On the existence of the logarithmic surface layer in the inner core of hurricanes

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We question recent studies invoking the existence of a traditional 'logarithmic surface layer', or log layer, in the boundary layer of the rapidly rotating core of a hurricane. One such study argues that boundary-layer parametrization schemes that do not include a log layer are 'badly flawed'. Another study assumes the existence of a log layer to infer drag coefficients at hurricane wind speeds. We provide theoretical reasoning supported by observational evidence as to why significant departures from the normally assumed logarithmic layer might be expected, questioning its use in the inference of the drag coefficient at high wind speeds and laying bare suggestions that hurricane models using boundary-layer schemes that do not represent the log layer should not be used. The ramifications of these findings for hurricane modelling are discussed. Finally, we draw attention to a study examining a range of boundary-layer schemes demonstrating that a recently articulated boundary-layer spin-up mechanism transcends the presence of a log layer.

Key Words: hurricanes; tropical cyclones; typhoons; surface layer; logarithmic layer, spin-up

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1. Introduction

The importance of the boundary layer in tropical cyclones has been recognized for several decades because the frictional breakdown of gradient wind balance leads to strong inflow in the layer. This inflow converges moisture evaporated from the sea surface to feed the deep convective clouds in the storm's inner core.

The boundary layer is a key element of Ooyama's seminal axisymmetric tropical cyclone model (Ooyama, 1969), Carrier's hurricane model (Carrier *et al.*, 1971; Carrier, 1971a,b) as well as Emanuel's (1986) model for an axially symmetric, steady-state hurricane. Emanuel's model became the basis for a widely used theory for the potential intensity (PI) of a hurricane, i.e. the maximum gradient wind speed (Bister and Emanuel, 1998; Emanuel and Rotunno, 2011). In all of these models, the boundary layer is treated as a layer of air of constant density. In the models of both Ooyama and Emanuel, the boundary layer is treated as a layer of constant depth and with vertically uniform properties, and explicitly or effectively assumes that the layer is in gradient wind balance (Smith *et al.*, 2008).

Recently, Smith *et al.* (2009) have demonstrated that the role of the boundary layer extends beyond that of converging moisture: it has a dynamical role in converging absolute angular momentum, M*. Although M is not materially conserved in the boundary layer, large tangential wind speeds can be achieved there if the radial inflow is sufficiently large to bring the air parcels to small radii with minimal loss of M. This spin-up mechanism, while coupled to the interior flow

*Defined as $rv + 1/2fr^2$, where r is the radius, v is the (azimuthally averaged, storm-relative) tangential wind speed, and f is the Coriolis parameter.

via the radial pressure gradient at the top of the boundary layer, is tied fundamentally to the dynamics of the boundary layer, where the flow is not in gradient wind balance over a substantial radial span. It was shown that this mechanism accounts for the occurrence of the maximum tangential wind in the boundary layer, a feature that has been found also in observational studies (Montgomery *et al.*, 2006; Kepert, 2006a,b; Bell and Montgomery, 2008; Sanger *et al.*, 2012).

The idealized numerical calculations of Smith *et al.* (2009) employed a relatively simple bulk boundary-layer parametrization scheme, albeit more sophisticated than Emanuel's slab model. For this reason, the calculations were repeated by Smith and Thomsen (2010) using a range of boundary-layer schemes having various degrees of sophistication. While the latter study showed quantitative differences in the intensification rate, mature intensity and certain flow features in the boundary layer for different schemes, in all cases the maximum tangential wind was found to occur close to the top of the inflow layer (Figure 2 of Smith and Thomsen, 2010; Figure 1 here), implying that the boundary-layer spin-up mechanism articulated by Smith *et al.* (2009) is robust and not dependent on a particular scheme. Similar results were obtained by Braun and Tao (2000) and Nolan *et al.* (2009a,b) in case-studies of two particular hurricanes, where different boundary-layer schemes were compared. While a range of schemes were investigated in all these studies, some relatively crude and others rather sophisticated, none of the studies went so far as recommending a particular scheme.

In an effort to address this issue, Kepert (2012) compared a range of boundary-layer parametrization schemes in the framework of a steady-state, height-resolving, boundary-layer model in which the tangential wind speed at the top of the boundary layer is prescribed and assumed to be in gradient wind balance. One outcome of his study as stated in his abstract is that '... one popular class of schemes is shown to be badly flawed in that it incorrectly predicts the near-surface wind profile, and therefore should not be used. Another is shown to be sensitive to diagnosis of the boundary-layer depth, a difficult problem in the core of the tropical cyclone, and caution is advised. The Louis boundary-layer scheme and a higher-order closure scheme are, so far as we can discern, without major problems, and are recommended'. In his conclusions, Kepert states that 'one class of schemes, representing the Bulk and Hi-Res parametrizations[†] available within MM5[‡], produces the strongest surface inflow, strongest supergradient jet, and fails to produce the observed near-surface logarithmic layer' and 'these features are due to the diffusivity being a maximum at the lowest model level, which in turn is due to an incorrect parametrization of the mixing length. These schemes are therefore significantly in error on observational and theoretical grounds' and that '... it would seem prudent that such studies be repeated with a more reasonable parametrization'. Kepert does not elaborate on what constitutes 'significantly in error' (presumably, this remark applies also to Emanuel's widely-used PI theory that assumes a slab boundary layer) and we question here the 'observational and theoretical grounds' that underpin his claim.

[†]Kepert's article gives a more detailed description of these schemes.
[‡]The Pennsylvania State University/National Center for Atmospheric Research mesoscale model.

The main basis of Kepert's critique of many schemes is that the log layer has to be satisfied to avoid 'significant error' and, for consistency with a constant stress layer, the associated mixing length and eddy diffusivity must increase linearly with depth near the surface. This assertion appears to be founded on an observational study of the hurricane boundary layer by Powell *et al.* (2003) and on laboratory measurements in non-rotating boundary layers in a turbulent fluid (Von Kármán, 1921; Schlichting, 1979; see also Stull, 1988; Garratt, 1992). Using a composite analysis of a large number of Global Positioning System (GPS) dropwindsonde soundings in the inner core of storms, Powell *et al.* showed that the logarithmic layer provides an acceptable fit to the wind speed data below about 200 m (his Figure 1), although there is a large scatter in the wind speed data and Powell *et al.* showed only data points at each height and not individual vertical profiles. The existence of such a layer is used by both Powell *et al.* (2003) and Holthuijsen *et al.* (2012) as a basis for estimating the drag coefficient at major hurricane wind speeds.

