On the hypothesized outflow control of tropical cyclone intensification

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We present a series of idealized, three-dimensional, convection-permitting numerical experiments to evaluate the premise of the revised theory of tropical cyclone intensification proposed by Emanuel (2012). The premise is that small-scale turbulence in the upper tropospheric outflow layer determines the thermal stratification of the outflow and, in turn, an amplification of the system-scale tangential wind field above the boundary layer. The aim of our paper is to test whether parameterized small-scale turbulence in the outflow region of the developing storm is an essential process in the spin up of the maximum tangential winds.
Compared to the control experiment in which the small-scale, shear-stratified turbulence is parameterized in the usual way based on a Richardson number criterion, the vortex in a calculation without a parameterized representation of vertical mixing above the boundary layer evolves in a similar way with no significant difference in upper-level outflow layer temperature, thermal stratification or vortex intensification rate with time. Richardson number near-criticality is found mainly in the upper-level outflow outside the eyewall. However, the present solutions indicate that eddy processes in the eyewall play a significant role in determining the structure of moist entropy surfaces in the upper-level outflow. In the three-dimensional model, these eddy processes are largely realizations of asymmetric deep convection and are not obviously governed by any Richardson number-based criterion. The experiments do not support the premise on which the new theory is based. The results would appear to have ramifications for recent studies that invoke the new theory.

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1. Introduction

The steady-state hurricane model formulated by Emanuel (1986) has been a corner stone in underpinning the theory of hurricane behaviour for the last three decades. In particular, it has formed the basis for constructing a theory for the maximum Potential Intensity (PI) that a storm may achieve at a particular location. In PI theory, intensity is defined as the maximum gradient wind at the top of the frictional boundary layer. Over the years, the model has been refined in several ways (Emanuel 1988, 1995; Bister and Emanuel 1998) and it has been extended to provide a theory for storm intensification (Emanuel 1997, hereafter E97). An appraisal of the steady-state model and its application to PI theory is provided by Montgomery and Smith (2017, see section 5). An appraisal of the unsteady version of the model and its relation to other paradigms for hurricane intensification, including a new alternative rotating-convection paradigm, is given by Montgomery and Smith (2014).

The E97 intensification theory highlighted the frontogenetic nature of eyewall formation, drawing upon the presumed (p. 1019) " ... crucial presence of downdrafts by reducing the entropy tendency there (outside the radius of maximum tangential wind (RMW), our insertion) by a factor β". The factor β is assumed to be a function of radius. The key element of the time-dependent model was the derivation of an expression for the time rate-of-change of the tangential wind at the top of the boundary layer (his Eqn. [20]). This expression equates the tangential wind tendency to the sum of three terms. One of these terms is always negative definite, one vanishes at the radius of maximum gradient wind (RMW), and the third is positive only if the radial gradient of the '*ad hoc*' function, $\beta(r)$, is sufficiently negative to offset the other two terms. The function $\beta(r)$ is introduced to "... crudely represent the effects of convective and large-scale downdrafts, which import low θ_e ([equivalent potential temperature - our insertion) air into the subcloud layer" (E97, p.1019, below Eqn. (16)). One problem with the theory is the lack of a rigorous basis for the formulation of $\beta(r)$. A second problem is the assumption that the boundary layer is in approximate gradient wind balance. As discussed in Smith et al. (2008), this assumption is difficult to defend in the inner-core region of a tropical cyclone.

In a series of idealized, cloud-permitting numerical experiments, Montgomery et al. (2009) showed that tropical cyclone intensification does not require downdrafts, unlike the E97 theory. Further, they showed that vortex intensification proceeds optimally in the pseudo-adiabatic case in which downdrafts are excluded altogether. These results call into question the so-called β formulation of tropical cyclone intensification. They show also that the widely held Wind-Induced-Surface-Heat-Exchange (WISHE) evaporation-wind feedback mechanism of tropical cyclone intensification is neither essential nor the dominant pathway of intensification in the prototype problem for intensification (see Section 2a).

A few years ago, Emanuel and Rotunno (2011) and Emanuel (2012) questioned the assumption of Emanuel's earlier hurricane models (the steady model of Emanuel (1986) and the time-dependent E97 model) that the air parcels rising in the eyewall exit in the lower stratosphere in a region of approximately constant absolute temperature. To quote Emanuel (2012, p. 989): "Emanuel and Rotunno (2011) demonstrated that in numerically simulated tropical cyclones, the assumption of constant outflow temperature is poor and that, in the simulations, the outflow temperature increases rapidly with angular momentum." These authors proposed a revised theory[1] postulating that "the entropy stratification is determined by a requirement that the Richardson number not fall below a critical value" and that the temperature stratification of the outflow is determined by small-scale, shear-stratified turbulence.

Ordinarily, the critical Richardson number demarcates the local boundary between stratified shear stability and instability/turbulence[2]. Here it seems that small-scale turbulence in the outflow layer is presumed to operate and bound the Richardson number to a near critical value.

In the revised intensification theory of Emanuel (2012), small-scale, shear-stratified turbulence in the upper-tropospheric outflow layer is presumed to determine the thermal stratification of the upper-level outflow and, in turn, an amplification of the system-scale tangential wind field above the boundary layer. The new theory represents a major shift in the way a storm is presumed to be influenced by its environment. In the earlier models, it was assumed that the near-isothermal structure of the lower stratosphere set the (constant) outflow temperature. In the revised time-dependent theory, the vertical structure of the outflow temperature is determined internally within the vortex so that, in principal, it no longer matches the temperature structure of the storm environment. While the revised steady-state theory of Emanuel and Rotunno (2011) was configured to ensure that the outflow temperature at the radius of maximum wind (in angular momentum coordinates) equals the temperature of the environment (p2245)[3], the time-dependent theory does not appear to impose such a constraint.

A key result of the revised analytical intensification theory of Emanuel (2012) (see his section 3) is the derivation of a new expression for the time rate-of-change of tangential wind (his Eqn. [16]) (analogous to E97's Eqn. [20]). The right-hand side of this tendency equation involves three terms: the first term vanishes at the radius of maximum gradient wind; the third term is always negative definite and can only contribute to spin down; the second term is positive and would appear to represent a new azimuthal force that depends on "the radial gradient of outflow temperature, which in turn, according to the results of Part I (Emanuel and Rotunno - our insertion), is a result of small-scale turbulence in the outflow region[4]."

On the face of it, the premise of the new intensification theory as articulated above appears implausible to us, at least from a fluid dynamics perspective, because of the tenuous link between small-scale mixing processes in the upper tropospheric outflow layer and the amplification of the system scale swirling wind at the top of the boundary layer. While the new theory makes numerous assumptions (some of which are highlighted later), to us the purported existence and nature of this tangential force is the most mysterious facet of the revised theory. Another puzzling assumption used to generate solutions presented

[1]In the revised theory, the maximum gradient wind is reduced by a factor of approximately $1/\sqrt{2}$ compared with the nominal potential intensity when the ratio of the bulk enthalpy exchange and drag coefficient is near unity ($C_k/C_D = 1$). The reduced intensity is a consequence of neglecting the pressure dependence of the saturation mixing ratio in the theory (see Emanuel and Rotunno 2011, pp. 2246-47).

[2]In fluid dynamics, a Richardson number of 0.25 defines the instability threshold for normal mode disturbances in the absence of moist processes (Drazin and Reid 1981). When turbulent processes are allowed for, the criticality boundary is usually extended to a value of unity based on simple energetics considerations (e.g., Cushman-Roisin 1994). In the revised steady state and intensification theories, this criticality boundary is tacitly assumed to hold true when moist processes are included.

[3]"... a shooting method is applied in which an outer radius [r_o - our insertion] is first specified, the system [defined by thier Equations (31) and (35) - our insertion] integrated, and the outflow temperature at the radius of maximum winds is noted. If it is not equal to T_t [the ambient tropopause temperature - our insertion], the integration is restarted with a new value of r_o, and so on, until the outflow temperature at the radius of maximum winds equals T_t".

[4]Later for pedagogical purposes we review in Appendix A the key assumptions and approximations underlying the new tendency equation and we derive the tendency equation from these assumptions. The derivation exposes *inter alia* the tangential force that is responsible for increasing the maximum tangential wind at the top of the boundary layer in the new theory.

by Emanuel (2012), was the choice of an unrealistically large value of 5000 m for the boundary layer depth, h.

For the foregoing reasons an immediate question arises as to whether or not the premise of the new theory is physically defensible, at least for realistic parameter values consistent with the latest observational guidance? This question is relevant in view of recent studies that invoke the revised theory for the determination of a universal tangential wind profile for a hurricane (Chavas and Lin 2016) and as support for the integrity of a redefined Wind-Induced-Surface-Heat-Exchange (WISHE) intensification theory (Zhang and Emanuel 2016).

As an additional remark, it is worth pointing out that, because of the underlying axisymmetric formulation of the revised intensification theory, the hypothesis of small-scale, shear-stratified turbulence control on *vortex intensification* tacitly assumes that the shear stratified turbulence can be meaningfully represented as ring-like eddy structures encircling the vortex axis. In reality, such turbulent mixing occurs locally in azimuth and the axisymmetric assumption is highly questionable. Moreover, given the intrinsic limitations of vortex intensification in strict axisymmetric geometry in comparison with companion three-dimensional simulations (Persing et al. 2013), one should have a healthy skepticism for an axisymmetric theory of vortex spin up that invokes an undocumented force in the tangential direction to spin up the maximum tangential wind. For these reasons, we will not use an axisymmetric model and use instead a three-dimensional model as the proper benchmark to evaluate the premise of the revised intensification theory.

