How important is the isothermal expansion effect to elevating equivalent potential temperature in the hurricane inner-core?

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We examine aspects of the thermodynamic structure of mature Atlantic Hurricane Earl (2010) based on airborne dropwindsondes released from the upper troposphere during the National Aeronautics and Space Administration (NASA), Genesis and Rapid Intensification Processes (GRIP) experiment. Vertical sounding profiles of the data raise questions concerning the relative roles of isothermal expansion and relative humidity increase in elevating the equivalent potential temperature of air parcels spiralling inwards to the eyewall convection region. The observational results obtained for two successive days of this Category 4 hurricane show that the isothermal expansion effect leads to roughly one half of the increment in equivalent potential temperature for boundary-layer air parcels moving between the region outside the eyewall and the eyewall and eye region. The analysis corroborates prior views on the importance of the dependence of reducing surface pressure in the determination of boundary-layer equivalent potential temperature.

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1. Introduction

The generally accepted axisymmetric paradigm for the zero-order structure a mature hurricane assumes that, as air parcels ascend along the eyewall, they conserve their absolute angular momentum, M, and saturation pseudo-equivalent potential temperature, θ_e^*, so that M and θ_e^* surfaces are congruent (Emanuel 1986, henceforth E86). In addition, the paradigm assumes explicitly that the tangential flow above the boundary layer is in gradient wind balance. An important constraint in the model is rate at which M and θ_e^* vary with radius in the boundary layer inside the radius of maximum tangential wind speed (r_m), which E86 assumes to be located at the outer edge of the eyewall (Figure 1). A brief summary of the model formulation is contained in section 2 of Smith *et al.* (2008).

While the steady-state model has undergone a number of reincarnations over the years (Emanuel 1988, Emanuel 1995, Bister and Emanuel 1998, 2002, Emanuel 2004, Emanuel and Rotunno 2011, Emanuel 2012), the foregoing aspects have remained unchanged. An important feature of the model is the increase in θ_e^* with diminishing radius in the vicinity of the eyewall updraught. Such a feature had been documented earlier from observational analyses (Hawkins and Imbembo 1976) and has been confirmed by more recent work (Montgomery *et al.* 2006, Marks *et al.* 2008, Bell and Montgomery 2008). Since the virtual temperature, θ_v, in cloud increases monotonically with θ_e^*, θ_v must increase also with decreasing radius at a given pressure level, consistent with the warm core structure of the vortex. Because the M and θ_e^* surfaces flare outwards with height, ascending air parcels move to larger radii, implying that the tangential wind speed decreases with height as required also by the thermal wind equation (E86).

It is commonly assumed that the increase in boundary-layer θ_e, and hence in θ_e^* above the boundary layer, with decreasing radius is dominated by high surface moisture fluxes (e.g., Rotunno and Emanuel 1987, their Section 4b).

It can be shown that, to a reasonable first approximation, the radial variation in near-surface θ_e can be written as[1]:

$$\Delta\theta_e = \Delta\theta + \frac{L}{c_p\pi}\Delta r_v, \quad (1)$$

where r_v is the water vapour mixing ratio, L is the coefficient of latent heat per unit mass, $\pi = (p/p_o)^\kappa$ is the Exner function, p is the pressure, p_o is a reference pressure, and $\kappa = R_d/c_p$, R_d is the specific gas constant for dry air and c_p is the specific heat of dry air per unit mass. Here Δ represents the increase in the indicated quantity between a given radius and the environment. If there were no heat or moisture sources, θ and r_v would be conserved and there would be no change in θ_e, but the temperature would decrease with decreasing pressure. Observations (including those to be presented) indicate that the low-level inflow into a hurricane is nearly isothermal, which implies that there must be a sensible heat flux from the ocean. It is this flux that elevates θ through the first term in Equation 1. Because the saturation mixing ratio, r_v^*, increases with decreasing pressure, isothermal expansion would lead to a reduction in the relative humidity in the absence of sufficient surface moisture fluxes. In reality, of course, the moisture flux is considerable and the second term on the right-hand-side of Equation 1 is not only positive, it may considerably exceed the first term.