We are unconvinced by this 'observational support' for the ubiquity of a log layer in the core region of a rapidly rotating vortex for several reasons articulated below. We are unconvinced also by the theoretical support for the log layer in tropical cyclones asserted by Powell *et al.*, Kepert and others, which is based on dimensional analysis and assumes horizontal homogeneity. The purpose of this article is to revisit the interpretations of Powell *et al.* (2003) regarding the log layer and to question some of the scientific conclusions reported in Kepert's study.

The structure of the article is as follows. In section 2 we review the derivation of the log layer and explain why it may be inapplicable in a rapidly rotating vortex. In section 3 we show examples of inner-core dropwindsonde soundings that do not support the existence of a log layer. Section 4 considers the implications of the issues raised for modelling the hurricane boundary layer and section 5 presents the conclusions.

2. Theoretical considerations

2.1. *The log layer revisited*

The derivation of the log layer for the atmospheric boundary layer is reviewed in a classical paper by Tennekes (1973) and is based on an asymptotic similarity theory expounded by Blackadar and Tennekes (1968). The starting point is the equations of motion for a stationary, horizontally homogeneous, barotropic boundary-layer flow with constant density ρ, which is forced by a geostrophic flow, $\overline{\mathbf{u}}_g$:

$$-f(\overline{v} - \overline{v}_g) = \frac{d}{dz}(\overline{-u'w'}), \tag{1}$$

$$f(\overline{u} - \overline{u}_g) = \frac{d}{dz}(\overline{-v'w'}), \tag{2}$$

where $\overline{u}$ and $\overline{v}$ are the standard Reynolds-averaged zonal and meridional wind components in the boundary layer, $\overline{u}_g$ and $\overline{v}_g$ are the corresponding geostrophic wind components at the top of the boundary layer, f is the Coriolis parameter, and z is the height above the surface. The expressions $\overline{-u'w'}$ and $\overline{-v'w'}$ are the vertical turbulent momentum fluxes of zonal and meridional momentum, respectively (primes denote a departure from the mean flow, w' being the perturbation of vertical velocity). Taking the magnitude of the geostrophic wind as $G = (\overline{u}_g^2 + \overline{v}_g^2)^{1/2}$, the surface roughness length as z_o, and the surface stress ρu_*^2 (here u_* is the surface friction velocity), it is possible to establish a relationship between the two non-dimensional quantities: u_*/G and $Ro = G/(fz_o)$, the surface Rossby number. Typically $Ro >> 1$.

Tennekes (1973) notes also that these equations admit two kinds of self-similar solutions:

1. zf/u_* finite (i.e. finite relative height in the boundary layer), but with $z/z_o \to \infty$ and $Ro \to \infty$. Then, to a first approximation, the wind profile is asymptotically independent of Ro, provided it is plotted as

$$\frac{\overline{u} - u_g}{u_*} = F_x\left(\frac{zf}{u_*}\right), \qquad \frac{\overline{v} - v_g}{u_*} = F_y\left(\frac{zf}{u_*}\right), \tag{3}$$

where F_x and F_y are some universal functions to be determined. This is the scaling for the part of the boundary layer above the surface layer.

2. z/z_o finite, but with $zf/u_* \to 0$ and $Ro \to \infty$. Again, to a first approximation, the wind profile is asymptotically independent of Ro, provided it is plotted as

$$\frac{\overline{u}}{u_*} = F_s\left(\frac{z}{z_o}\right), \qquad \frac{\overline{v}}{u_*} = 0, \tag{4}$$

where F_s is another universal function to be determined and it has been assumed that the surface stress points in the x-direction. This is the scaling for *the surface layer*.

Tennekes notes that, although Eq. (3) is valid only well outside the surface layer ($z/z_o \to \infty$) and Eq. (4) is valid only inside the surface layer (z/z_o finite), they must have a region of common validity if Ro is large enough. This region of overlap, in which $z/z_o \to \infty$ and $zf/u_* \to 0$, is called the *matching layer*, or *inertial sublayer*. In this layer, Eqs (3) and (4) and all their derivatives have to agree with each other. Blackadar and Tennekes (1968) showed that the matching is possible only if the wind profile is logarithmic with height and that, if the coordinate axes are chosen so that the surface stress is in the x-direction, Eqs (3) and (4) have the forms:

$$\frac{\overline{u} - u_g}{u_*} = \frac{1}{\kappa}\log\left(\frac{zf}{u_*}\right) + \frac{B}{\kappa}, \qquad \overline{v} = 0, \quad \frac{v_g}{u_*} = \frac{-A}{\kappa}, \tag{5}$$

and

$$\frac{\overline{u}}{u_*} = \frac{1}{\kappa}\log\left(\frac{z}{z_o}\right), \qquad \overline{v} = 0, \tag{6}$$

where A, B and κ are constants, the latter being the Von Kármán constant.

Tennekes (1973, section 4) notes that 'from a theoretical point of view, the inertial sublayer (represented here by Eq. (6)) is a constant-stress layer in the asymptotic sense, provided $zf/u_* \to 0$ as $Ro \to \infty$'. He estimates that the stress stays within 1% of its surface value only below $zf/u_* = 10^{-3}$, which, in typical conditions, amounts to $z = 3$ m. He goes on to point out that 'the logarithmic law is useful and accurate well above that height if the boundary layer is an adiabatic one'. This statement would appear to suggest that the wind profile in the direction of the surface stress continues to remain logarithmic for some distance above the matching layer, but since the wind component transverse to the stress direction is not determined above the matching layer, *it would not follow that the total wind continues to increase logarithmically with height*.