We defer further discussion of the foregoing issues until later after we have summarized the results of a series of idealized, three-dimensional, experiments designed to test the new outflow control premise on vortex intensification for realistic parameter settings based on the latest observational guidance.

In the next section we describe the setup of the numerical experiments that are used to test the premise of the new intensification theory. The key results are discussed in Section 3 and a discussion of these and conclusions are the topic of Section 4.

2. The Model and Experiments

2.1. Model core and pertinent parameter settings

The model configuration relates to the prototype problem for tropical cyclone intensification. This problem considers the evolution of an initially cloud free, axisymmetric vortex of near tropical storm strength in thermal wind balance, embedded in a mean tropical environment without any ambient flow. The simulations are carried out using the numerical model of Bryan and Fritsch (2002), CM1 version 14. The reference sounding is the near-neutral sounding of Rotunno and Emanuel (1987).

The model set up follows closely the formulation of Persing et al. (2013) and we give below a brief description of the model set up for the control experiment. A detailed listing of the common and relevant numerical parameters and their definitions in the FORTRAN code is provided in Appendix B.

The simplest physics options are chosen to provide the cleanest possible comparison to the idealized framework of Emanuel (2012)'s new axisymmetric intensification theory. The effects of radiation are represented by Newtonian damping towards the reference sounding, with the damping rate capped at 2 K day^{-1}. The upper boundary includes a Rayleigh damping layer in the height range 20-25 km to control gravity wave noise reflected from the top boundary. Precipitation is represented by the simple scheme of Rotunno and Emanuel (1987) with a fixed fall speed for liquid water of 7 m s^{-1}. Ice microphysical processes are neglected. The foregoing options, while arguably difficult to justify for prolonged (multiple week) simulations (Persing et al. 2016), are adequate to simulate the intensification of an initial cyclonic vortex for the prototype problem on a realistic forecast time scale of order 5 days.

The calculations are carried out on an f-plane with the Coriolis parameter $f = 5 \times 10^{-5}$ s^{-1}, corresponding to 20°N. The sea surface temperature is fixed at 299.3 K (26.15 C).

2.2. Sub grid-scale turbulence parameterization

A bulk aerodynamic formulation for heat and enthalpy is used to model the turbulent momentum and enthalpy transfer at the surface and, for simplicity, the generally wind-speed dependent values of these exchange coefficients are taken to be constant. The enthalpy transfer coefficient is $C_k = 1.29 \times 10^{-3}$ and the drag coefficient is $C_\mathrm{D} = 2.58 \times 10^{-3}$. These constant values of surface exchange coefficients represent the best mean estimates from the latest in-situ observations under major hurricane conditions (Bell et al. 2012a).

The subgrid-scale turbulence is represented by choosing option "iturb=3" in the model, which is designed for problems that do not resolve any part of the turbulent Kolmogorov inertial range. This choice requires the specification of the horizontal mixing length $l_\mathrm{h} = 700$ m and the vertical mixing length $l_\mathrm{v} = 50$ m. These values are based on the recent observational findings of Zhang and Montgomery (2012) and Zhang et al. (2011b), respectively, and the resulting vertical and horizontal eddy diffusivities that are output in the model simulations. These values are also close to the values recommended by Bryan (2012) in order to produce realistic hurricane structure. For simplicity, these mixing lengths are assumed constant in both space and time.

The subgrid scale scheme follows the traditional formulation of Smagorinsky (1963) and Lilly (1962), except that different eddy viscosities are employed for the horizontal and vertical directions. As per the CM1 documentation, the flow-dependent momentum diffusivities in the horizontal and vertical directions are specified as follows: $K_\mathrm{m,h} = l_\mathrm{h}^2 S_\mathrm{h}$ and $K_\mathrm{m,v} = l_\mathrm{v}^2 S_\mathrm{v}\sqrt{1 - \mathrm{Ri}/\mathrm{Pr}}$, where the m subscript refers to momentum and the second subscript h or v refer to the horizontal and vertical directions, S_h and S_v denote the parts of the total deformation, S, that involve the horizontal and vertical strain components, $\mathrm{Ri} = N_m^2/S_\mathrm{v}^2$ is the moist Richardson number, N_m^2 is the moist Brunt-Väisälä frequency, and Pr is the Prandtl number (set to unity in this option) (see Bryan and Fritsch 2002 for complete definitions of these parameters). In this scheme, the heat and momentum diffusivities are taken to be identical, i.e. $K_\mathrm{h} = K_\mathrm{m}$ and the vertical eddy diffusivity is proportionally reduced in regions with positive moist Richardson number (< 1). Whenever Ri exceeds unity, $K_\mathrm{h,v}$ and $K_\mathrm{m,v}$ are set to zero.

Table I. The six numerical experiments carried out.

Experiment	K_v above $z > 1$ km	initial moisture perturbation	vertical grid mesh
EX-1	unmodified	unperturbed	stretched
EX-2	$K_v = 0$	unperturbed	stretched
EX-3	unmodified	60 mg kg^{-1} in BL	stretched
EX-4	$K_v = 0$	60 mg kg $^{-1}$ in BL	stretched
EX-5	unmodified	unperturbed	fixed grid spacing (250 m)
EX-6	$K_v = 0$	unperturbed	fixed grid spacing (250 m)

2.3. *Initial conditions*

The same initial cyclonic vortex is used for all simulations. The initial radial and vertical velocity are set to zero. The initial tangential velocity is taken to be in thermal wind balance. This tangential velocity has a maximum of 13 m s^{-1} at the surface at a radius of 100 km radius. It varies smoothly in space, declining to zero at a radius of 400 km radius, beyond which it is set to zero. It is set to zero also above a height of 20 km. Appendix B gives the mathematical formula for the tangential wind as per the CM1 model code and displays its radius-height structure.

2.4. *The experiments*

We summarize the results of six numerical experiments, focusing primarily on the spin up phase of the vortex evolution as noted in the Introduction.

The control experiment, EX-1, is the same as the three-dimensional 3D3k simulation described by Persing et al. (2013) (with the same horizontal and vertical grid spacing described there, see Appendix B). Experiment EX-2 is similar to EX-1, but it is designed to suppress sub-grid scale mixing in the vertical direction above the boundary layer. Here, the vertical diffusivity K_v is set equal to zero at each time step above a height of 1 km. The suppression of vertical mixing by sub grid scale turbulence short circuits any presumed link between small-scale turbulence above the boundary layer and the amplification of tangential wind on the vortex scale. This remark applies, in particular, to parameterized small-scale turbulence in the upper-troposphere. If the new intensification theory of Emanuel (2012) is correct, then the vortex with zero vertical mixing should not intensify.

Two 'moisture' sensitivity simulations are presented, like EX-1 and EX-2, but with a small, horizontally homogeneous change in the initial moisture field of 60 mg kg^{-1} in the boundary layer to provide a small perturbation of each simulation with otherwise identical characteristics. These perturbation experiments are referred to as EX-3 and EX-4, respectively, and provide a simple estimate of the stochastic variability of EX-1 and EX-2 in association with a small perturbation of moisture in the boundary layer.

Two additional 'vertical resolution' sensitivity experiments, EX-5 and EX-6, are presented also. The experiment EX-5 is like EX-1, but employs a uniform vertical grid spacing of 250 m. Experiment EX-6 uses the same uniform vertical grid mesh as EX-5, but like EX-2 suppresses vertical diffusion above 1 km height. Simulations EX-5 and EX-6 are conducted to address a potential concern raised by an anonymous reviewer that the vertical structure of the outflow layer may not be adequately resolved in simulations EX-1 through EX-4 and hence these experiments

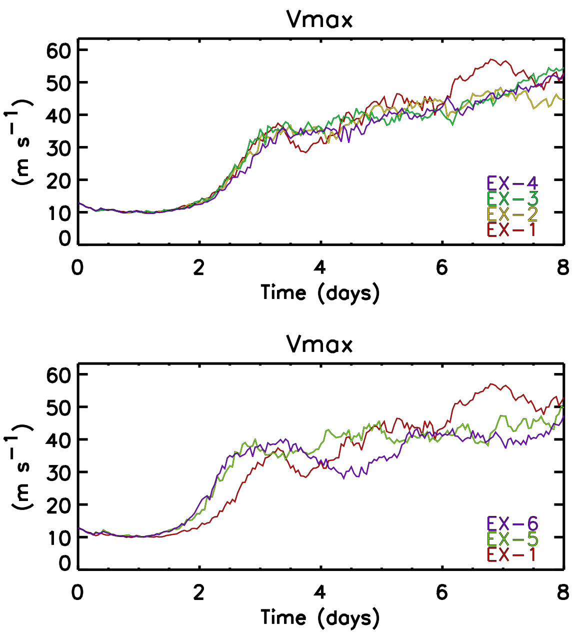

Figure 1. Time-series of maximum azimuthally-averaged tangential wind $V_{\max}$ for the numerical experiments carried out in this study. At top, the control experiment EX-1 (red) from Persing et al. (2013); the suppressed vertical diffusivity experiment EX-2 (yellow) ($K_v = 0$ everywhere above 1 km height). Also shown are $V_{\max}$ time series for the perturbation moisture experiments EX-3 (green, a perturbation of EX-1) and EX-4 (blue, a perturbation of EX-2) as described in the main text. At bottom, EX-1 is again shown (red); the experiment with fixed, 250 m vertical grid spacing EX-5 (green); and the suppressed vertical diffusivity with fixed, 250 m vertical grid spacing EX-6 (blue).

may not properly represent the parameterized turbulence in the outflow region and its premised role in amplifying the system-scale tangential wind field at the top of the boundary layer[5]. These two experiments feature better resolution of the outflow layer than the foregoing experiments (by a factor of three, and a little better than the 312.5 m grid spacing simulation experiments employed by Emanuel and Rotunno 2011). These experiments have fewer grid points in the boundary layer than EX-1 through EX-4.