At this stage it is insightful to write $r_v = RHr_v^*$, where RH is the relative humidity. Then Equation (1) becomes

$$\Delta\theta_e = \Delta\theta + \frac{L}{c_p\pi}RH \times \Delta r_v^* + \frac{L}{c_p\pi}r_v^* \times \Delta RH, \quad (2)$$

We refer to the contributions from the three terms on the right-hand-side of this equation as $\Delta\theta_{e1}$, $\Delta\theta_{e2}$, and $\Delta\theta_{e3}$, respectively. One can envisage a situation in which the surface moisture flux is just sufficient to keep the relative humidity constant. Then $\Delta\theta_{e3} = 0$ and $\Delta\theta_{e2}$ represents the increase in θ_e from the moisture flux in this situation. Clearly, then, $\Delta\theta_{e3}$ must be positive in order to raise the relative humidity of inflowing air.

The premise of the air-sea interaction model of Malkus and Riehl (1960) and E86 (and later refinements) is that isothermal expansion, by itself (i.e. $\Delta\theta_{e3} = 0$), cannot provide a sufficient increment in θ_e to support a strong hurricane. In other words, latent heat transfer over and above that required to maintain the relative humidity in the presence of isothermal expansion is assumed to be crucial for storm maintenance. This view was supported by the numerical model calculations of Rotunno and Emanuel (1987, their Sec. 4b) who concluded that "... latent heat transfer beyond that due to isothermal expansion is responsible for more than half the inward increase in θ_e." It provided also a foundation for the so called Wind Induced Surface Heat Exchange (WISHE) mechanism of intensification and maintenance. This theory, is based on the idea that surface enthalpy fluxes increase with the surface wind speed so that as the storm becomes more intense, so do the surface fluxes, leading to a feedback process (Emanuel

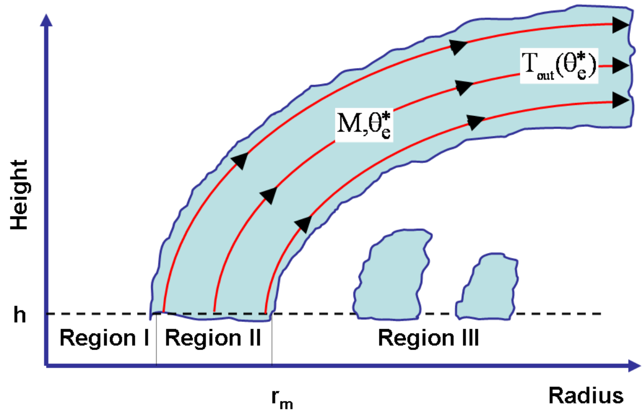

Figure 1. Schematic diagram of Emanuel's 1986 model for a mature steady-state hurricane. The boundary layer is assumed to have constant depth h and is divided into three regions as shown: the eye (Region I), the eyewall (Region II) and outside the eyewall (Region III) where spiral rainbands and shallow convection emanate into the vortex above. The absolute angular momentum per unit mass, M, and equivalent potential temperature, θ_e of an air parcel are conserved after the parcel leaves the boundary layer and ascends in the eyewall cloud. The precise values of these quantities depend on the radius at which the parcel exits the boundary layer. The model assumes that the radius of maximum tangential wind speed, r_m, is located at the outer edge of the eyewall cloud, whereas recent observations (e.g. Marks et al. 2008, Fig. 3) indicate it is closer to the inner edge.

et al. 1994, Montgomery *et al.* 2009, Figure 1). There are a number of caveats in this assumed feedback, which are discussed and appraised by Montgomery *et al.* (2009)[2].