Alternative derivations of the logarithmic velocity profile in a layer adjacent to surface are common in the literature for the case of *a homogeneous flow* on an f-plane (e.g. Brown, 1974; Panofsky and Dutton, 1984; Stull, 1988; Garrett, 1992; McWilliams, 2006). Most of these derivations are based on a scale analyses of the layer alone, without considering a formal matching to the boundary layer above, although Panofsky and Dutton and Garratt do discuss also the so-called Rossby similarity theory summarized above and Brown presents a detailed analysis of matching in a subsequent chapter. The derivations assume that the flow in the surface-based layer is unidirectional and independent of f. For example, the starting point for McWilliams' derivation is based on the idea that the mean vector velocity profile $\mathbf{u}(z)$ has a large shear with a profile shape governed by the boundary stress (characterized by the friction velocity u_*) and the near-boundary turbulent eddy size (effectively the height z) in the following way:

$$\frac{d\overline{\mathbf{u}}}{dz} = \frac{u_*}{\kappa z}\widehat{\mathbf{s}}, \tag{7}$$

where $\widehat{\mathbf{s}}$ is a unit vector in the direction of the surface shear stress and other quantities are defined above. This equation may be integrated to yield

$$\overline{\mathbf{u}}(z) = \frac{u_*}{\kappa}\log\left(\frac{z}{z_o}\right)\widehat{\mathbf{s}}. \tag{8}$$

Accordingly, the wind is unidirectional in the direction of the surface shear stress and increases logarithmically in magnitude with height. *Note that, if expressed as wind components in any locally orthogonal coordinate system, the magnitude of both components must increase with height.*

In the case of a steady axisymmetric vortex with tangential wind speed $v_g(r)$ in gradient wind balance above the boundary layer, the equations analogous to Eqs (1) and (2) are

$$\overline{u}\frac{\partial \overline{u}}{\partial r} + \overline{w}\frac{\partial \overline{u}}{\partial z} - \frac{\overline{v}_a^2}{r} - \left(f + \frac{2v_g}{r}\right)\overline{v}_a = \frac{\partial}{\partial z}(\overline{-u'w'}), \tag{9}$$

$$\overline{u}\frac{\partial \overline{v}_a}{\partial r} + \overline{w}\frac{\partial \overline{v}_a}{\partial z} + (f + \zeta_g)\overline{u} = \frac{\partial}{\partial z}(\overline{-v'w'}), \tag{10}$$

where now $\overline{u}$ and $\overline{v}$ are the radial and tangential components of the Reynolds-averaged wind, respectively, and $\overline{v}$ has been replaced by the agradient wind $\overline{v}_a = \overline{v} - v_g$. The derivation makes the normal boundary-layer approximation in which the radial pressure gradient is uniform across the boundary layer and the radial derivative of the turbulent shear stress is neglected. Now, in the steady-state case, the radial flow at the top of the boundary layer is zero[§] (i.e. $\overline{u}_g = 0$)

[§]If the mean radial flow above the boundary layer were not zero, the tangential flow would evolve with time on account of the material conservation of absolute angular momentum, except in the special case of a vortex in which the flow above the boundary layer is along absolute angular momentum surfaces.

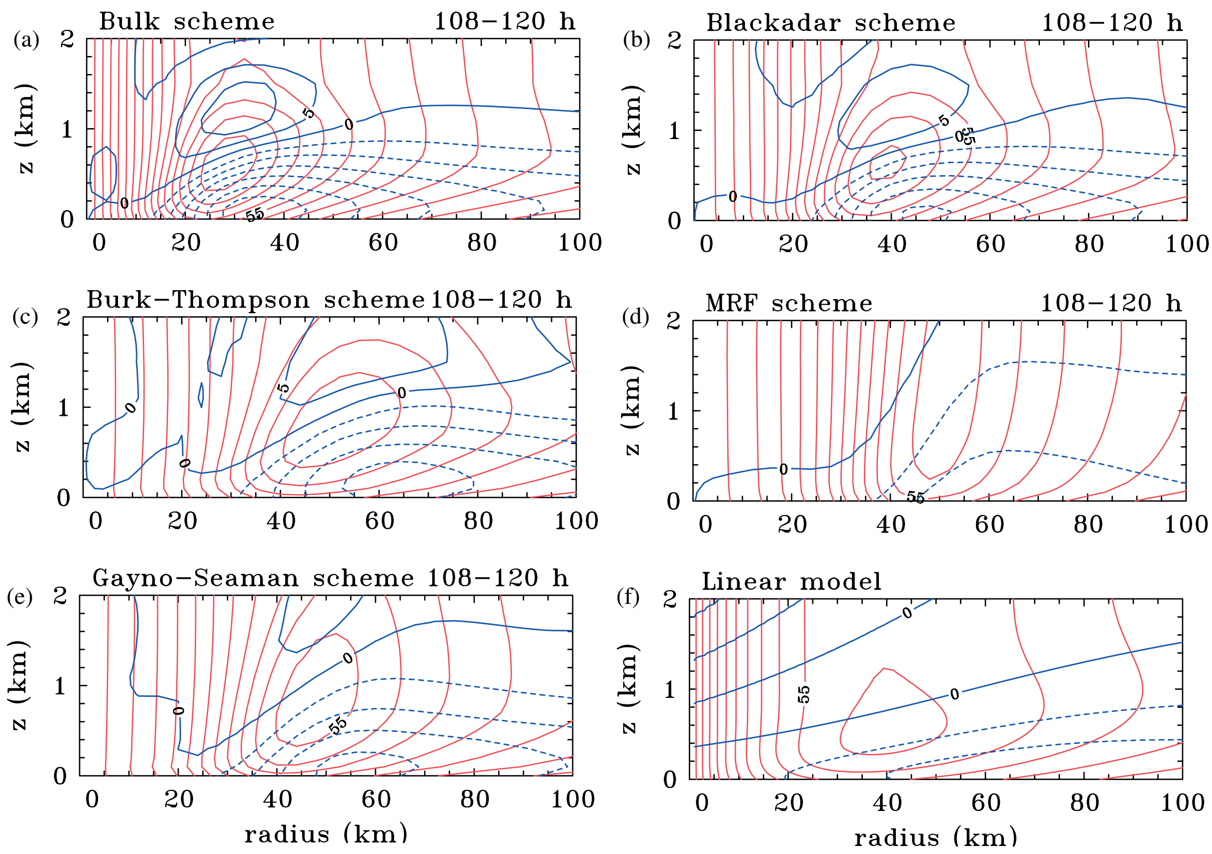

Figure 1. Radius–height cross-sections of azimuthally averaged radial (thin contours, with negative values dashed) and tangential (bold contours) wind components in the lowest 2 km averaged at 15 min intervals during the period 108–120 h for the different boundary-layer schemes: (a) bulk scheme, (b) Blackadar scheme, (c) Burk–Thompson scheme, (d) MRF scheme, (e) Gayno–Seaman scheme, and (f) the steady linear model. The contour interval is 5 m s^{-1}. This figure is available in colour online at wileyonlinelibrary.com/journal/qj

while the tangential flow is a function of radius, r. Also, $\zeta_g = (1/r)\mathrm{d}(rv_g)/\mathrm{d}r$ is the vertical component of relative vorticity of the gradient wind.