3. Results from idealized numerical experiments

3.1. *Vortex intensification compared*

Figure 1 shows a time series over an 8 day interval of the azimuthally-averaged maximum tangential velocity in the

[5]If, indeed, the vertical resolution of prior experiments EX-1 through EX-4 is inadequate to represent the parameterized mixing processes in the upper tropospheric outflow layer, it would follow that the same remark is true of all current forecast models that are used for routine prediction of tropical cyclones.

main experiments, EX-1 and EX-2. This time interval spans more than a typical forecast time scale of 3 to 5 days. The evolution of intensity until 80 hours, when hurricane intensity (~ 34 m s^{-1}) is achieved, is approximately the same for all simulations[6]. During this time the vortices intensify rapidly with a maximum spin up rate of around 1 m s^{-1} h^{-1} at 70 hours.

After 80 h, the vortices continue to intensify with a reduced time-mean rate and achieve peak intensities between 50 and 60 m s^{-1}. The differences in vortex intensity at a given time between experiments EX-1 and EX-2 are not significant, as suggested by the intensification time series for the vortices with small moisture perturbations in the boundary layer (EX-3 and EX-4, Fig. 1). These intensity differences reflect the stochastic nature of deep convection as discussed by Nguyen et al. (2008) and Shin and Smith (2008). These results do not support the hypothesis that the intensification is controlled to leading order by small-scale, vertical, turbulent mixing anywhere above the boundary layer.

Experiment EX-5 intensifies to 35 m s^{-1} about 12 h sooner than EX-1. Experiment EX-6 has a similar intensity evolution as EX-5, except with a temporary decrease in intensity at around 4.5 days. Accounting for the 12 h lag in the start of the spin up process, the rapid intensification period is similar in EX-5 and EX-6 to EX-1. Furthermore, after 3 days, the intensity time series of EX-5 and EX-6 falls within the range of stochastic variability shown by EX-1 and EX-3. The higher vertical resolution experiments demonstrate that a factor of three increase in the vertical resolution of the outflow region does not fundamentally change the main findings of the experiments EX-1 through EX-4. For these reasons we will restrict our analysis to the first two experiments, EX-1 and EX-2.

3.2. *Outflow temperature evolution*

Figure 2 shows radius-time plots of the azimuthally-averaged temperature deviation from the environmental temperature at 13.0 km height in EX-1 and EX-2. The 13 km height is chosen because it corresponds approximately to the level of maximum radial outflow in both experiments. This height is where the shear-stratified (parameterized turbulent) mixing is hypothesized to have a strong influence on the thermal structure of the vortex. The far-field environmental temperature at this altitude is 213.8 K, and remains near its initial value for the full simulation.

Both experiments exhibit a progressively growing warm anomaly ($>$ 2K) inside a 50 km radius. Beyond this radius, the warm anomaly diminishes in strength until the 900 km radius where the outflow jet terminates (not shown). Emanuel's revised intensification theory presumes a modification of the thermal structure (stratification) that emerges from the eyewall so that the thermal structure there controls the spin up of the vortex below. In EX-2, without vertical subgrid scale mixing, the evolution of the outflow temperature is very similar to that of EX-1.

To investigate the possible dependence of the upper-tropospheric lapse rate on the parameterized vertical mixing, Figure 3 shows the deviation of the upper-level lapse rate from the environment for both the control (EX-1) and zero diffusivity experiment (EX-2). As above, the same height (13.0 km) is chosen for these lapse rate calculations. The lapse rate deviation shows a gradual radial variation, becoming somewhat more statically-stable than the environment at 150 km radius, then becoming somewhat less stable near 600 km, then reaching the environmental value by 900 km radius. These variations are similar between the two experiments at this level of maximum outflow. It is unclear how small-scall (parameterized) turbulence might govern this specific radial arrangement. Looking more closely at the evolving lapse rate of EX-1, specific events showing less static stability (more negative values in the figure) can be traced to outward propagating features that move with a radial speed of approximately 150 km day^{-1} (1.7 m s^{-1}). This diagnosis suggests that the radial structure of evolving lapse rate may be more-readily explained by outward advection of discrete features generated from the eyewall than a local subgrid scale mixing parameterization in the outflow region exterior to the eyewall.

The foregoing results do not support the proposition that small-scale (parameterized) turbulence in the outflow layer controls the spin up of the vortex.

3.3. *Richardson number structure*

In the CM1 model, the shear-stratified turbulence parameterization scheme is activated when the Richardson number has a value between zero and one. The gradient Richardson number in this model is implemented in Cartesian coordinates as follows

$$\mathrm{Ri}_{\mathrm{CM1}} = \frac{N_{\mathrm{m}}^2}{2\left(\frac{\partial w}{\partial z}\right)^2 + \left(\frac{\partial w}{\partial x} + \frac{\partial u_x}{\partial z}\right)^2 + \left(\frac{\partial w}{\partial y} + \frac{\partial u_y}{\partial z}\right)^2}, \quad (1)$$

with u_x, u_y, and w being the components of the wind in the x-, y-, and z-directions, respectively. Here, we evaluate this in storm-centered cylindrical coordinates using

$$\mathrm{Ri}_{\mathrm{CM1}} = \frac{N_{\mathrm{m}}^2}{2\left(\frac{\partial w}{\partial z}\right)^2 + \left(\frac{1}{r}\frac{\partial w}{\partial \lambda} + \frac{\partial v}{\partial z}\right)^2 + \left(\frac{\partial w}{\partial r} + \frac{\partial u}{\partial z}\right)^2}, \quad (2)$$

where u, v, and w are the r-, λ-, and z-components of the velocity vector, respectively, and N_{m}^2 is the local moist static stability as computed by the numerical model (Durran and Klemp 1982). In the diagnosis presented here, the Richardson number is computed locally at every grid point on a (r, λ, z, t) grid, at one-hour intervals between 95 and 105 hours, during the intensification phase (see Fig. 1)[7]. Figure 4 shows the azimuthal-time average of $\mathrm{Ri}_{\mathrm{CM1}}$ during this period for experiments EX-1 and EX-2. It shows also the relative frequency at which near criticality (1.5 or below)[8] occurs in each experiment at each (r, z) point

[6]For all of the experimental results presented here, the term 'intensity' is used in a more general sense than in PI theory and is defined as the azimuthally-averaged maximum tangential velocity. This maximum generally occurs in the frictional boundary layer at a height of approximately 600 m.

[7]For the computation of $\mathrm{Ri}_{\mathrm{CM1}}$ shown here, the potential temperature, pressure, vapor and liquid mixing ratios, and tangential and radial wind components are interpolated to the cylindrical grid from the original Cartesian computational grid. The Richardson number is computed at each point on the cylindrical grid at each time, and then averaged.

[8]Since the local computation of Richardson number is computed here on a three-dimensional cylindrical grid, rather than the original Cartesian grid mesh, a near-criticality of 1.5 is used for estimating the percentage of criticality occurrence, rather than 1.0. This modest increase in the criticality threshold is used to account for the re-interpolation of a variable that shows skewness in distribution.

