine also why an ensemble of numerical experiments is required to investigate it further.

Li et al.’s misunderstanding appears to result from the use of the word “supergradient”, perhaps because the second author has played down the nonlinear dynamics that generate such winds in the boundary layer as well as the role of these winds elsewhere (Heng and Wang 2016; Heng et al. 2017). They do acknowledge that as air parcels move radially outwards while approximately conserving their absolute angular momentum, their tangential wind component will slow down. In particular, they argue that, if air parcels have any supergradient component of motion as they exit the boundary layer, they will rapidly adjust back to close gradient balance before ascending far into the eyewall. On this basis, and supported by their interpretations of their ensemble experiments, they argue that the upward advection of high tangential momentum associated with the *supergradient component of the winds* from the boundary layer “should *not* (our emphasis) be a dominant mechanism of TC intensification” (see their conclusions). Here, however, it is unclear whether by “intensification” they refer to spin up of the eyewall or simply to the spin up of the near surface winds.

The fact that the eyewall can spin up *without* the winds exiting the boundary layer being supergradient is not in doubt and has been demonstrated by the occurrence of vortex spin up in prognostic axisymmetric balance models such as those described by Ooyama (1969); Schubert and Alworth (1982); Emanuel (1989); Smith et al. (2018); Smith and Wang (2018)[2]. However, this spin up has to be accomplished by an inward flux of absolute vorticity in the lower troposphere, since the tangential winds in the boundary layer of a balance model are strictly subgradient (e.g. Smith and Montgomery 2008) so that the vertical advection of tangential momentum from this layer will make a negative contribution to spinning up the eyewall.

In a further effort to show that the eyewall can be spun up without the winds exiting the boundary layer being supergradient, Li et al. carry out an ensemble of axisymmetric model experiments in which the upward advection of the supergradient part of the tangential momentum ascending out of the boundary layer is suppressed. However, they do not appear to have noticed that by suppressing this supergradient component, they are, in effect, introducing a ring of negative impulsive torque to the tangential momentum equation. This torque would appear as an extra term on the right hand side of Eq. (1).

Li et al.’s description of their ensemble simulations is perfectly consistent with what one might anticipate such a torque would do. For example, the eyewall in these simulations is close to upright in the lower troposphere rather than being tilted with height as is observed (e.g., Marks et al. 2008, Fig. 2a), a result of suppressing the outward component of agradient force associated with the tangential wind being supergradient. For this eyewall to spin up, the gradient wind in the boundary layer would need to strengthen with time, but Li et al. do not explain by what mechanism this strengthening occurs.

It is difficult to see what one can learn about the real world by such unrealistic thought experiments, since air ascending in real storms does not experience a ring of negative torque as it exits the boundary layer. In fact, in a three-dimensional configuration, Eq. (1) would be one for the azimuthally-averaged tangential wind tendency and would include eddy advection terms on the right-hand-side. Numerical model simulations show that these terms make a positive contribution to eyewall spin up and the vertical advection of eddy tangential momentum is the dominant positive term (Persing et al. 2013, Montgomery et al. 2020). This effect amounts to a positive torque after the eddy terms are multiplied by radius rather than the tacit negative torque imposed by Li et al.. Thus Li et al.’s experiments are perturbing the tangential momentum equation by the wrong sign in reference to the proper three-dimensional benchmark.

If Li et al.’s conclusion that the supergradient component of the winds from the boundary layer “should *not* (our emphasis) be a dominant mechanism of TC intensification” refers also to the eyewall, it would be reasonable to ask what then is the “main mechanism” for the spin up of the eyewall in axisymmetric hurricanes when the air exiting the boundary layer is flowing outwards? We could not find a convincing answer to this question in their paper.

As a final remark, we draw attention to the results of several studies showing that the tangential wind in the eyewall is supergradient through the depth of the troposphere (Zhang et al. 2001, their Fig. 7b; Montgomery et al. 2020, Figs. 4a,b; Wang et al. 2020, Fig. 5b). All these studies showed that the agradient force is positive throughout most of the eyewall and the assumption that the supergradient winds adjust rapidly back to gradient wind balance just as the air exits the top of the boundary layer during storm spin up and maturity is not correct.

Acknowledgements

GK acknowledges support of the German Research Council (DFG) under grant number KI-2248. MTM acknowledges the support of NSF grant AGS-1313948, IAA-1656075, ONR grant N0001417WX00336, and the U. S. Naval Postgraduate School. The views expressed herein are those of the authors and do not represent sponsoring agencies or institutions.

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[2] Curiously, none of these references are cited in Li et al.!

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