Comments on: Revisiting the balanced and unbalanced aspects of tropical cyclone intensification, by J. Heng, Y. Wang and W. Zhou

Michael T. Montgomery,

Department of Meteorology, Naval Postgraduate School, Monterey, CA USA

and Roger K. Smith

[*] *Meteorological Institute, Ludwig-Maximilians University of Munich, Munich, Germany*

---

[*] *Corresponding author address:* Prof. M. T. Montgomery, Department of Meteorology, Naval Postgraduate School, Monterey, CA USA.

E-mail: mtmontgo@nps.edu

1. Introduction

Recent work has led to the development of a new paradigm of tropical cyclones intensification, called the rotating convection paradigm (Smith and Montgomery 2016; Montgomery and Smith 2017; Smith et al. 2017 and refs.) The new paradigm represents an overarching framework for interpreting the complex vortex-convective phenomenology in simulated and observed tropical cyclones. The paradigm explicitly recognizes the presence of localized, rotating deep convection whose vorticity is amplified greatly over initial values by vortex-tube stretching and tilting processes in the cyclonic seed environment of an incipient storm. The paradigm includes both the azimuthally-averaged fluid dynamics and thermodynamics (with eddy covariance terms) and local asymmetric or eddy processes. One attribute of the new model is an explanation for how the maximum tangential winds generally reside in the boundary layer. The explanation highlights the important role of nonlinear boundary layer processes in the rapidly rotating core of a developing storm. The paradigm highlights also the progressive control exerted by the boundary layer as a storm intensifies (Kilroy et al. 2016, 2017, 2018a,b; Kilroy and Smith 2017). The importance of the nonlinear boundary layer spin up mechanism has been challenged in the recent paper by Heng et al. (2017). The purpose of our comment is to critically evaluate this challenge and to refute it.

2. A critique of the Heng et al. (2017) study

In their paper, Heng et al. (2017) report on an idealized numerical simulation of a tropical cyclone in a quiescent environment, which they use to investigate the degree to which the azimuthally-averaged circulation can be interpreted in terms of balance dynamics. The stated motivation is to re-examine the results of Bui et al. (2009), who compared the azimuthally-averaged solutions derived from an idealized numerical simulation of a tropical cyclone in a likewise quiescent environment with those obtained by solving the Sawyer-Eliassen balance equations forced by diabatic and tangential frictional forcing distributions diagnosed from the numerical simulation. In Bui et al. and Heng and Wang, the term *balance* refers to an axisymmetric flow regime comprising gradient wind balance in the radial coordinate direction and hydrostatic balance in the vertical coordinate direction. The Sawyer-Eliassen balance formulation used by these (and other) authors assumes that these force balances prevail throughout the vortex, including the frictional boundary layer.

Bui et al. (2009) showed that the balanced calculations capture a major fraction of the azimuthally-averaged secondary circulation of the three-dimensional simulation except in the boundary layer, where the gradient balance assumption breaks down and where there is an inward agradient force. In particular, the Sawyer-Eliassen balance theory was shown to significantly underestimate the low-level radial inflow and therefore the maximum azimuthal-mean tangential wind tendency.

Heng et al. (2017) claim to demonstrate that Bui et al. (2009)'s findings are incorrect and that " ... balanced dynamics can well capture the secondary circulation in the full-physics model simulation *even in the inner-core region in the boundary layer* (italics are our emphasis)". This is a surprising claim in itself, since it is generally well known that strong inflow in the inner core boundary layer is a result of gradient wind imbalance (Smith 1968; Carrier 1971; Anthes 1971; Kuo 1971; Anthes 1974; Shapiro 1983; Kepert and Wang 2001; Slocum et al. 2014; Montgomery et al. 2014), an imbalance that is strongly supported by a scale analysis of the boundary layer equations (e.g. Smith and Montgomery 2008; Vogl and Smith 2009). In contradiction to this fact, as early as the first paragraph of their Introduction, Heng et al. (2017) assert that " ... the primary circulation [of a tropical cyclone: our insertion] remains nearly in gradient wind balance as a slowly evolving system". No caveat is given to point out that this statement is invalid for the boundary layer or the upper-tropospheric outflow region of the developing vortex.

Since the foregoing claims are a major departure from the currently accepted understanding of tropical cyclone dynamics, they call for close scrutiny. Indeed, if Heng et al. (2017)'s findings were correct, it would call into question the veracity of a large number of studies which have found otherwise (e.g. Zhang et al. 2001; Huang et al. 2012; Rotunno and Bryan 2012; Gopalakrishnan et al. 2013a,b).