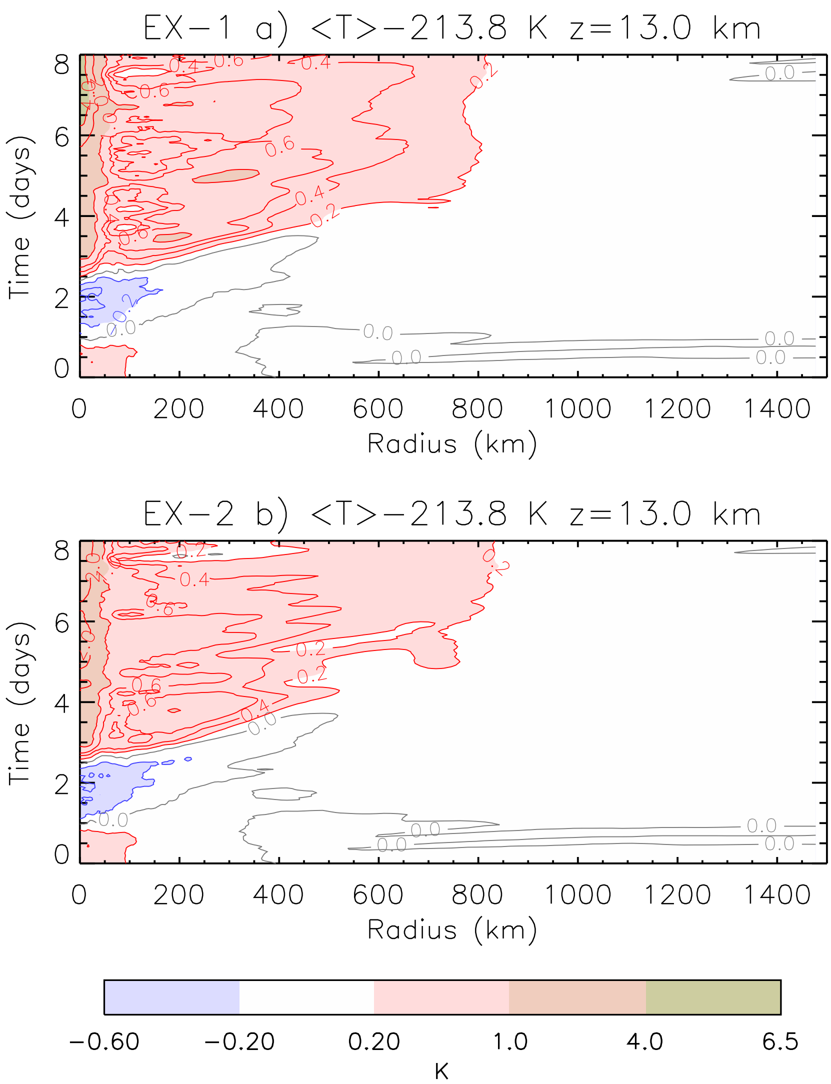

Figure 2. Radius-time contour plots of azimuthally-averaged temperature deviation from the far-field at 13.0 km height. The far-field temperature at this height is 213.8 K. Contours shown are $\pm\{0.2, 0.4, 0.6, 1.0, 2.0, 4.0\}$ K. Red contours denote positive values and blue contours denote negative values. Shading indicated in color bar. Top panel (a) for the control experiment EX-1, bottom panel (b) for experiment EX-2. The data shown are smoothed in time using a 5 hour boxcar smoother.

within the 360 degree azimuth and during the 10 hour time averaging period.

During a period of intensification in the control simulation, near-criticality is found (Fig. 4a) in the time-azimuth mean below the outflow around 11 km height and outside 60 km radius. A similar analysis for EX-2 (Fig. 4b) shows less near-criticality. The low-level eyewall in both EX-1 and EX-2 has large spatial shears in the flow, from which near-critical values of Richardson number are found extending up to 2 km height. The mid-level has relatively larger values of Richardson number. Since the Richardson number tends to be a quantity with large positive skew, the occurrence of near critical values of Richardson number is examined also. Frequencies of occurrence of near-criticality greater than 50% (red in Figs. 4c,d) are limited to the identified regions, below the outflow jet and in the low-level eyewall, plus throughout the planetary boundary layer and a smaller region above the outflow jet.

3.4. Is there a relation between upper-level mixing and Richardson number?

Comparison of our computation of $\mathrm{Ri}_{\mathrm{CM1}}$ (Fig. 4a) with that of Figure 6a of Emanuel and Rotunno (2011) show a few differences which, for completeness, need to be carefully considered. First, they display $\sqrt{\mathrm{Ri}}$. Second, they compute Richardson number based on the 24 hour averaged fields from an axisymmetric model versus our display of the time-azimuth average of Richardson number computed locally in space and time. Third, Emanuel and Rotunno (2011) display values from the mature stage, while we show values from an intensifying stage. Our Figure 4a shows a larger region of criticality in the outflow region than does Emanuel and Rotunno (2011) using the same metric[9]. We can confirm the large region of negative values of Ri through

[9]Our previous paper Persing et al. (2013) provided a preliminary analysis of the Richardson number in three-dimensional and axisymmetric hurricane simulations. We discovered a coding error in our previous diagnosis of the gradient Richardson number that over-reported the value of this number that we correct here.

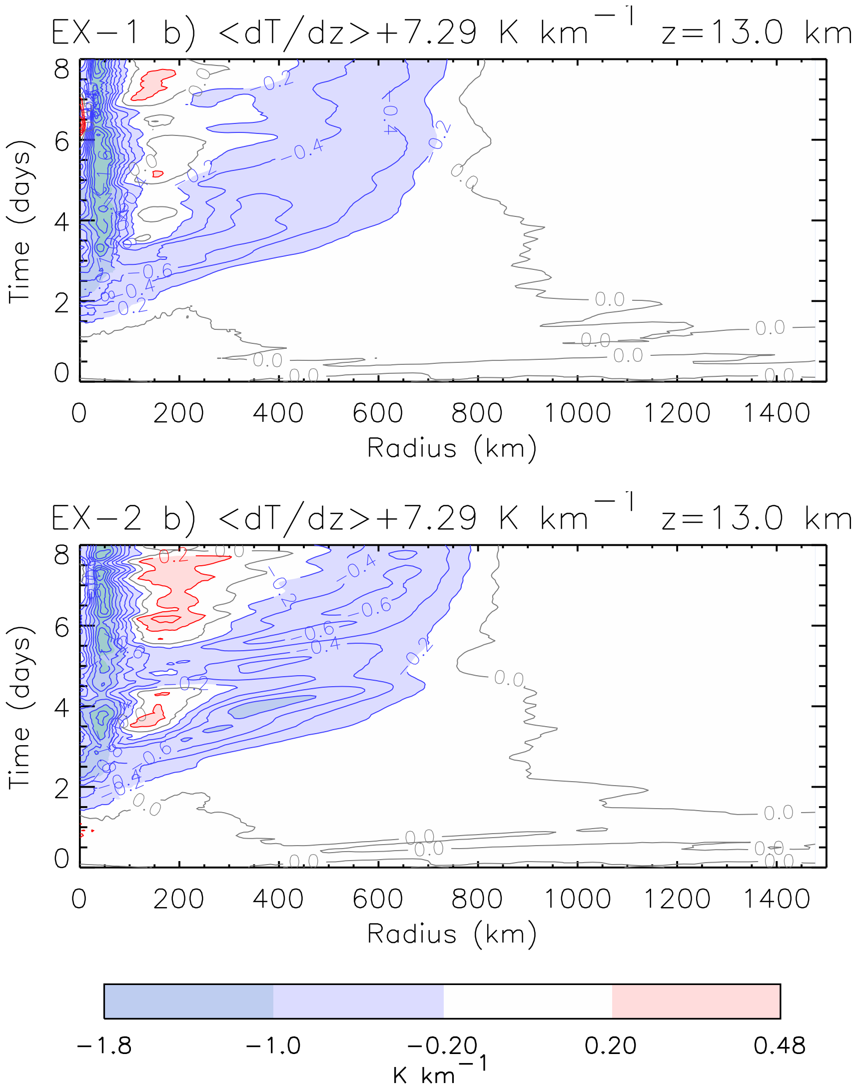

Figure 3. Radius-time contour plots of azimuthally-averaged lapse rate deviation from the far-field at 13.0 km height. The far-field lapse rate at this height is -7.292 K km^{-1}. Contours are shown with an interval of 0.2 K km^{-1}. Blue contours denote lapse rates that are greater (more negative) than the far-field and red contours denote lapse rates smaller (less negative) than the far-field. Shading indicated in the color bar. Top panel (a) for the control experiment EX-1, bottom panel (b) for experiment EX-2. The data shown are smoothed in time using a 5-hour boxcar smoother.

the mid-to-lower troposphere when we performed a separate computation with the ER11 formula, hereafter $\mathrm{Ri}_{\mathrm{ER11}}$[10]. In the tropical atmosphere, one would expect to find a reversal of sign with height of $\partial s^*/\partial z$ (vertical gradient of saturated entropy, e.g. Holton 2004, Fig 11.1). The mean of $\mathrm{Ri}_{\mathrm{CM1}}$ does not show such a region of negative values through the mid-to-lower troposphere as does $\mathrm{Ri}_{\mathrm{ER11}}$. This difference is attributable in part to how the CM1 model uses a separate dry calculation for uncloudy grid cells and to the somewhat different formulation of N_m^2 in the cloudy cells. Where the water vapor content is small in the upper troposphere, the CM1 formulation (based on Durran and Klemp (1982)) and Emanuel and Rotunno (2011) formulation should (and do) agree.

In CM1, by design, the quantity $\mathrm{Ri}_{\mathrm{CM1}}$ controls the activation of a sub-grid scale vertical mixing parameterization when $\mathrm{Ri}_{\mathrm{CM1}} < 1$, thereby activating subgrid scale mixing associated with shear-stratified turbulence. Activation of shear-stratified turbulence is prevalent in the outflow of EX1, but in EX2, where this process is artificially suppressed, there is little difference in evolution of intensity from EX1. The EX2 simulation shows greater spatio-temporal variability of the secondary circulation and in turn a more extensive region of criticality than EX1. While Figure 4b gives the impression of reduced criticality (in the average sense), Figure 4d affirms that the frequency of criticality is more widespread in the upper troposphere. We conclude that mixing by shear-stratified turbulence is not essential for tropical cyclone intensification in the prototype problem using standard values of the model parameters and using the latest observational guidance for horizontal and vertical turbulence mixing lengths in real tropical cyclones.