The more recent study by Montgomery *et al.* (2009) questioned the need for greatly augmented latent heat fluxes and, in particular, the need to allow surface fluxes to increase with wind speed beyond some nominal Trade-Wind value, say 10 m s^{-1}, showing that a vortex in both a three-dimensional and axisymmetric model simulation still intensifies to a mature vortex, but at a somewhat reduced rate. The mean intensity was found to be only slightly less than that in the un-capped flux experiments.

These studies motivate a fundamental question: framed in the context of Equation (2), what is the relative contribution of the increase in eyewall θ_e arising from isothermal expansion and the elevation of the boundary layer relative humidity? The data presented here provide an opportunity to estimate the relative contribution of the various terms in this equation from high-density observations of a major hurricane.

2. Data

In the late summer of 2010, a trio of field experiments[3] was conducted by the National Aeronautics and Space

[1]The approximate formula for θ_e is $\theta_e = \theta\exp(Lr_v/(c_pT)$, whete T is the temperature at the lifting condensation level and other quantities are defined in the text. Since $L/(c_pT)$ is $O(1)$ and $r_v << 1$, the exponential terem can be linearized to a first approximation.

[2]As a note of caution, the mechanism proposed by Rotunno and Emanuel (1987) and Emanuel *et al.* (1994) is quite different from that described by Kepert (2010, p13), who interprets WISHE in the context of a steady-state vortex as "The role of the surface enthalpy fluxes in making the expansion of the inflowing boundary layer air isothermal rather than adiabatic ... ." In the context of Equations 1 or 2, Kepert associates the elevation of boundary-layer θ_e with just the first term on the right-hand side. Despite this, Kepert does not mention the necessity of the wind-speed dependence of the fluxes of latent and sensible heat and does not make a distinction between dry and moist enthalpy in his discussion.

[3]The experiments included the Genesis and Rapid Intensification Processes (GRIP) project of NASA, the Intensity Forecasting Experiment (IFEX) of the NOAA, and the Pre-Depression Investigation of Cloud Systems in the Tropics (PREDICT) experiment of NSF.