Although a scale analysis shows that the nonlinear terms in Eqs (9) and (10) cannot be neglected in the boundary layer of a tropical-cyclone-strength vortex (e.g. Smith, 1968; Carrier, 1971a; Vogl and Smith, 2009), let us suppose for the sake of the current discussion that they can. Then the equations are similar in structure to Eqs (1) and (2), but the presence of the radially variable coefficients involving the absolute angular velocity, $f + 2v_g/r$, and the absolute vorticity, $f + \zeta_g$, of the gradient wind invalidates the scaling analysis discussed above. In a rapidly rotating vortex, these terms are dominated by the contributions $2v_g/r$ and ζ_g, which typically are unequal and have a strong radial variation, except possibly close to the centre, where the flow may be in approximate solid-body rotation. In this case, the assumption of horizontal homogeneity in the analysis of the Eqs (1) and (2) for constant f is no longer valid. Hence, even if the nonlinear terms are ignored, it is by no means obvious to us that a similar scaling analysis can be applied, since additional scales including the radius, the absolute angular velocity and relative vorticity of the gradient flow have emerged. These scales reflect the presence of a net, radially inward, pressure-gradient force which is a maximum at the surface where the tangential flow is reduced the most by the azimuthal frictional stress. *In other words, the vertical gradient of horizontal velocity in Eqs (9) and (10) does not depend simply on the distance from the surface (and neither does the eddy diffusivity)*. Thus the existence of a net transverse pressure gradient force with components along and normal to the surface stress vector would invalidate the assumption of a constant stress throughout a surface-based layer.

As far as we are aware, the validity of the near-surface constant stress assumption in the turbulent boundary layer of a rapidly rotating vortex has not been questioned. Indeed, it has been advocated by Kepert (2012) as an essential ingredient of any plausible boundary-layer scheme. We present evidence below from various numerical calculations as well as observations suggesting that the assumption cannot be justified in the inner core of a hurricane.

2.2. *Near-surface wind structure in hurricane models*

There is both observational and theoretical support to suggest that the vertical gradients of the radial and tangential wind components have different signs near the surface in the inner core of a hurricane, a feature that is not compatible with one property of the log layer noted above. The tendency to produce the maximum radial inflow at the surface is evident for all the schemes investigated by Braun and Tao (2000), Nolan *et al.* (2009a,b) and Smith and Thomsen (2010), whether or not the formulation of the scheme incorporated a log layer. As an illustration of this feature, we show in Figure 1 vertical cross-sections of the azimuthally averaged radial and tangential wind speed components in the idealized hurricane simulations described by Smith and Thomsen (2010) for five different boundary-layer schemes. The cross-sections encompass the lowest 2 km in height with the velocity fields being averaged at 15 min intervals during the mature stage of vortex evolution (the period 108–120 h). The boundary-layer schemes include the bulk scheme, the Blackadar scheme, the Burk–Thompson scheme, the MRF scheme, and the Gayno–Seaman scheme, details of which are summarized in Smith and Thomsen (2010) with references. Kepert (2012) gives an erudite summary of the essential features of these different schemes.

For comparison, Figure 1(f) shows an example of cross-sections obtained by solving the quasi-linear boundary-layer model with a prescribed tangential wind profile¶ just below the top of the layer. In this example, the maximum tangential wind speed at large height is taken to be 60 m s^{-1} and the eddy diffusivity is taken to be a constant, equal to 100 m^{2}s^{-1}. The quasi-linear model is locally analogous to the classical Ekman layer model‖ (Eliassen and Lystadt, 1977; Kepert, 2001). Although it has been shown that the quasi-linear approximation becomes invalid in the inner core of a hurricane (Carrier, 1971a,b; Vogl and Smith, 2009), this model shows also the tendency to produce the maximum radial wind component at the surface.

As an aid to comparing the schemes in Figure 1, we show in Figure 2 the vertical profiles of the radial and tangential wind components at the radius ($r_{\max}$) of the maximum azimuthally averaged tangential wind speed ($v_{\max}$). Included also are the corresponding profiles for the quasi-linear solution shown in Figure 1(f). Again these profiles highlight the fact that in all cases, $v_{\max}$ occurs near the top of the inflow layer. *It is particularly noteworthy that for all schemes, and for the quasi-linear solution, the maximum radial wind speed occurs at or very close to the surface.*

When interpreting the first five panels in Figure 1, it should be borne in mind that the lowest model level in the MM5 calculations is at a height of approximately 40 m and that the plotted surface wind components are obtained by quadratic extrapolation using wind component values at this level and the two above it in each grid column. While some boundary-layer schemes in MM5 (e.g. the Bulk scheme) apply a quadratic stress law at the lowest model level, and do not extrapolate the winds to the surface, more sophisticated schemes (e.g. the Blackadar and Gayno-Seaman schemes) assume implicitly or explicitly the presence of a log layer. For the latter schemes, the nominal 'ocean surface' would be at the roughness height z_o of the assumed log layer and the wind speed would be zero at this level. However, it seems to us physically unrealistic to plot zero wind speed at this height in the figure, recognizing that the ocean surface is ill-defined in a major hurricane due to wave-breaking, spume and emulsion processes and the fact that waves may be many metres in height. Nevertheless, the MM5 profiles in Figure 2 should be interpreted with caution below a height of 40 m.

¶Profile 3 in Figure 1 of Smith (2003).

‖The quasi-linear model for the steady boundary layer is obtained by neglecting the nonlinear acceleration terms for the agradient wind in the horizontal momentum equations (9) and (10) and the centrifugal and Coriolis terms are linearized about the gradient wind of the bulk vortex at the top of the boundary layer (Vogl and Smith, 2009). The radial diffusion of momentum is neglected also. These equations have the form $-\zeta_a v' = (\partial/\partial z)(K\partial u/\partial z)$ and $-\xi u = (\partial/\partial z)(K\partial v'/\partial z)$, where u and v' are the radial and tangential components of the agradient wind, ζ_a is the absolute vorticity of the gradient wind, $\xi = 2v/r + f$ is twice the absolute angular velocity of the gradient wind, K is the vertical eddy diffusivity and z is the height. Simple closed-form solutions at a given radius may be obtained if the diffusivity is assumed to be constant with height.