Heng et al. (2017) take issue not only with the results of Bui et al. (2009), but also with those in a related paper by Smith et al. (2009)[1]. These last authors pointed out that the classical model for tropical intensification, which is based on balance dynamics, cannot explain the spin up of the maximum tangential winds in the boundary layer, a feature of tropical cyclones that is found in both numerical models and observations. Smith et al. (2009) introduced the idea of a boundary layer spin up mechanism to complement the classical spin up mechanism as articulated in the pioneering paper by Ooyama (1969). As it turns out, the underlying idea was not new. It had been anticipated long ago by Anthes (1974) (p. 506) and articulated in the context of a slab boundary layer model by Smith and Vogl (2008) (see section 4.2 therein). In essence, the mechanism explains how the tangential wind in the boundary layer can be accelerated to the point of exceeding that above the boundary layer, even in a boundary layer that is steady, but it requires a temporal strengthening of the boundary layer inflow to produce a temporal increase in the tangential wind maximum and for this reason requires the classical spin up mechanism to operate in tandem.

As a part motivation for their approach, Heng et al. (2017) invoke results of Stern et al. (2015) who they claim "challenged the hypothesized positive contribution of surface friction to TC [tropical cyclone: our insertion] intensification proposed by Smith *et al.* (2009) based on the results from a linearized vortex model", but they then go on to recount the critique of Smith and Montgomery (2015) who pointed out that the linear model used in Stern et al. (2015) has its limitations and cannot be used to isolate the contribution of surface friction to producing inflow from that of diabatic forcing as the boundary dynamics is intrinsically nonlinear. It is not clear where Heng et al. (2017) stand on this issue[2].

---

[1] In their Introduction, Heng et al. (2017) make many incorrect statements concerning Bui et al. (2009)'s claims as well as those of a related paper by Smith et al. (2009). So as not to detract from our critique of Heng et al. (2017)'s results, but to put the record straight, these are noted in an Appendix.

[2] Indeed, having pointed out the limitation of Stern et al.'s study as a motivation for that of Heng and

Heng et al. (2017) then discuss their previous paper (Heng and Wang 2016), which purports to "include the nonlinearity", but the pitfalls of their thought experiment have been pointed out by Smith and Montgomery (2016) and Kilroy *et al.* (2017). For one thing, Heng and Wang do not appear to recognize the distinction between the global effects of friction and the local effect that leads to an amplification of the tangential wind relative to the gradient wind in the inner core boundary layer (Slocum et al. 2014) and a few kilometers above the boundary layer in the developing eyewall of the storm (see e.g., Montgomery et al. 2014, section 5.4). As we pointed out in our critique of Heng and Wang (2016), one of their concluding statements (on p1331) that "The negative contribution of surface friction to TC intensification found in this study contradicts the positive contribution hypothesis of Smith et al. (2009)" is misconstrued as these authors demonstrate nicely with their model simulation with friction included that the boundary layer spin up mechanism is operating to generate supergradient winds. We know of no other mechanism for producing supergradient winds other than by the vertical diffusion of momentum (as in the Ekman layer), where the effect is only weak.

Taken together, to the casual reader, the results of Heng and Wang (2016) and Heng et al. (2017) would appear to refute the validity of the boundary layer spin-up mechanism in tropical cyclones as well as the notion that gradient wind imbalance in the boundary layer is important. However, the caveats presented at the end of Heng et al.'s conclusions as well as the fact that the strong boundary layer inflow is there because the flow in the boundary layer is *not* in gradient wind balance, are reasons alone to be cautious of their conclusions. For a start, Heng et al. admit that the initial state incorporated in their Sawyer-Eliassen equation is NOT in balance! Specifically, they use the azimuthally-averaged potential temperature and tangential wind field from the output of their full-physics model to define

---

Wang (2016), they go on in their Conclusions to invoke the results of Stern et al. as support for their conclusion that "the boundary layer spinup mechanism of a TC ... should not be a primary mechanism of TC intensication". They do not say what they regard as "a primary mechanism".

the variable coefficients (static stability, inertial stability, baroclinicity) that comprise the Sawyer-Eliassen equation for the balanced meridional circulation to the diagnosed heating rate, tangential momentum and related eddy forcing terms. In other words, the basic state flow in the boundary layer, about which the secondary circulation response is being computed, is NOT in gradient and thermal wind balance as should be assumed for a strictly balanced calculation according to the Sawyer-Eliassen formulation. The consequences of this limitation on their results are hard to foresee!