The question remains as to what degree the postulated shear-stratified turbulent signature is identifiable in the CM1 model? The revised PI theory of Emanuel and Rotunno (2011) requires mixing of entropy to occur across M-surfaces to determine the outflow stratification. To examine

[10] We presume that Emanuel and Rotunno (2011) also computed negative values of $\mathrm{Ri}_{\mathrm{ER11}}$, and that as an expedient they have displayed imaginary values of $\sqrt{\mathrm{Ri}_{\mathrm{ER11}}}$ as zero.

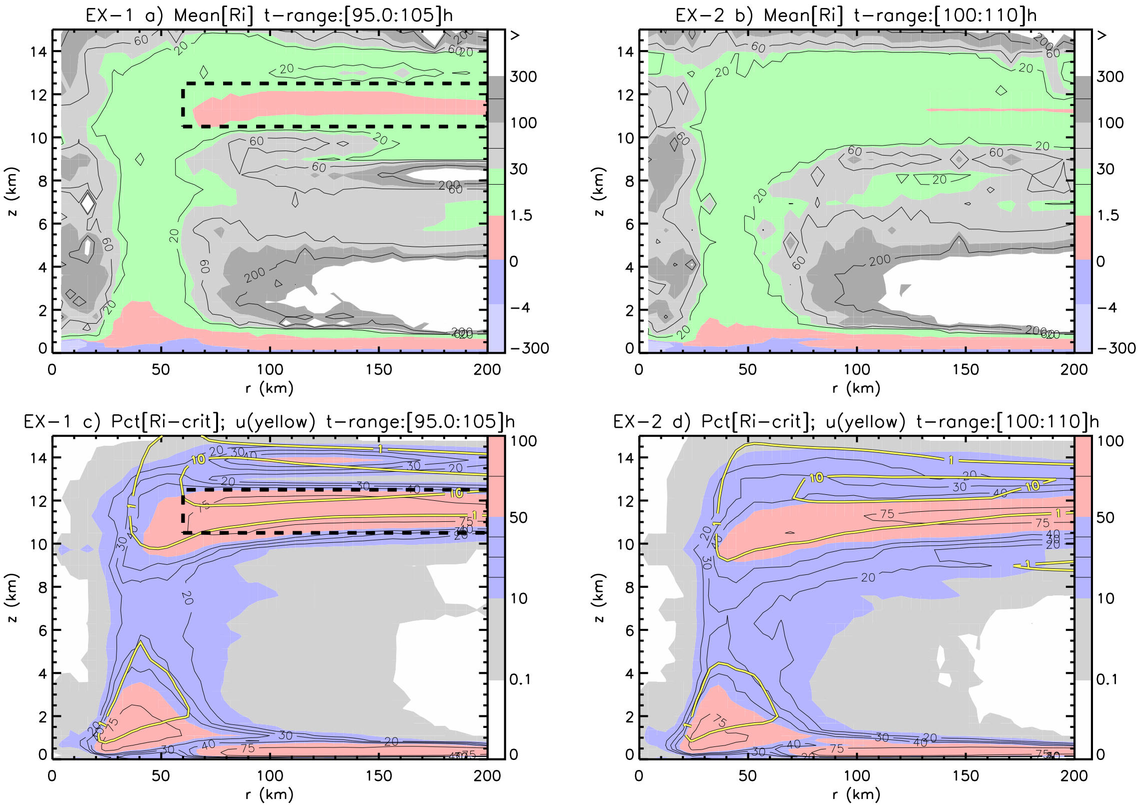

Figure 4. Panels (a) and (b) show radius-height structure of azimuthal-time-mean gradient Richardson number $\mathrm{Ri}_{\mathrm{CM1}}$ for experiments EX-1 and EX-2, respectively. Values are shown by shading indicated in the color bar. Supplemental contours of 20, 60, and 200 are shown also. Panels (c) and (d) show the relative frequency ('PCT') of occurrence of $0 \leq \mathrm{Ri}_{\mathrm{CM1}} \leq 1.5$ (marking near-criticality) at each (r, z) coordinate during the time intervals $95 \leq t \leq 105$ h and $100 \leq t \leq 110$ h for experiments EX-1 and EX-2, respectively. Frequency is shown by shading indicated in the color bar. Supplemental contours of 20, 30, 40, and 75 % are shown also. The azimuthal-time mean outflow jet is indicated by the yellow curves with contours of 1 and 10 m s^{-1}. The dashed box denotes an annulus of near-criticality referred to in the text.

this question, we use equivalent potential temperature θ_e as a proxy for moist entropy. Figure 5 quantifies the eddy and diffusive fluxes of θ_e that contribute to the redistribution of θ_e across M-surfaces in EX-1 for the same time period as Figure 4. All quantities plotted in this figure are time averaged over the time interval $95 \leq t \leq 105$ h for experiment EX-1 and averaged azimuthally also. Two challenges that arise in applying the ER11 theory should be noted here. First, the core of the outflow jet (Fig. 5a), presumably originating from the strongest updrafts found near the radius of maximum winds, is a region of large values of Richarson number (values up to 20 in Fig. 4a), thus spoiling a basic tenet of ER11. Second, the explicit assumption in ER11 (p. 2243) that there is a one-to-one relationship between M and saturation entropy is complicated in the simulation by the folding of the M-surfaces in the core of the outflow jet (Fig. 5c,d).

As evident in Figure 4, there is an annulus of criticality ($60 \leq r \leq 200$ km, $10.5 \leq z \leq 12.5$ km). Accordingly, in this region one would expect to find some footprint of the proposed mixing process. In Figure 4a, we see that this annulus is located in the outflow jet below the jet maximum. The two principal signals of mixing of entropy are the resolved radial (Figure 5e) and resolved vertical (Figure 5f) eddy fluxes of θ_e. The corresponding diffusive flux terms (Figures 5g and 5h) are found to be at least an order of magnitude smaller. The eddy fluxes are largest near the eyewall and are of lesser magnitude in the upper-level outflow (note the smaller contour values of panels g and h). Moreover, the region of Ri-criticality in the highlighted annulus shows no particularly strong signal in either the resolved eddy or diffusive flux terms.

On the basis of the foregoing evidence, it would seem that the rearrangement of entropy occurs mainly in the eyewall region and near-criticality, which is limited to the outflow, bears little relation to this rearrangement process. The identified eddy processes in the eyewall are largely realizations of asymmetric deep convection in the model and are not obviously governed by any Richardson number-based criterion.

4. Discussion and conclusions

We have used a small set of idealized, three-dimensional, numerical experiments to evaluate the premise of the revised theory of tropical cyclone intensification proposed by Emanuel (2012). In the revised theory, small-scale turbulence in the upper tropospheric outflow layer is hypothesized to determine the distribution of moist entropy and thermal stratification of the outflow and, in turn, *an amplification of the system-scale tangential wind field above the boundary layer*.

As noted in the Introduction, there are intrinsic differences between the behaviour of tropical cyclone vortices in axisymmetric and three-dimensional configurations (e.g. Persing et al. 2013). Since small-scale, shear-stratified turbulence is a local phenomenon and one that is not well approximated by axisymmetric rings, one should be cautiously skeptical of results from axisymmetric simulations that project this process into axisymmetric rings. The three