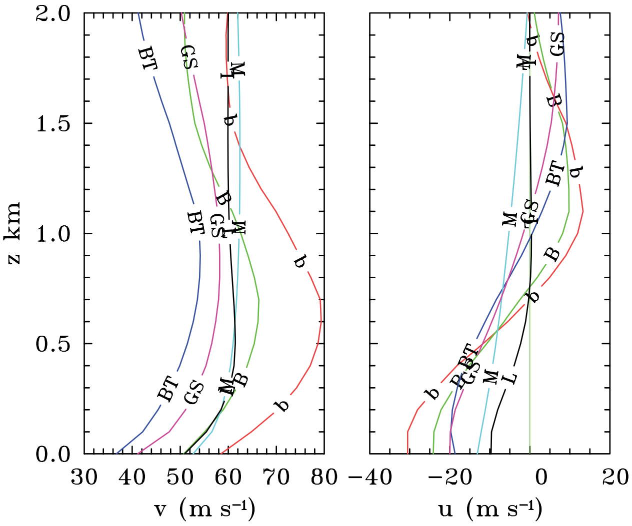

Figure 2. Vertical profiles of azimuthally averaged radial and tangential wind components in the lowest 2 km at the radius of maximum tangential wind speed for the different schemes shown in Figure 1: b = bulk scheme, B = Blackadar scheme, BT = Burk–Thompson scheme, M = MRF scheme, GS = Gayno–Seaman scheme, and L = steady quasi-linear model. This figure is available in colour online at wileyonlinelibrary.com/journal/qj

It is significant that, for the region inside the radius of maximum tangential winds, Kepert's (2012) solutions have also the tendency to produce the maximum radial inflow at the surface. This is a feature of all the schemes he investigated, whether or not a log layer was 'imposed' by the choice of the linear variation of near-surface eddy diffusivity with height (e.g. his Figures 3 to 6). As noted above, such a feature is inconsistent with the existence of a log layer. Even so, it is pertinent to mention that the numerical model used by Kepert, as well as the quasi-linear boundary-layer model summarized above, have an issue in that, as pointed out by Smith and Montgomery (2010), they effectively *prescribe* the tangential wind at the top of the boundary layer *where the flow is upwards*. Such a prescription at an outflow boundary makes the physical problem ill-posed as the boundary layer itself should be allowed to determine the tangential momentum that it expels into the bulk vortex aloft (see also Rotunno and Bryan, 2012, p 17).

As noted above, a property of the layer defined by the solution (6) is the strict unidirectional nature of the wind within it. However, if the solution (5) continues to hold for some distance *above* the matching layer, the wind profile in the direction of the surface stress may remain logarithmic with height while there may be some cyclonic turning with height (Blackadar and Tennekes, 1968).

To show that a unidirectional surface-based layer is not a feature of any of the parametrization schemes in Figure 1, irrespective of whether they represent a log layer in the traditional sense (i.e. they have an eddy diffusivity increasing linearly with height implying a constant stress layer), we show in Figure 3 the hodographs of the wind profiles in Figure 2. Except in the MRF scheme (which, as noted by Smith and Thomsen, predicts a rather diffuse boundary layer) and in the quasi-linear solution, the schemes indicate that a significant turning of the wind vector with height occurs in the lowest few hundred metres, a property that cannot be represented by the traditional log layer. Note that, except in Figure 3(c), the radial wind component remains approximately constant *or actually decreases* in magnitude with height in the lowest 50 m. This feature is certainly not