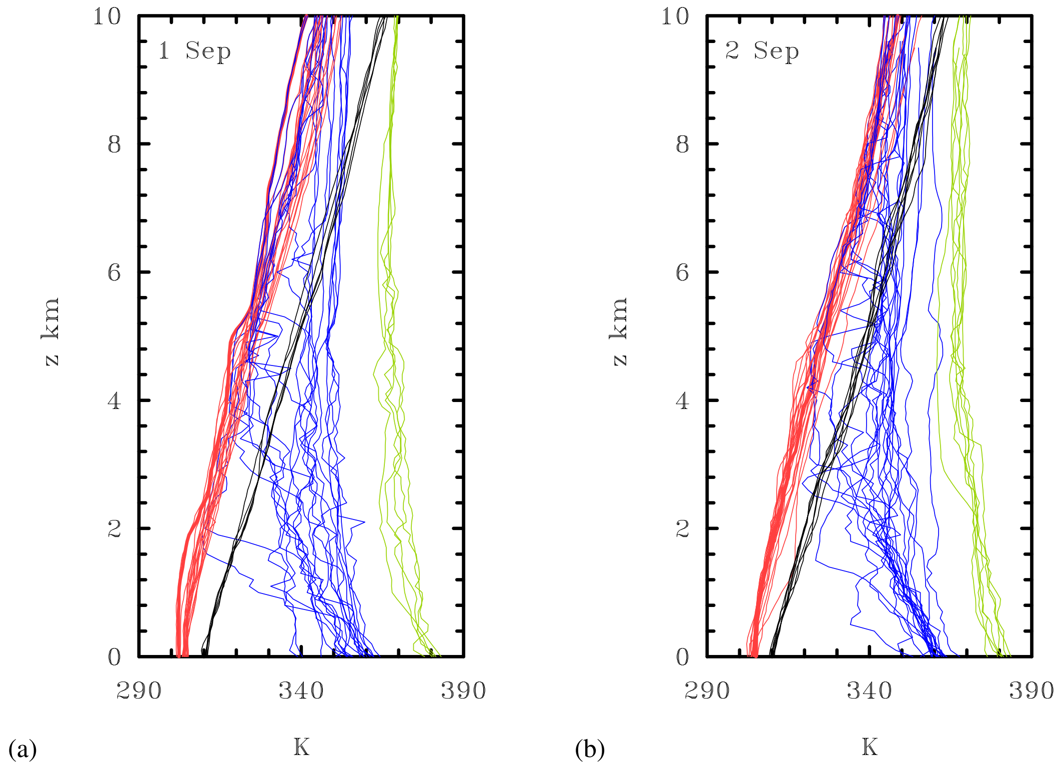

Figure 2. All soundings of θ_v (red/black curves) and θ_e (blue/green curves) for Hurricane Earl on (a) 1 september, and (b) 2 September, 2010. The black and green profiles are those for the eye/eyewall region while the red and blue profiles are for soundings made at larger radius.

Administration (NASA), National Oceanic and Atmospheric Administration (NOAA) and National Science Foundation (NSF) to investigate a range of questions related to the genesis, rapid intensification and mature structure of Atlantic and Carribean hurricanes. Two of these, the IFEX and GRIP experiments carried out a series of airborne measurement missions to obtain data in Hurricane Earl. Some of the measurements were made by the NASA DC8 research aircraft, which, has the capability to release dropwindsondes from moderately high altitudes in the troposphere ($\approx 10-11$ km). These data provide an unprecedented set of measurements during multiple penetrations of the storm on two days and are sufficient to allow an examination of the fundamental question articulated above.

3. Hurricane Earl: 1-2 September

Figure 2 shows the vertical profiles of all dropwindsonde soundings made by the NASA DC-8 during the two missions into Hurricane Earl on 1 and 2 September 2010. There were 25 soundings on 1 September and 29 soundings on 2 September. Additional soundings were made by the NOAA P3 and G IV aircraft, but the former were from a much lower altitude and the latter only in the storm environment, typically beyond 250 km from the centre. These additional soundings are not used here as the DC8 soundings are believed to give a sufficiently large sample for the analysis described. On each day, the figure indicates a natural division of the soundings into two bins: those in the eyewall or eye, which have significantly higher values of θ_e and are distinctly warmer than the latter in terms of virtual potential temperature, and those at larger radius. The eyewall profiles of θ_e can be distinguished from those in the eye as they are almost vertical, a feature that is suggestive of moist adiabatic ascent up to flight level, bearing in mind that eyewall tends to flare outwards with height. The soundings at larger radii were made within a radius of about 250 km from the storm centre.

Taking the subdivision of soundings suggested by Figure 2, one can construct ‘bin-means’ of various quantities in the two ‘sounding bins’. Table I compares differences in ‘bin-mean’ values of various thermodynamic quantities at the surface and at a height of 200 m above the surface. Note the consistency in the various quantities on the two successive days of observation. In particular, the surface temperature on 1 September decreases by 0.4 C between the outer region and eyewall region and on 2 September it increases very slightly by 0.2 C. At a height of 200 m, there is no temperature change on 1 September, but a 0.7 C increase on 2 September. These data affirm the approximate isothermal nature of the expansion of inflowing air parcels. In the absence of sensible heat transfer, adiabatic cooling would result in a temperature decrease of about 5-6 degrees Celsius. The corresponding increase in surface mixing ratio is 6 g kg^{-1} on 1 September and 4.5 g kg^{-1} on 2 September and the relative humidity increases from 79% to 99% on 1 September and from 87% to 99% on 2 September. The corresponding increases at a height of 200 m are 5.7 g kg^{-1} in the mixing ratio and from 81% to 99% in relative humidity on 1 September and 4.5 g kg^{-1} in the mixing ratio and from 89% to 99% in relative humidity on 2 September, i.e. practically the same as at the surface. The surface pressure reduction is on the order of 60 mb on 1 September and 50 mb on 2 September. These data are used to estimate the terms in Equations (1) and (2) in the next section.