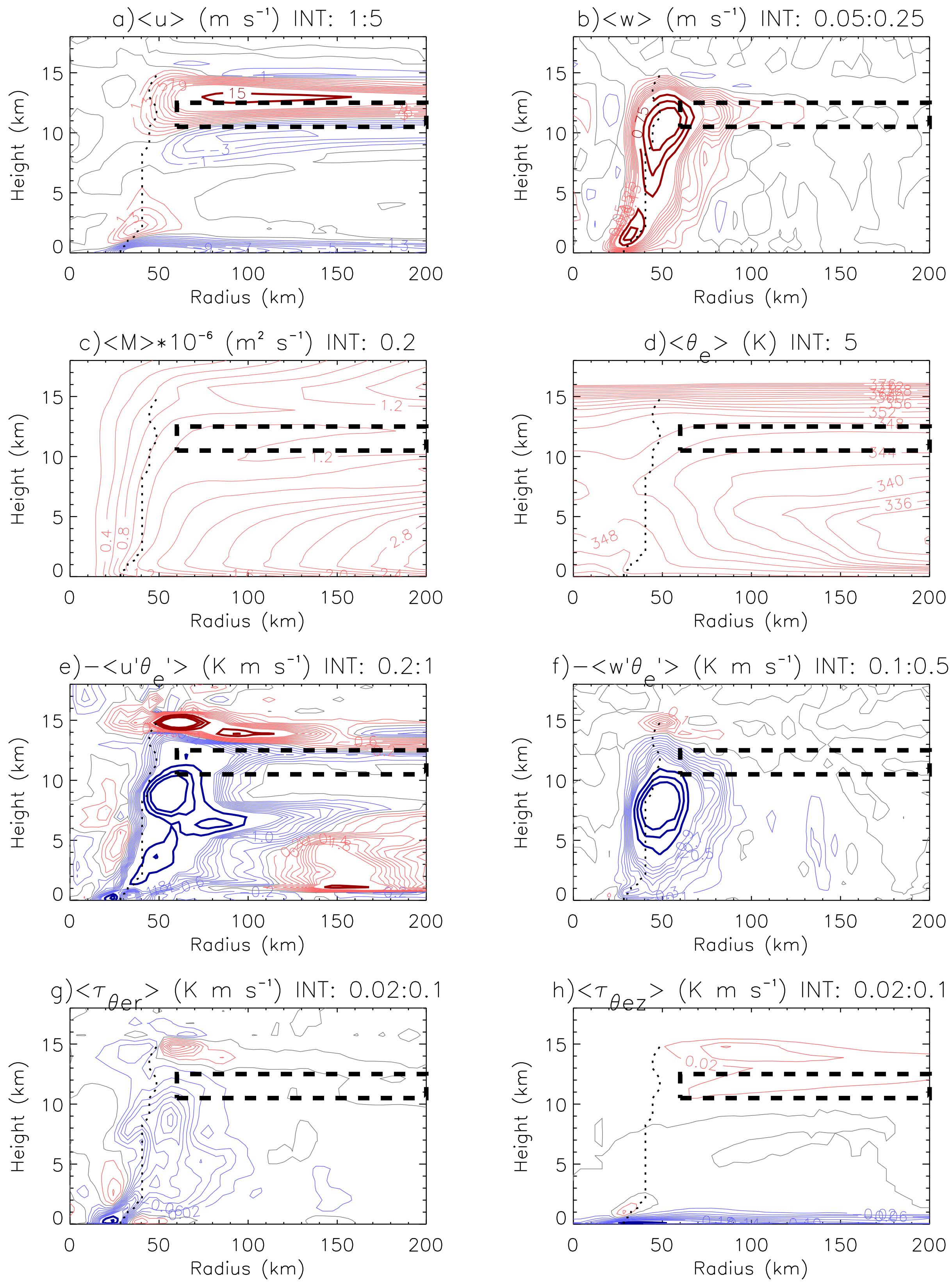

Figure 5. Contours of azimuthal-time-mean quantities (indicated by a bracket symbol) from EX-1 for a time period during vortex intensification, 95 to 105 h, plotted as a function of radius and height, with positive contours red, zero contour in grey, and negative contours in blue. The dotted curve shows the radius of the maximum azimuthally-averaged tangential wind as a function of height below 15 km height. (a) mean radial wind $\langle u \rangle$ with contour interval of 1 m s^{-1}. (b) mean vertical velocity $\langle w \rangle$ with contour interval of 0.05 m s^{-1} (thin) and 0.25 m s^{-1} (thick) beginning at 0.75 m s^{-1}. (c) mean absolute angular momentum $\langle M \rangle$, shown with values divided by 10^6, with contour interval of 0.2 m^2 s^{-1}. d) mean equivalent potential temperature $\langle \theta_e \rangle$ with contour interval of 5 K. (e) $-\langle u'\theta_e' \rangle$, the mean radial eddy flux of θ_e, with contour interval 0.2 K m s^{-1} (thin) and 1 K m s^{-1} (thick) starting at $\pm$3 K m s^{-1}. (f) $-\langle w'\theta_e' \rangle$, the mean vertical eddy flux of θ_e, with contour interval 0.1 K m s^{-1} (thin) and 0.5 K m s^{-1} (thick) starting at $\pm$1.5 K m s^{-1}. (g) $\langle \tau_{\theta_e r} \rangle$, the parameterized mean radial flux of θ_e corresponding to (e) with contour interval 0.02 K m s^{-1}. (h) $\langle \tau_{\theta_e z} \rangle$, the parameterized mean vertical flux of θ_e corresponding to (f) with contour interval 0.02 K m s^{-1}. The black dotted box is the same as that shown in Figure 4.

dimensional model should be regarded as the proper benchmark.

Compared to the control experiment in which the small-scale, shear-stratified turbulence is parameterized in the usual way based on a standard Richardson number criterion, the vortex in a calculation without any representation of vertical diffusion above the boundary layer evolves in a similar way with no significant difference in the intensification rate of the maximum azimuthally-averaged tangential velocity, upper-level outflow temperature, or outflow thermal stratification.

Despite the statement of Emanuel (2012) that "... the critical Richardson number hypothesis leads to predictions of storm evolution that are also, for the most part, in good accord with (axisymmetric, our insertion) numerical simulations", our three-dimensional calculations using plausibly realistic values of the subgrid-scale turbulence parameters cast strong doubt on the premise of the revised theory that small-scale, shear-stratified turbulence in the upper-level outflow of the developing vortex controls the intensification of a tropical cyclone. Even if the apparent force that is shown here to be a critical element of the revised intensification theory exists, it does not appear to manifest itself in the control experiment for realistic subgrid scale parameters.

We have offered an explanation for the different conclusions from the different numerical models and Richardson number closure formulations used by Emanuel (2012) and the present study. For the three-dimensional control experiment conducted herein, the rearrangement of moist entropy is found to occur mainly in the eyewall region. The diagnosed eddy processes in the eyewall are largely realizations of asymmetric deep convection in the model and are not obviously governed by any Richardson number-based criterion.

These findings are believed to be significant in the light of recent studies that invoke the revised theory for the determination of a universal tangential wind profile in hurricanes and as support for the integrity of a redefined Wind-Induced-Surface-Heat-Exchange (WISHE) intensification theory.

Acknowledgements

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Appendix A: Derivation of revised tendency equation and azimuthal force

Here we review the key approximations and assumptions underpinning the tendency equation of Emanuel's (2012; hereafter E12) revised intensification theory. In particular, we derive the tendency equation from the stated assumptions[11]. The derivation exposes the tangential force that is responsible for amplifying the maximum tangential wind at the top of the boundary layer in the new axisymmetric theory. Amidst the derivation, some additional questions arise about the theory and its proffered analytical solution.

The starting point of the revised theory is the axisymmetric equation for the depth-averaged boundary layer moist entropy s_b in absolute angular momentum ($M = rv + 1/2fr^2$) and pressure coordinates (Eq. (12) in E12):

$$h\frac{\partial s_b}{\partial \tau} - C_D r|\mathbf{V}|V\frac{\partial s_b}{\partial M} = C_k|\mathbf{V}|(s_0^* - s_b) + C_D\frac{|\mathbf{V}|^3}{T_s}, \tag{3}$$

where C_D and C_k are the surface exchange coefficients for momentum and enthalpy, $\mathbf{V}$ is the Reynolds-averaged velocity vector averaged across the boundary layer, s_0^* is the saturation moist entropy (defined below) at the sea surface temperature, T_s, and h is the boundary layer depth, assumed to be constant. Here τ is the dimensional time variable wherein partial derivatives with respect to τ hold M and p constant. The boundary layer depth $h = \Delta p_b/\rho g$, where Δp_b is the boundary layer depth (in pressure units), ρ is the density averaged over the boundary layer and g is the gravitational acceleration.

The two terms on the left hand side of the entropy equation are the local time tendency of s_b and the depth-averaged radial advection of s_b in M-coordinates. (The latter uses the fact that in M-coordinates the radial velocity u is defined by $u = DM/Dt$ and $hDM/Dt = -C_D r|\mathbf{V}|V$ from the angular momentum equation integrated over the boundary layer). The right hand side of the entropy equation consists, respectively, of the bulk-aerodynamic parameterization of the vertical transfer of moist entropy between the underlying ocean and the moist air at anemometer level and a bulk representation of dissipative heating (and corresponding entropy production). By definition, radial and vertical eddy entropy and angular momentum fluxes are zero in the axisymmetric theory.

The saturation moist entropy is given by

$$s^* = c_p \ln T - R_d \ln p + \frac{L_v q^*}{T} \tag{4}$$

where c_p is the specific heat of dry air at constant pressure, T is the temperature, R_d is the gas constant of dry air, p is the pressure, q^* is the saturation water vapour mixing ratio, and L_v is the latent heat of condensation.

The quantity s_0^* denotes the saturation entropy at the sea surface temperature

$$s_0^* = c_p \ln T_s - R_d \ln p_0 + \frac{L_v q_{v0}^*}{T_s}. \tag{5}$$

where p_0 is the surface pressure and q_{v0}^* is the surface saturated vapor mixing ratio. It will prove useful later to

[11]Although we accept here the starting equations for the purposes of gaining basic understanding of the E12 theory, this should not be interpreted as our endorsement of the starting assumptions when applied to real or simulated storms in a three-dimensional configuration. We would step back from endorsing the starting point of the derivation on the grounds that intensification is intrinsically non-axisymmetric and that M and θ_e surfaces only approach a state of congruence *after* the intensification process is already well under way (e.g., Kilroy *et al.* 2018, their Figure 6).

note that if we neglect the pressure dependence of s_0^* and use the environment values to evaluate this quantity, then the M derivative of s_0^* will vanish (see below for more).

The three principal approximations that are invoked to derive the tendency equation in the new theory are the following:

- (a) neglect the pressure dependence of s_0^*;
- (b) neglect dissipative heating;
- (c) approximate $|\mathbf{V}|$ and V by V_g

where V_g is the gradient wind.