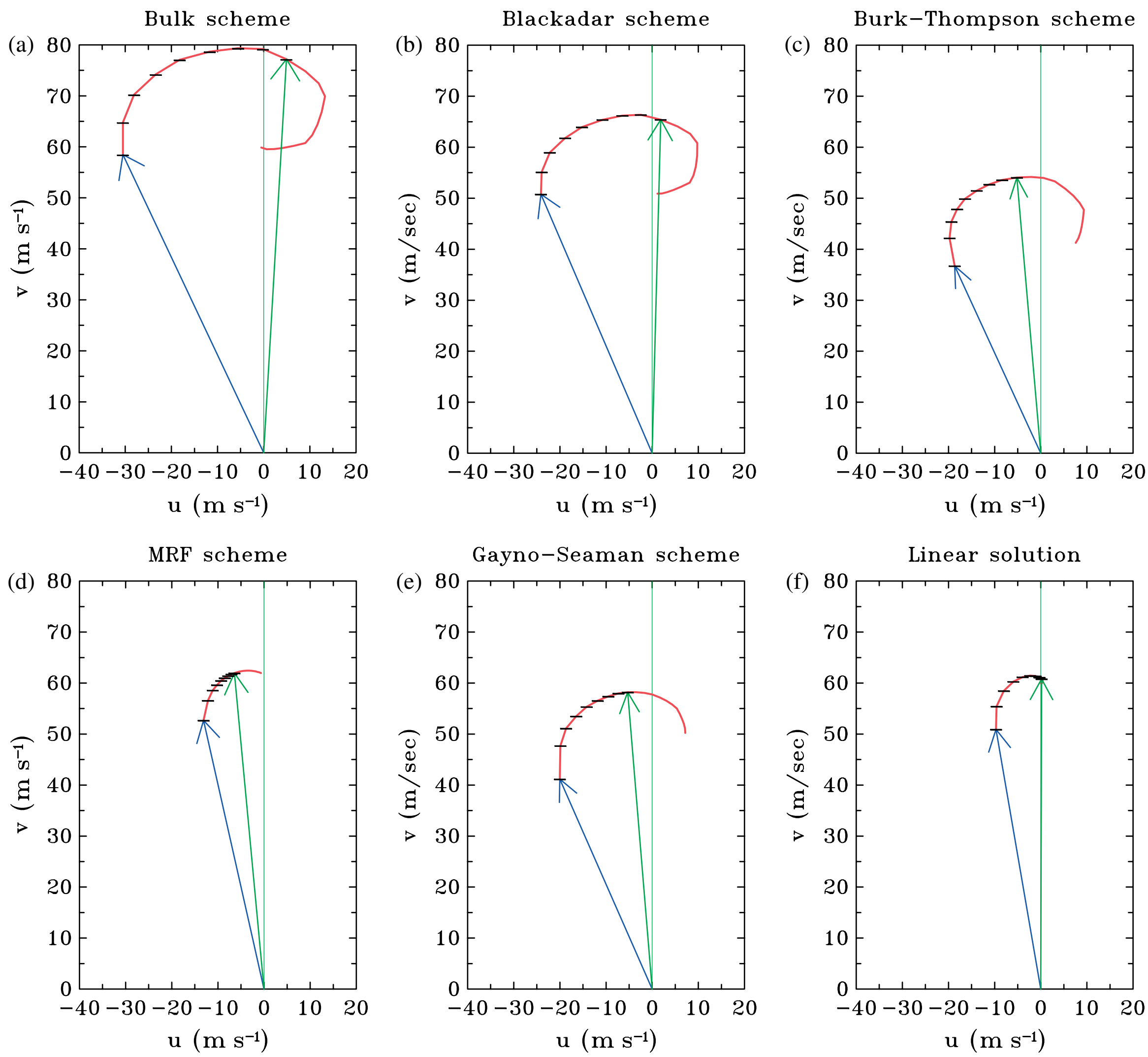

Figure 3. Wind hodographs in the lowest 2 km corresponding to the vertical profiles in Figure 2. (a) bulk scheme, (b) Blackadar scheme, (c) Burk–Thompson scheme, (d) MRF scheme, (e) Gayno–Seaman scheme and (f) quasi-linear model. The tick marks on the curves indicate height intervals every 50 m starting at the surface and ending at 400 m. The two lines with arrows represent the wind vectors at the surface (left/blue) and at a height of 400 m (right/green), respectively. The vertical line marks $u = 0$. This figure is available in colour online at wileyonlinelibrary.com/journal/qj

a property of the traditional log layer, where the magnitude of both components must increase with height.

3. Observations of the hurricane boundary layer

The data used by Powell *et al.* (2003) to justify the presence of a log layer have a great deal of scatter and individual profiles are not shown. However, other studies indicate that many individual inner-core wind soundings do not exhibit the structure of a traditional log layer. In fact, GPS dropwindsonde data in the inner core of hurricanes and typhoons *frequently show radial wind profiles that have a maximum inflow at the surface*. It may be worth noting that Powell *et al.* assume a traditional, strict, constant-stress log layer as defined above and do not invoke the Blackadar and Tennekes formulation embodied in Eq. (5). Thus, even if the component of flow in the direction of the stress remains logarithmic for some height range above the inertial sub-layer, it is unclear to what degree the *total* wind speed might remain logarithmic, because the asymptotic theory does not determine the corresponding formula for $\overline{v}$ in this region. It seems possible that the uncertainty in the applicability of the asymptotic theory to heights above the inertial layer could account for at least a part of the scatter in the data.

Composite dropwindsonde soundings in the eyewall of individual storms can provide a useful perspective on the vertical structure of the boundary-layer winds there. Two such examples are illustrated here using data from typhoon *Jangmi* (Figure 4) presented by Sanger *et al.* (2012), and hurricane *Isabel* (2003) (Figure 5) presented by Montgomery *et al.* (2006) and Bell and Montgomery (2008). In both cases, the maximum storm-relative tangential wind speed (v_{max}) occurs within the layer of relatively strong inflow (u). Without exception, the tangential wind component (v) is a minimum at the surface. While the magnitude of v in *Jangmi* increases with height near the surface, that of u *decreases with height*, except in a very shallow layer (below 50 m) in the supertyphoon stage. In *Isabel*, a negative vertical gradient of radial velocity is evident throughout the boundary layer on two out of three days, except in a very shallow layer below 50 m. On 14 September, the maximum inflow resides at the surface.

In those profiles where the radial wind speed increases with height below 50 m, we cannot definitively rule out the existence of a shallow log profile, but we can rule out a log

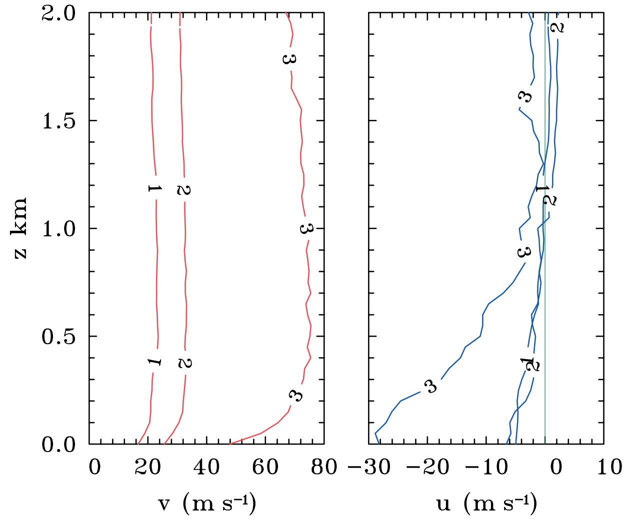

Figure 4. Vertical profiles of the storm-relative tangential (v) and radial (u) wind components in the eyewall composites for typhoon *Jangmi* (2008). Indices 1, 2, 3 on the curves denote the tropical storm, typhoon and supertyphoon stages, respectively. These data have a vertical resolution of 50 m. Data courtesy of N. T. Sanger. This figure is available in colour online at wileyonlinelibrary.com/journal/qj

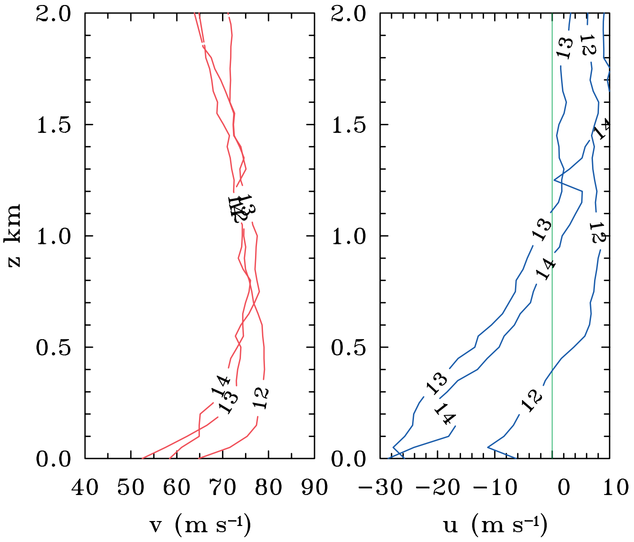

Figure 5. Vertical profiles of the storm-relative tangential (v) and radial (u) wind components in the eyewall composites for hurricane *Isabel* (2003) on three consecutive days of observations (12–14 September). Numbers on curves denote the date. These data have a vertical resolution of 50 m. Data courtesy of M. A. Bell. This figure is available in colour online at wileyonlinelibrary.com/journal/qj

layer extending 100–200 m in depth as proposed by Powell *et al.* (2003) for inferring drag coefficients at major hurricane wind speeds. However, the subsequent decrease in the radial wind component above this height is *not* consistent with a traditional log layer.