As it turns out, Bui et al. (2009) showed in their section 5.2 and Appendix that with such a relaxed prescription of the basic state vortex, the inflow in the boundary layer was significantly enhanced relative to the strictly balanced calculation. This apparent 'improvement' in reproducing the full numerical model solution is an illusion, resulting from the unbalanced basic state vortex. It comes at the cost of a slowly convergent (or possibly locally divergent) iterative solution (p1720, right col.), a marked dependence of the solution on the vertical resolution of the grid mesh (p1728, right col.; found also by Heng et al.) and small-scale subsidiary meridional circulations in and around regions of symmetric instability corresponding to large vertical shear of tangential wind (their Fig.11).

In view of the results presented by Bui et al., the approximate agreement between the 'pseudo-balanced' calculations of Heng et al. and their companion nonlinear simulation is not entirely surprising. However, such an agreement is being misapplied to argue against the importance of the boundary layer spin up mechanism as articulated and demonstrated by Smith and Vogl (2008); Smith et al. (2009); Bui et al. (2009); Montgomery and Smith (2014, 2017); Slocum et al. (2014); Abarca and Montgomery (2015) and Abarca et al. (2015). In this context, it is interesting that Heng et al. (2017) find that "the unbalanced dynamics contributes to inward penetration of boundary layer inflow into the eye and thus the contraction of the RMW". Given that the spin up and contraction of the tangential wind field, itself, is in part the result of a spatial concentration of absolute vertical vorticity by the induced inflow from the nonlinear aggregate of deep convection and surface friction, it is physically implausible that the unbalanced dynamics that "contributes to the inward penetration of boundary layer inflow" is generally unimportant.

Another possible factor, in addition to the foregoing, that may help explain Heng et al.'s finding is that the diffusivity in the model, either the vertical or horizontal diffusivity, is unrealistically large. It is unclear from their paper (or from Heng and Wang 2016) what diffusion coefficients they have used. It was shown by Smith and Thomsen (2010) that an unrealistically large vertical diffusivity leads to only small departures from gradient wind balance in the boundary layer. (See also Gopalakrishnan et al. (2013a) and Zhang et al. (2015) who demonstrated a significant dependence of the spin up and maximum intensity over realistic forecast time scales on the vertical diffusivity.) It was shown also by Bryan and Rotunno (2009) and Bryan (2012) that there is a strong dependence of the simulated intensity and departure from gradient wind theory on the horizontal mixing length (and related diffusivity) used to parameterize asymmetric mixing and small-scale turbulence.

As a final point, if Heng et al.'s conclusions are correct that the boundary layer spin up mechanism is unimportant, it would follow that the classical balanced spin up mechanism of Ooyama (1969), in conjunction with the vertical advection of absolute angular momentum, M, is sufficient to describe the spin up of the tangential winds in the eyewall region in real or simulated tropical cyclones when there is radial outflow everywhere above the boundary layer. In specific relation to this latter situation, which is found in numerous tropical cyclone simulations, Smith et al. (2018) concluded: "Clearly, for spin up to occur anywhere where there is radial outflow, there must be a negative vertical gradient of M to permit the vertical advection of M to dominate the spin down tendency accompanying radial advection. In a strictly balanced model (such as the time-dependent Sawyer-Eliassen balance model in which gradient wind balance is assumed strictly throughout the vortex including the boundary layer, or in a generalized balanced flow consisting of gradient wind balance above the boundary layer and Ekman-like balance in the boundary layer (e.g. Abarca et al. 2015), our insertion), the vertical gradient of M must be positive in the boundary layer so that spin up in the lower troposphere above the boundary layer requires the classical mechanism to operate to spin up the eyewall there. If there is outflow over the whole depth of the eyewall, spin up requires a source of M in the boundary layer. It has been shown in recent work that the spin up of supergradient tangential winds in the boundary layer can provide the necessary negative vertical gradient of M to spin up the eyewall (Schmidt and Smith 2016; Montgomery and Smith 2017)."

3. Conclusion

In this comment we have refuted the claim that the axisymmetric balance dynamics comprising the linear Sawyer-Eliassen balance equation for the overturning circulation can well capture the secondary circulation and the implied tangential wind tendency in the nonlinear boundary layer of an intensifying tropical cyclone.