In the eyewall region, moist air rises out of the boundary layer, rapidly condenses, and ascends in cloud along M surfaces (slantwise convective neutrality). Thus, along an M surface, the saturated entropy above the boundary layer is assumed equal to the originating boundary layer entropy, $s^* = s_b$, whereupon

$$V^2 = -(T_b - T_o)M\frac{\partial s^*}{\partial M}, \tag{6}$$

where T_b is the boundary layer temperature and T_o is the outflow temperature. This equation is a form of the thermal wind equation. The variation of the outflow temperature with M is then assumed to be controlled by the action of shear-stratified turbulence wherein the gradient Richardson number is bound to a near critical value

$$\frac{\partial T_o}{\partial M} = -\frac{\mathrm{Ri}_c}{r_t^2}\left(\frac{\partial s^*}{\partial M}\right)^{-1} \tag{7}$$

which is a turbulence closure assumption on the outflow. In this equation r_t denotes the radius where the gradient Richardson number first becomes critical in the outflow layer.

The moist entropy equation at the top of the boundary layer is then

$$h\frac{\partial s^*}{\partial \tau} - C_D V M\frac{\partial s^*}{\partial M} = C_k V(s_0^* - s^*) \tag{8}$$

Now $\frac{\partial}{\partial M}$ of Eq. (8) gives

$$h\frac{\partial}{\partial \tau}\frac{\partial s^*}{\partial M} - C_{\mathrm{D}}\frac{\partial}{\partial M}\left(VM\frac{\partial s^*}{\partial M}\right) = C_k\frac{\partial}{\partial M}[V(s_0^* - s^*)] \tag{9}$$

but Eq. (6) gives

$$\frac{\partial s^*}{\partial M} = \frac{-V^2}{(T_b - T_o)}\frac{1}{M}$$

whereupon, the first term on the left hand side of Eq. (9) becomes[12]

$$-\frac{h}{M}\frac{\partial}{\partial \tau}\left(\frac{V^2}{T_b - T_o}\right) \tag{10}$$

and the second term on the left hand side becomes

$$\begin{aligned} &- C_{\mathrm{D}}\frac{\partial}{\partial M}\left(VM\frac{\partial s^*}{\partial M}\right) \\ &= C_{\mathrm{D}}\frac{\partial}{\partial M}\left(\frac{V^3}{T_b - T_o}\right) \\ &= \frac{3C_{\mathrm{D}}V^2}{T_b - T_o}\frac{\partial V}{\partial M} + C_{\mathrm{D}}V^3\frac{\partial}{\partial M}\left(\frac{1}{T_b - T_o}\right). \end{aligned} \tag{11}$$

[12] Note that M is an independent variable so that $\partial M/\partial \tau = 0$.

Now E12 makes the additional approximation

$$\frac{\partial}{\partial M}\left(\frac{1}{T_b - T_o}\right) \approx \frac{1}{(T_b - T_o)^2}\frac{\partial T_o}{\partial M}$$

This approximation follows from the assumption that T_b, the temperature after integrating across the boundary layer, is assumed to be a constant in radius (and hence M).

At this point the turbulence closure assumption, Eq. (7), is applied, i.e.

$$\frac{\partial T_o}{\partial M} = -\frac{\mathrm{Ri}_c}{r_t^2}\left(\frac{\partial s^*}{\partial M}\right)^{-1}$$

and $\frac{\partial s^*}{\partial M}$ is substituted using Eq. (6) so that

$$\begin{aligned} \frac{\partial}{\partial M}\left(\frac{1}{T_b - T_o}\right) &= \frac{-1}{(T_b - T_o)^2}\frac{\mathrm{Ri}_c}{r_t^2}\left(\frac{\partial s^*}{\partial M}\right)^{-1} \\ &= \frac{-1}{(T_b - T_o)^2}\frac{\mathrm{Ri}_c}{r_t^2} \times -\frac{M(T_b - T_o)}{V^2} \\ &= \frac{M}{V^2(T_b - T_o)}\frac{\mathrm{Ri}_c}{r_t^2} \end{aligned} \tag{12}$$

The term on the right of Eq. (9) becomes

$$\begin{aligned} &C_k\frac{\partial}{\partial M}[V(s_0^* - s^*)] \\ &\quad = C_k(s_0^* - s^*)\frac{\partial V}{\partial M} + C_k V\frac{\partial}{\partial M}(s_0^* - s^*) \end{aligned} \tag{13}$$

and E12 writes the second term on the right hand side of this equation as

$$C_k V\frac{\partial}{\partial M}(s_0^* - s^*) \approx -C_k V\frac{\partial s^*}{\partial M} = \frac{C_k V^3}{M(T_b - T_o)}, \tag{14}$$

using Eq. (6) and neglecting the pressure dependence of s_0^* (as foreshadowed above).

Collecting the terms in Eq. (9) together now gives (using (10) - (14)),

$$\begin{aligned} -\frac{h}{M}\frac{\partial}{\partial \tau}\left(\frac{V^2}{T_b - T_o}\right) = &-\frac{3C_{\mathrm{D}}V^2}{T_b - T_o}\frac{\partial V}{\partial M} \\ &- \frac{C_{\mathrm{D}}VM}{(T_b - T_o)}\frac{\mathrm{Ri}_c}{r_t^2} \\ &+ C_k(s_0^* - s^*)\frac{\partial V}{\partial M} \\ &+ \frac{C_k V^3}{M(T_b - T_o)} \end{aligned} \tag{15}$$

or, cleaning up,

$$\begin{aligned} \frac{h}{M}\frac{\partial}{\partial \tau}\left(\frac{V^2}{T_b - T_o}\right) = &\frac{\partial V}{\partial M}\left[\frac{3C_{\mathrm{D}}V^2}{T_b - T_o} - C_k(s_0^* - s^*)\right] \\ &+ \frac{C_{\mathrm{D}}VM}{(T_b - T_o)}\frac{\mathrm{Ri}_c}{r_t^2} - \frac{C_k V^3}{M(T_b - T_o)} \end{aligned} \tag{16}$$

Multiplying the last equation by $M(T_b - T_o)/(hV)$ gives Eq. (16) of E12:

$$\frac{T_b - T_o}{V}\frac{\partial}{\partial\tau}\left(\frac{V^2}{T_b - T_o}\right) = \frac{M}{hV}\frac{\partial V}{\partial M}\left[3C_{\mathrm{D}}V^2 - C_k(T_b - T_o)(s_0^* - s^*)\right] + \frac{C_{\mathrm{D}}}{h}\frac{\mathrm{Ri}_c}{r_t^2}M^2 - \frac{C_k}{h}V^2 \quad (17)$$

At V_{m} (the maximum tangential wind), $\partial V/\partial M = 0$, whereupon Eq. (17) simplifies to

$$\frac{T_b - T_o}{V}\frac{\partial}{\partial\tau}\left(\frac{V^2}{T_b - T_o}\right) = \frac{C_{\mathrm{D}}}{h}\frac{Ri_c}{r_t^2}M^2 - \frac{C_k}{h}V^2. \quad (18)$$

E12 (p. 992) assumes that the outflow temperature at the RMW equals the tropopause temperature, i.e., $T_o = T_t$, with the latter assumed constant in time. This implies that the time derivative of T_o vanishes at the RMW. We can thus simplify the time derivative in the foregoing equation to obtain the tangential velocity tendency equation at the RMW:

$$\frac{\partial V_m}{\partial\tau} = \underbrace{\frac{C_D}{2h}\frac{\mathrm{Ri}_c}{r_t^2}M^2}_{>0} - \underbrace{\frac{C_k}{2h}V_m^2}_{>0}, \quad (19)$$

where V_m denotes the maximum tangential velocity at the top of the boundary layer.

The foregoing is a tendency equation for V_m forced by two terms on the right hand side. The first term is positive and denotes a new force per unit mass that increases V_m with time. This term is proportional to the drag coefficient and the critical gradient Richardson number. The second term is negative and arises in association with the depletion of tangential momentum in the boundary layer. Curiously, however, this second term is proportional to the enthalpy transfer coefficient C_k. One would ordinarily expect this term to be proportional to the drag coefficient C_{D}. E12 notes however that the equation is not yet closed and argues that it is possible that the global solution dependence on C_{K} may be different than would be apparent solely from an examination of this term at this stage in the derivation.

E12 proceeds to make an additional (and, in our view, unsubstantiated) assumption that the RMW always lies on the same M surface. Combining this assumption with an algebraic relation deduced from the revised steady-state theory (not written here), E12 derives an analytical solution for the evolution of V_m (his Eq. (19)). The novelty of the result notwithstanding, the apparent elegance of the analytical solution conceals the essential role of the azimuthal force in amplifying V_m.