Figure 6 shows the hodographs of the eyewall wind composites for *Jangmi* presented in Figure 4. As in the numerical calculations shown in Figure 3, much of the turning of the wind occurs within the lowest few hundred metres. While the turning of the wind does not, by itself, rule out the existence of a logarithmic wind *profile* as noted in section 2, it does challenge the existence of a traditional surface-based log layer in which the wind and shear stress vector are unidirectional.

The turning of the wind is particularly marked in the supertyphoon stage of *Jangmi*, for which the hodograph is quite similar to that in the bulk scheme (compare Figure 3(a) with Figure 6(c)). *This finding would indicate that the bulk scheme is not necessarily as poor as Kepert's (2012) critique of it might suggest.*

It may be argued that typhoon *Jangmi* and hurricane *Isabel* are only two storms and that the structures shown in Figures 4–6 may not be general. For this reason, we show in Figure 7 composite plots of storm-relative radial and tangential velocity for the eyewall region of thirteen Atlantic hurricanes. These eyewall composite profiles were constructed from data used to characterize the mean boundary-layer structure of the near-core vortex region contained within a radius of about four times the radius of maximum tangential winds (Zhang *et al.*, 2011a). The eyewall composites consist of several hundred dropwindsondes. In these composites, the tangential velocity component increases in magnitude with height near the surface. The radial velocity component increases slightly in magnitude with height within the first 50 m, and subsequently decreases rapidly. The increase of the two wind components in the lowest 50 m would not rule out the existence of a log layer there. To examine this possibility, we plot in Figure 8 the total wind speed from these components as a function of height in the lowest kilometre and also the wind hodograph to a height of 2 km. It is seen that, while the profile of *total* wind is approximately logarithmic in the layer between 100 and 400 m, this logarithmic profile *does not extend all the way to the surface*. Moreover, as in the hodographs shown in Figures 3 and 6, the wind vector turns through an appreciable angle within this layer, ruling out that the layer behaves as a constant-stress layer. While it might be argued that the logarithmic behaviour would be consistent with solution (5), we note again that this equation refers to the wind component in the direction of the surface stress. Moreover, the decrease in the radial component of flow above 50 m is *strong evidence that the net radial pressure gradient with height is important in the near-surface layer.*

In summary, the foregoing observations indicate a significant turning of the wind vector with height in the lowest few hundred metres of the inner-core boundary layer, generally accompanied by a *decrease* in the radial wind component with height. These features, which support the modelling results discussed in section 2, cannot be represented by the traditional surface log layer. Even in the eyewall composite for many hurricanes, the vertical profile of storm-relative wind speed does not strictly follow a logarithmic profile throughout the lowest 200 m.

4. Ramifications for hurricane modelling

Kepert (2012) criticizes the bulk and 'high-res' schemes on one ground that they do not produce the observed near-surface logarithmic layer. As argued above, the basis for a log layer in the inner core of a hurricane is not compelling, either on theoretical or observational grounds. Even so, the question remains: how important are the structural details of the shallow surface layer on the prediction of vortex evolution, provided that the surface stress and surface heat fluxes are adequately represented?

From an elementary perspective, in the classical Ekman solution (including the version where the surface stress is assumed to be in the direction of the surface wind), the

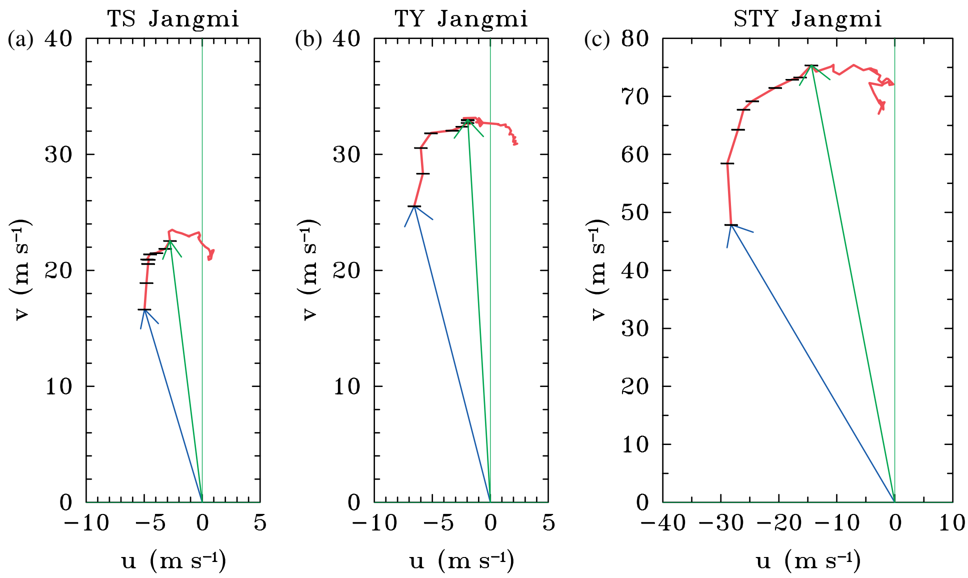

Figure 6. Wind hodographs in the lowest 2 km corresponding with the vertical wind profiles of *Jangmi* shown in Figure 4. The tick marks on the curves indicate height intervals every 50 m starting at the surface and ending at 400 m. The two lines with arrows represent the wind vectors at the surface (left/blue) and at a height of 400 m (right/green), respectively. This figure is available in colour online at wileyonlinelibrary.com/journal/qj

radial volume transport depends only on the surface stress and not on the details of the shallow surface layer. This result follows directly by integrating the steady linearized tangential momentum equation with respect to height and assuming that the tangential wind approaches the gradient wind at large heights. A similar result is true for the quasi-linear model of the boundary layer discussed above.