Appendix

This Appendix responds to numerous incorrect statements in reference to our own work in the Introduction of Heng et al. (2017).

- "Bui et al. (2009) have criticized the classic understanding of TC intensification based on balanced dynamics, ... ". Bui et al. (2009) did not "criticize" the "classic understanding", which is based on balance dynamics, but merely pointed out its limitations in regards to the boundary layer, which is intrinsically unbalanced.

- "They [Bui et al. (2009): our insertion] thus concluded that the balanced dynamics significantly underestimates the boundary layer inflow and thereby the spinup of tangential wind in the inner-core region, and thus the unbalanced dynamics should be largely responsible for TC intensification." Nowhere did Bui et al. say that "the unbalanced dynamics should be largely responsible for tropical cyclone intensification". Moreover, subsequent work has been careful to stress that the boundary layer spin up mechanism cannot act alone (Montgomery and Smith 2014, p56, right col.; Montgomery and Smith 2017, section 3.5; Smith and Montgomery 2015, p3027, right col.).

- “They seemed to suggest that the occurrence of unbalanced supergradient wind in the interior of the boundary layer as a result of surface friction plays an important role in spinning up the inner core of a TC.” Bui et al. do not “suggest” this: they argue that it is true.

- “This argument gave the impression that surface friction and its associated unbalanced processes can dominate the balanced dynamics in spinning up the TC in the inner core in the boundary layer.” It doesn’t give that impression to us. Again, we have pointed out that the classical spin up mechanism and the boundary layer spin up mechanism must act in tandem (see references in the foregoing item). Also, in the classical spin up mechanism as originally envisioned by Ooyama (1969), the boundary layer tangential flow is not spun up in the traditional sense of a positive tendency in the tangential velocity in the boundary layer; rather, it is assumed to be slaved (and equal) to the balanced tangential flow at the top of the boundary layer.

- “Although surface friction can substantially enhance the boundary layer inflow, the net dynamical effect of surface friction is negative because the positive tangential wind tendency as a result of frictionally induced inflow could not offset the direct spindown by surface friction.” As noted earlier, Heng et al. (2017) do not recognize the distinction between the global effects of friction and the local effect that leads to an amplification of the tangential winds in the inner core boundary layer. This is clear from the statement on p2576 (right col.) that “Since the supergradient wind is well above the surface, it does not directly contribute to the surface energy production or loss and the TC intensity”. The boundary layer spin up mechanism does not address “surface energy production or loss” since it does not appear in the global energetics and can only be understood by an examination of the coupled horizontal momentum equations, the associated generalized Coriolis and frictional forces acting in the radial and tangential directions, and the changes in the swirling wind field and radial pressure gradient force in the interior flow above the boundary layer.

- "Note that Stern et al. (2015) also confirmed the importance of vertical shear of tangential wind in the boundary layer to the frictionally induced inflow in the balanced response, as shown in Bui et al. (2009)." The meaning of this sentence is unclear because, for one thing, Stern et al. actually nullified the vertical shear of the tangential wind in the boundary layer in all of their calculations using the 3DVPAS model. (This was Stern et al.'s way of averting the representation of symmetric instability and roll-like instabilities in the their linearized, axisymmetric, model of the tropical cyclone boundary layer.) Further, Bui et al. did not implicitly or explicitly argue for "the importance of vertical shear of tangential wind in the boundary layer to the frictionally induced inflow in the balanced response."

- "This is in sharp contrast to the traditional view that surface friction is the major energy sink of a TC system, thus contributing negatively to TC intensification and maximum intensity". Again, Heng et al. appear to have missed the very important result that the effects of friction, in conjunction with the deep convection in the emerging eyewall of the storm, can lead to a local amplification of the tangential winds in the inner core boundary layer, the mechanism for which is not explicitly evident in the global energetics that forms the basis of maximum intensity theory for the gradient wind according to Emanuel (1986, 1989, 1995) and later revisions (Emanuel and Rotunno 2011).

Acknowledgments.

MTM acknowledges the support of NSF AGS-1313948 and the U.S. Naval Postgraduate School. RKS acknowledges financial support for this research from the US Office of Naval

Research Global under Grant No. N62909-15-1-N021.