The critical role of the tangential force may be exposed by repeating the foregoing derivation while discarding the closure equation for the outflow temperature. In this case, one obtains at the RMW

$$\frac{\partial V_m}{\partial\tau} = \frac{C_D {V_m}^2 M}{2h(T_b - T_t)}\frac{\partial T_o}{\partial M} - \frac{C_k}{2h}V_m^2, \quad (20)$$

where T_t denotes the tropopause temperature at the RMW (assumed equal to T_o and independent of time). While other outflow closures are conceivable that would, in turn, change the specification of $\partial T_o/\partial M$, if one employs the traditional Emanuel formulation of a constant outflow temperature, then $\partial T_o/\partial M$ would be identically zero. The resulting tendency equation for V_m in this case consists only of the second term on the right hand side of Equation (20), which is negative definite. Thus without the new force in the traditional formulation of the upper boundary condition, the vortex spins down! Notwithstanding this fact, we have a more fundamental issue with the physics encompassed by Equation (20). We find it puzzling how, in reality, the stratification of the outflow layer ($\partial T_o/\partial M$) would act to move the M surfaces inwards in a way to amplify the tangential wind at the top of the boundary layer. A similar remark would apply to Equation (19), in which $\partial T_o/\partial M$ has a specific parameterization.

Appendix B: Model parameters used in numerical experiments

This appendix documents the common numerical parameters used in the numerical experiments presented in this study and those parameters that relate to specific decisions made in specifying the EX-1 simulation. Although there is some repetition of material with that of Section 2, we list all of the pertinent model parameters for completeness. This documentation would allow the reader to download the numerical model and repeat the experiments presented herein. The numerical model, CM1, is publicly available at George Bryan's UCAR webpage (www2.mmm.ucar.edu/people/bryan). Version 14 (CM1v14) is used in this study; most of these parameters are common to later versions of CM1, others though are no longer supported. A name list file is used to set many parameters of the simulation at the time of execution. Simulations were performed on a Red Hat Linux cluster, kernel release 2.6.32-358.13.1.e16.x86_64 dated 17 June 2013 for x86_64 architecture using the Portland Group compiler with NetCDF support.

The grid mesh is a stretched grid in the horizontal and vertical with origin at the center of the domain (*iorigin=2*). In the middle of the domain in the horizontal is a fine-mesh grid region 405 × 405 km square with fixed grid spacing of 3 km in both the x- and y-directions. The grid configuration is established in the name list file with 185 grid points in the x- and y- directions (*nx=185, ny=185*) with stretching (*stretch_x=1, stretch_y=1*), an inner grid spacing of 3 km (*dx_inner=3000.0, dy_inner=3000.0*), an outer grid spacing of 100 km (*dx_outer=100000.0, dy_outer=100000.0*), a no-stretch length of 405 km (centered on the origin at the middle of the domain) (*nos_x_len=405000.0, nos_y_len=405000.0*), and total domain size of 2980 × 2980 km (*tot_x_len=2980000.0, tot_y_len=2980000.0*).

The vertical grid mesh has 40 points (*nz=40*), the separation of which is stretched in the vertical (*stretch_z=1*) with a top boundary of 25 km (*ztop=25000.0*). The nominal vertical grid spacing at the lowest level is 50 m (*dz_bot=50.0*) and at the top is 1200 m (*dz_top=1200.0*). The vertical grid stretching is applied throughout the full depth of the model atmosphere (*str_bot=0.0, str_top=25000.0*).

The equations of Bryan and Fritsch (2002) with a Runge-Kutta integrator with condensation adjustment is used (*neweqts=2*). The horizontal and vertical advection use a 5th-order scheme (*hadvorder=5, vadvorder=5*) and diffusion uses the recommended 6th-order scheme with diffusion coefficient 0.04 (*difforder=6, kdiff6=0.040*) which is in addition to the parameterized turbulence below. A vertically implicit Klemp and Wilhelmson (1978) time-splitting is used for acoustic modes (*psolver=3*), with six small time steps for each large time step (*nsound=6*). The vertically-implicit acoustic solver uses an off-centering coefficient of 0.60, which is slightly forward in time (*alph=0.60*). Potential temperature is not integrated on the small time steps (*thsmall=0*). The coefficient for a divergence damper is 0.1 (*kdiv=0.10*). As discussed in section 2, parameterized turbulence is used (*iturb=3*) with vertical and horizontal mixing length scales $l_v = 50$ m and $l_h = 700$ m (*l_v=50.0, l_h=700.0*) based on the recent observational findings of Zhang et al. (2011a) and Zhang and Montgomery (2012), and the resulting vertical and horizontal eddy diffusivities that are output in the model simulations. These values are also close to the values recommended by Bryan (2012) in order to produce realistic hurricane structure.

The fixed Coriolis parameter on an f-plane is 5×10^{-5} s^{-1} (*fcor=0.00005*).

The sea-surface temperature is 26.14 C (*tsurf=299.29*) with environmental surface pressure of 1015.1 mb (*psurf=101510.0*). The use of a simple bulk aerodynamic drag scheme (*idrag=1*) requires the use of a no-slip lower and upper boundary condition (*bcturbu=3*) setting; the treatment by the bulk aerodynamic scheme of the lower boundary in the CM1 code overrides the no-slip condition there.

A zero-flux condition is imposed at the top and bottom boundary for scalars (*bcuturbs=1*); the treatment by the bulk exchange scheme of the lower boundary in the CM1 code (*isfcflx=1*) overrides the zero-flux condition. For simplicity, we employ constant values for the drag coefficient (C_{D}) and exchange coefficient (C_k)(*cecd=1*): $C_{\mathrm{D}} = 2.58 \times 10^{-3}$ and $C_k = 1.29 \times 10^{-3}$ (*cnstcd=0.00258, cnstce=0.00129*). The value for C_k is close to the mean value (1.2×10^{-3}) derived from the Coupled Boundary Layers/Air-Sea Transfer (CBLAST) experiment (Fig. 6 of Black et al. (2007); Fig. 4 of Zhang et al. (2009)), a recent laboratory study (Fig. 1 of Haus et al. (2010)) near and slightly above marginal hurricane wind speeds, and an energy and momentum budget analysis of the lower-tropospheric eyewall region at major hurricane wind speeds (Bell et al. 2012b). The value C_D is set to be twice the enthalpy exchange coefficient $C_{\mathrm{D}} = 2 \times C_k = 2.58 \times 10^{-3}$, and is close to the estimated mean value of $C_{\mathrm{D}} = 2.4 \times 10^{-3}$ from observations derived from CBLAST for major hurricane wind speeds by Bell et al. (2012b).

Open radiative boundary conditions are used on the lateral boundaries (*wbc=2, ebc=2, sbc=2, nbc=2*) employing the Durran and Klemp (1982) scheme (*irbc=4*); the outward flux is not restricted (*roflux=0*).

Rayleigh damping is applied at the upper boundary (*irdamp=1*) above a height of 20 km (*zd=20000.0*) with an inverse e-folding time scale of $1/300$ s^{-1} (*rdalpha=3.333e-3*). Rayleigh damping is turned off at the lateral boundaries (*hrdamp=0*). Dissipative heating is not included (*idiss=0*).

The simple Rotunno and Emanuel (1987) rainfall scheme is used (*ptype=6*) with a fixed fall speed of 7 m s^{-1} (*v_t=7.0*). Positive definiteness of moisture is ensured by a redistribution of moisture from neighboring cells (*pdscheme=1*).

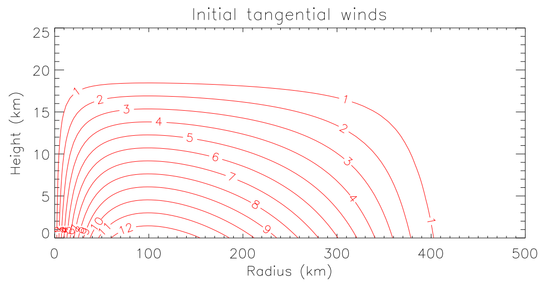

Figure 6. The initial tangential wind field (m s^{-1}) used in all numerical experiments presented.

As an expedient for radiative cooling, we follow Rotunno and Emanuel (1987) and choose simple Newtonian relaxation to the initial basic state sounding of potential temperature (*rterm=1*).

For EX-5 and EX-6, the foregoing settings are the same except that the vertical grid mesh has 100 points (*nz=100*) and the vertical grid is unstretched (*stretch_z=0*). Also, the fixed vertical grid spacing is set to 250 m (*dz=250.0*; the stretched grid mesh does not use the *dz* setting).

The initial vortex (Fig. 6) is provided in the public distribution of CM1v14, using name list option *iinit=7*. We use parameters for the nominal maximum tangential wind $V = 15$ m s^{-1}, the nominal radius of maximum tangential wind $R = 82.5$ km (although in practice the wind maximum is slightly weaker and the radius of maximum tangential wind is larger than the nominal values), the outer radius $R_0 = 412.5$ km, and the upper height for the vortex of $Z = 20$ km. The initial vortex is defined over the region $(r < R_0, z < Z)$ by

$$v(r,z) = \frac{Z-z}{Z}\left[\sqrt{V^2 \frac{r}{R}\left[G(r) + \frac{f^2 r^2}{4}\right]} - \frac{fr}{2}\right\}, \tag{21}$$

where

$$G(r) = \left(\frac{2R}{r+R}\right)^2 - \left(\frac{2R}{R_0+R}\right)^2 \tag{22}$$

The basic state sounding is specified from an input file (*isnd=7*). The sounding used is obtained following the method of Rotunno and Emanuel (1987) for producing a near-neutral sounding.

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