Date	location of mean	p mb	T^{o}C	q_v g kg^{-1}	RH %	θ K	θ_v K	θ_e K
Sep 1	outside eyewall	1003.1	27.6	18.7	79	300.5	303.9	355.5
	in eyewall	938.3	27.2	24.7	99	305.9	310.5	380.8
Sep 2	outside eyewall	997.6	27.4	20.4	87	300.7	304.5	361.1
	in eyewall	948.0	27.6	24.9	99	305.4	310.0	380.8
Sep 1	outside eyewall	980.8	25.9	17.7	81	300.7	304.0	353.0
	in eyewall	917.5	25.9	23.4	99	306.6	310.9	377.6
Sep 2	outside eyewall	975.5	25.8	19.5	89	301.2	304.7	358.8
	in eyewall	927.0	26.5	24.0	99	306.2	310.7	379.1

Table I. Mean surface data in the eye and eyewall and outside the eyewall in Hurricane Earl on 1 and 2 September 2010. Upper values are for the surface, and lower values are for 200 m above the surface.

Date	$\Delta\theta_{e1}$ K	$\Delta\theta_{e2}$ K	$\Delta\theta_{e3}$ K	$\Delta\theta_e$ K	$\Delta\theta_{e\,iso}$ K	$\Delta\theta_{e\,iso}/\Delta\theta_e$ %	$\Delta\theta_{e\,obs1}$ K	$\Delta\theta_{e\,obs2}$ K
Sep 1	5.4	3.8	12.1	21.2	9.2	43	21.5	25.3
Sep 2	4.7	3.0	7.3	15.0	7.7	51	15.0	19.7
Sep 1	5.9	3.6	10.2	19.6	9.5	48	21.3	24.6
Sep 2	5.0	2.9	5.9	13.8	7.9	57	17.2	20.3

Table II. Estimates of $\Delta\theta_e$ in Equation 1 and the three contributions thereto: $\Delta\theta_{e1}$, $\Delta\theta_{e2}$, and $\Delta\theta_{e3}$, using the values of relavant quantities in Table I. These estimates are based on the use of the lineaar approximation for θ_e. Listed also is the isothermal contribution, $\Delta\theta_{e\,iso} = \Delta\theta_{e1} + \Delta\theta_{e2}$, the ratio of this to the total contribution $\Delta\theta_{e\,iso}/\Delta\theta_e$ expressed as a percentage. The two right columns give the observed change, $\Delta\theta_{e\,obs1}$, calculated using the linear approximation to θ_e, and $\Delta\theta_{e\,obs2}$, calculated using the more accurate formula of Bolton. Upper values are for the surface, lower values for 200 m above the surface.

4. Results and Interpretation

Table II shows estimates of $\Delta\theta_e$ in Equation (2) and the three contributions thereto: $\Delta\theta_{e1}$, $\Delta\theta_{e2}$, and $\Delta\theta_{e3}$ defined above, using the values of relavant quantities[4] in Table I. It lists also the isothermal contribution to the total change, $\Delta\theta_{e\,iso} = \Delta\theta_{e1} + \Delta\theta_{e2}$, and the fractional contribution of this term as a percentage. These estimates are based on the use of the linear approximation for θ_e. The total change is compared with those computed directly from the observations, $\Delta\theta_{e\,obs1}$, which uses the linear approximation for θ_e, and $\Delta\theta_{e\,obs2}$, which uses the more accurate Bolton's formula (Bolton 1980).

The increase in θ_e with decreasing radius on account of sensible heat input during the isothermal expansion of air parcels ($\Delta\theta_{e1}$) is about 5-6 K, while the contribution by latent heat input through surface evaporation to maintain the relative humidity ($\Delta\theta_{e2}$) is slightly less, about 3-4 K. The increase in θ_e associated with the moisture contribution that boosts the relative humidity ($\Delta\theta_{e3}$) is about 10-12 K at the surface, but only 6-7 at a height of 200 m. The total isothermal contribution ($\Delta\theta_{e\,iso}$) accounts for between 40% and 60% of the total change.