REFERENCES

Abarca, S. F. and M. T. Montgomery, 2015: Departures from axisymmetric balance dynamics during secondary eyewall formation. *J. Atmos. Sci.*, **72**, 82–87.

Abarca, S. F., M. T. Montgomery, and J. C. McWilliams, 2015: The azimuthally-averaged boundary layer structure of a numerically simulated major hurricane. *J. Adv. Model. Earth Syst.*, **07**, 10.1002/.

Anthes, R. A., 1971: Iterative solutions to the steady-state axisymmetric boundary-layer equations under an intense pressure gradient. *Mon. Wea. Rev.*, **99**, 261–268.

— 1974: The dynamics and energetics of mature tropical cyclones. *Rev. Geophys. Space Phys.*, **12**, 495–521.

Bryan, G. H., 2012: Effects of surface exchange coefficients and turbulence length scales on the intensity and structure of numerically simulated hurricanes. *Mon. Wea. Rev.*, **140**, 1125–1143.

Bryan, G. H. and R. Rotunno, 2009: Evaluation of an analytical model for the maximum intensity of tropical cyclones. *J. Atmos. Sci.*, **66**, 3042–3060.

Bui, H. H., R. K. Smith, M. T. Montgomery, and J. Peng, 2009: Balanced and unbalanced aspects of tropical-cyclone intensification. *Quart. Journ. Roy. Meteor. Soc.*, **135**, 1715–1731.

Carrier, G. C., 1971: Swirling flow boundary layers. *J. Fluid Mech.*, **49**, 133–144.

Emanuel, K. A., 1986: An air-sea interaction theory for tropical cyclones. Part I: Steady state maintenance. *J. Atmos. Sci.*, **43**, 585–604.

— 1989: The finite amplitude nature of tropical cyclogenesis. *J. Atmos. Sci.*, **46**, 3431–3456.

— 1995: Sensitivity of tropical cyclone to surface exchange coefficients and a revised steady-state model incorporating eye dynamics. *J. Atmos. Sci.*, **52**, 3969–3976.

Emanuel, K. A. and R. Rotunno, 2011: Self-stratification of tropical cyclone outflow. Part I: Implications for storm structure. *J. Atmos. Sci.*, **68**, 2236–2249.

Gopalakrishnan, G., F. Marks, J. A. Zhang, X. Zhang, J. Bao, and V. Tallapragada, 2013a: Corrigendum. *J. Atmos. Sci.*, **70**, 2336.

— 2013b: A study of the impacts of vertical diffusion on the structure and intensity of the tropical cyclones using the high resolution HWRF system. *J. Atmos. Sci.*, **70**, 524–541.

Heng, J. and Y. Wang, 2016: Nonlinear response of a tropical cyclone vortex to prescribed eyewall heating with and without surface friction in TCM4: Implications for tropical cyclone intensification. *J. Atmos. Sci.*, **73**, 1315–1333.

Heng, J., Y. Wang, and W. Zhou, 2017: Revisiting the balanced and unbalanced aspects of tropical cyclone intensification. *J. Atmos. Sci.*, **74**, 2575–2591.

Huang, Y.-H., M. T. Montgomery, and C.-C. Wu, 2012: Concentric eyewall formation in Typhoon Sinlaku (2008). Part II: Axisymmetric dynamical processes. *J. Atmos. Sci.*, **69**, 662–674.

Kepert, J. D. and Y. Wang, 2001: The dynamics of boundary layer jets within the tropical cyclone core. Part II: Nonlinear enhancement. *J. Atmos. Sci.*, **58**, 2485–2501.

Kilroy, G., M. T. Montgomery, and R. K. Smith, 2018a: The role of boundary layer friction on tropical cyclogenesis and subsequent intensification. *Quart. Journ. Roy. Meteor. Soc.*, **144**, 1–15.

Kilroy, G. and R. K. Smith, 2017: The effects of initial vortex size on tropical cyclogenesis and intensification. *Quart. Journ. Roy. Meteor. Soc.*, **143**, 2832–2845.

Kilroy, G., R. K. Smith, and M. T. Montgomery, 2016: Why do model tropical cyclones grow progressively in size and decay in intensity after reaching maturity? *J. Atmos. Sci.*, **73**, 487–503.

— 2017: A unified view of tropical cyclogenesis and intensification. *Quart. Journ. Roy. Meteor. Soc.*, **143**, 450–462.

— 2018b: The role of heating and cooling associated with ice processes on tropical cyclogenesis and intensification. *Quart. Journ. Roy. Meteor. Soc.*, **144**, 99–114.

Kuo, H. L., 1971: Axisymmetric flows in the boundary layer of a maintained vortex. *J. Atmos. Sci.*, **28**, 20–41.

Montgomery, M. T. and R. K. Smith, 2014: Paradigms for tropical cyclone intensification. *Aust. Met. Ocean. Soc. Journl.*, **64**, 37–66.

— 2017: Recent developments in the fluid dynamics of tropical cyclones. *Annu. Rev. Fluid Mech.*, **49**, 1–33, doi:10.1146/annurev-fluid-010816-060022.

Montgomery, M. T., J. A. Zhang, and R. K. Smith, 2014: An analysis of the observed low-level structure of rapidly intensifying and mature Hurricane Earl (2010). *Quart. Journ. Roy. Meteor. Soc.*, **140**, 2132–2146, doi:doi:10.1002/qj.2283.

Ooyama, K. V., 1969: Numerical simulation of the life cycle of tropical cyclones. *J. Atmos. Sci.*, **26**, 3–40.

Rotunno, R. and G. Bryan, 2012: Effects of parameterized diffusion on simulated hurricanes. *J. Atmos. Sci.*, **69**, 2284–2299.

Schmidt, C. J. and R. K. Smith, 2016: Tropical cyclone evolution in a minimal axisymmetric model revisited. *Quart. Journ. Roy. Meteor. Soc.*, **142**, 1505–1516.

Shapiro, L. J., 1983: The asymmetric boundary-layer flow under a translating hurricane. *J. Atmos. Sci.*, **40**, 1984– 1998.

Slocum, C. J., G. J. Williams, R. K. Taft, and W. H. Schubert, 2014: Tropical cyclone boundary layer shocks. *arXiv*, **1405**, 1–19.

Smith, R. K., 1968: The surface boundary layer of a hurricane. *Tellus*, **20**, 473–483.

Smith, R. K. and M. T. Montgomery, 2008: Balanced depth-averaged boundary layers used in hurricane models. *Quart. Journ. Roy. Meteor. Soc.*, **134**, 1385–1395.

— 2015: Towards clarity on tropical cyclone intensification. *J. Atmos. Sci.*, **72**, 3020–3031.

— 2016: Understanding hurricanes. *Weather*, **71**, 219–223.

Smith, R. K., M. T. Montgomery, and H. Bui, 2018: Axisymmetric balance dynamics of tropical cyclone intensification and its breakdown revisited. *J. Atmos. Sci. Conditionally accepted*, **74**, xxx–xxx.

Smith, R. K., M. T. Montgomery, and S. V. Nguyen, 2009: Tropical cyclone spin up revisited. *Quart. Journ. Roy. Meteor. Soc.*, **135**, 1321–1335.

Smith, R. K. and G. L. Thomsen, 2010: Dependence of tropical-cyclone intensification on the boundary layer representation in a numerical model. *Quart. Journ. Roy. Meteor. Soc.*, **136**, 1671–1685.

Smith, R. K. and S. Vogl, 2008: A simple model of the hurricane boundary layer revisited. *Quart. Journ. Roy. Meteor. Soc.*, **134**, 337–351.

Smith, R. K., J. Zhang, and M. T. Montgomery, 2017: The dynamics of intensification in an hwrf simulation of hurricane earl (2010). *Quart. Journ. Roy. Meteor. Soc.*, **143**, 293–308.

Stern, D. P., J. L. Vigh, D. S. Nolan, and F. Zhang, 2015: Revisiting the relationship between eyewall contraction and intensification. *J. Atmos. Sci.*, **72**, 1283–1306.

Vogl, S. and R. K. Smith, 2009: Limitations of a linear model for the hurricane boundary layer. *Quart. Journ. Roy. Meteor. Soc.*, **135**, 839–850.

Zhang, D.-L., Y. Liu, and M. K. Yau, 2001: A multi-scale numerical study of Hurricane Andrew (1992). Part IV: Unbalanced flows. *Mon. Wea. Rev.*, **61**, 92–107.

Zhang, J. A., D. S. Nolan, R. F. Rogers, and V. Tallapragada, 2015: Evaluating the impact of improvements in the boundary layer parameterization on hurricane intensity and structure forecasts in HWRF. *Mon. Wea. Rev.*, **143**, 3136–3155.