At the surface, the total increments in θ_e in Equation 1 are about 3-5 K smaller than those determined directly from the data using Bolton's formula, consistent with the underestimate in θ_e provided by the linear formula in the range 8-12 K for the range of values found in the boundary layer of a hurricane.

Note that if one uses the subdivision of terms represented in Equation (1), the contribution to the elevation of θ_e by evaporation, effectively $\Delta\theta_{e2} + \Delta\theta_{e3}$, is substantially larger than the contribution from the sensible heat flux, $\Delta\theta_{e1}$, being in the range 64%-75% of the total change, $\Delta\theta_e$, in the data presented in Table II.

While the values in Table II are very similar at a given height on the two days of observation, there is likely to be some variation with storm intensity and from storm to storm. For example, in their control calculations, Rotunno and Emanuel (1987, p557) reported a value for $\Delta\theta_e$ of 11.6 K in going from a radius of 150 km to 20 km in a mature storm with a maximum *tangential* wind speed of about 45 m s^{-1}, with corresponding values $\Delta\theta_{e1} = 2.8$ K, $\Delta\theta_{e2} = 2.1$ K, and $\Delta\theta_{e3} = 6.7$ K. Then $\Delta\theta_{e\,iso} = 4.9$ K, or 42% of $\Delta\theta_e$, which is at the lower end of values found in Hurricane Earl. Correspondingly, $\Delta\theta_{e3}$ is 58% of $\Delta\theta_e$, which is at the upper end of the values obtained for Hurricane Earl.

5. Discussion

The elevation of the boundary-layer mixing ratio is a key feature of the widely accepted air-sea interaction paradigm (WISHE) of tropical-cyclone intensification referred to in section 1. Although an increase in mixing ratio is found observationally in the present study, it cannot be interpreted directly as support for the WISHE feedback mechanism, since an elevation of the mixing ratio by surface evaporation does not require the moisture flux to increase with wind speed.

The fact that the second term on the right-hand side of Equation 1, $\Delta\theta_{e2}$, is a comparatively small fraction of the total contribution $\Delta\theta_e$ (typically 20%) supports an approximation made in a recent revised version of the E86 steady-state model (Emanuel and Rotunno 2011).

Our analysis here raises an interesting question, namely: is the boost in near-surface relative humidity represented by the third term on the right-hand-side of Equation 2 necessary for tropical-cyclone intensification, or does it suffice that there is merely some elevation of the boundary-layer θ_e with diminishing radius as discussed in the Introduction? We are unaware of any physical principle that

[4] For the purpose of computing the saturation mixing ratio in the expression for $\Delta\theta_{e2}$, we used the mean temperature of the eye/eyewall region and the region outside.

requires such an increase in near-surface relative humidity to support intensification in theory, although such an elevation might be needed to maintain convective instability in the presence of a developing warm core aloft.

In the context of his steady-state model, E86 argues that: “isothermal expansion by itself could not provide a large enough increase in θ_e to support an intense cyclone.” In his paper, he derive an expression for the central pressure (his Equation (26)), “which shows that a transfer of heat above and beyond that associated with isothermal expansion is needed to support a tropical cyclone.” In fact, the corresponding formula for the square of the maximum tangential wind speed is proportional also to the increment in boundary-layer relative humidity between the environment and just inside the radius of maximum tangential wind speed (RMW) (see his Equation (43)). However, based on the idea that convective downdraughts limit the increase in relative humidity beyond this radius, Emanuel *assumed* that the relative humidity is constant beyond the RMW, so that the foregoing deduction from his Equations (26) or (43) would seem to be built into the theory. Even though the theory was revised by Emanuel (1995), the formula for the square of the maximum tangential wind speed remained identical.

While Emanuel’s assertions might be interpreted as answering the foregoing question, at least for the maximum intensity in the steady state, we believe that the question merits further investigation. We think that an examination of this question might be relevant to obtaining an improved understanding of processes responsible for rapid intensification.

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