In the classical Ekman solution, the vertical velocity at the top of the boundary layer is simply proportional to the radial gradient of the volume transport in the layer, and hence to the radial gradient of the surface shear stress. This result follows immediately by integrating the continuity equation with respect to height (Gill, 1982, section 9.4). Like the volume transport, the vertical velocity at large height does not depend on the details of the surface layer. These same remarks apply also to the quasi-linear vortex boundary-layer model discussed in section 2.

While a scale analysis shows that neither the Ekman model nor the quasi-linear model are valid in the inner-core region of a hurricane and that the nonlinear acceleration terms in the boundary-layer equations are important in this region (Smith, 1968; Smith and Montgomery, 2008; Vogl and Smith, 2009), it has yet to be demonstrated that the details of the shallow surface layer have a profound effect on the volume of air converging in the boundary layer and hence on the vertical motion out of the boundary layer, provided that the surface stress is represented appropriately.

Of course, the magnitude and vertical distribution of the radial velocity component depends in part on the assumed vertical profile of diffusivity, even in the classical Ekman solution. In particular, the bulk magnitude of the diffusivity together with the Coriolis parameter determines the depth of the inflow layer (e.g. Gill, 1982) and hence the depth over which the volume flux is distributed. This dependence on diffusivity extends therefore to the radial advection of absolute angular momentum. Clearly, the efficacy of the boundary-layer spin-up mechanism articulated by Smith *et al.* (2009) will depend quantitatively on the particular parametrization scheme as confirmed by the calculations of Smith and Thomsen (2010) summarized in Figures 1–3 here.

As noted above, the calculations of Smith and Thomsen (2010) demonstrate that a recently articulated boundary-layer spin-up mechanism for the hurricane by Smith *et al.* (2009) transcends the presence of a log layer. Independently, Bryan (2012, his Figure 16) has shown that the incorporation of a reduced vertical mixing length near the surface (using the Blackadar formulation for vertical mixing length) yields an essentially similar dependence of maximum tangential winds on the ratio of enthalpy and momentum surface exchange coefficients, while the simulated v_{max} tended 'to be slightly lower with the Blackadar formulation for vertical mixing length'. Together, these results suggest that the essence of tropical cyclone spin-up and the dependence of maximum winds on the ratio of enthalpy and momentum exchange coefficients is captured without a log layer.

It is evident from the foregoing discussion that uncertainties in the optimum scheme for use in operational hurricane models remain. In general, such models do not have the vertical grid resolution to resolve the putative log layer, which is used merely to extrapolate the wind from the lowest model grid level to a standard height (normally 10 m) where the normal aerodynamic drag formulation (in terms of C_D) is applicable. In view of the large uncertainties in the formulation of the eddy diffusivity above the lowest model level, the possible sensitivity to the precise formulation of the surface layer may be over-exaggerated. Indeed, a major issue confronting hurricane modellers is the lack of observational data on the radial and vertical structure of eddy diffusivity in the strong-wind region of hurricanes to guide the formulation of this quantity in models. One step in this direction has been taken in two recent articles by Zhang *et al.* (2011b) and Zhang and Montgomery (2012).

Despite Kepert's (2012) critique of the bulk and Blackadar schemes, the results of section 2 show that the predictions using the Blackadar scheme do not deviate significantly from the other schemes and even those of the bulk scheme are not totally unrealistic compared with some of the

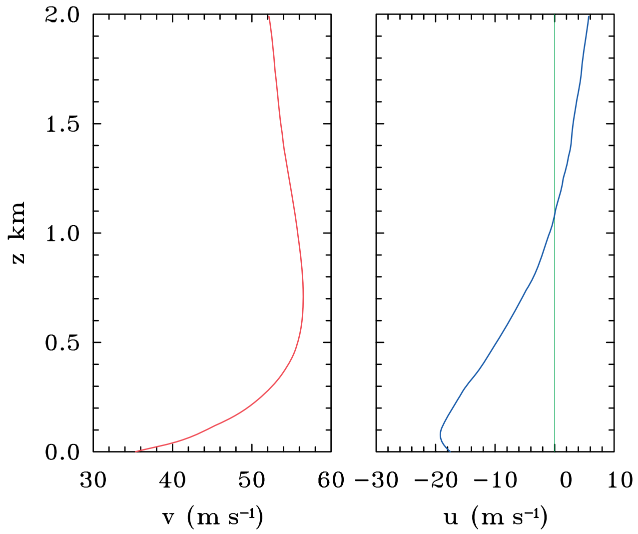

Figure 7. Vertical profiles of the tangential (v) and radial (u) wind components in the eyewall composites of many hurricanes. Data courtesy of J. A. Zhang. This figure is available in colour online at wileyonlinelibrary.com/journal/qj

observations shown above. In particular, we have shown that the wind speed profiles and hodographs in the vicinity of the eyewall region using the Bulk and Blackadar schemes are not inconsistent in magnitude with those in major hurricanes such as *Isabel* and *Jangmi*. While we do not wish to defend the use of simple boundary-layer schemes for their accuracy in operational prediction models, we do believe that they have an important role in generating understanding of tropical cyclone spin-up and maximum potential intensity. We believe that Kepert's erudite comparison of the different schemes is an important step in attempts to determine an optimum scheme for use in operational prediction models. Nevertheless, for the reasons articulated herein, we do not subscribe to his assertion that the absence of a log layer should be a criterion for rejecting a scheme.

Another issue raised by our results is the validity of assuming a constant-stress layer with a logarithmic wind speed profile for estimating the drag coefficient at major hurricane wind speeds (e.g. Powell *et al.*, 2003; Holthuijsen *et al.*, 2012). The basis of this assumption is that the flow can be treated as horizontally homogeneous, which we have shown here to be untenable on both observational and theoretical grounds. In particular, the nonlinear inertial effects are shown to be important near the surface, where the effective radial pressure gradient force is largest. A consequence is that the radial flow tends to be a maximum at or near the surface, as seen in the observations, and the vertical gradients of the magnitude of the radial and tangential wind components tend to have opposite signs. A method that avoids these assumptions in estimating the drag coefficient at high wind speeds is discussed by Bell *et al.* (2012), although this method has its own limitations as well. This method is based on a control volume analysis of absolute angular momentum and total energy around the eyewall region in the lower troposphere.

It is beyond the scope of this article to quantify the errors that might arise from the assumption of a log layer when estimating the drag coefficient at high wind speeds, but we regard this as a legitimate question for further study following the concerns we have raised about the existence of such a layer itself.

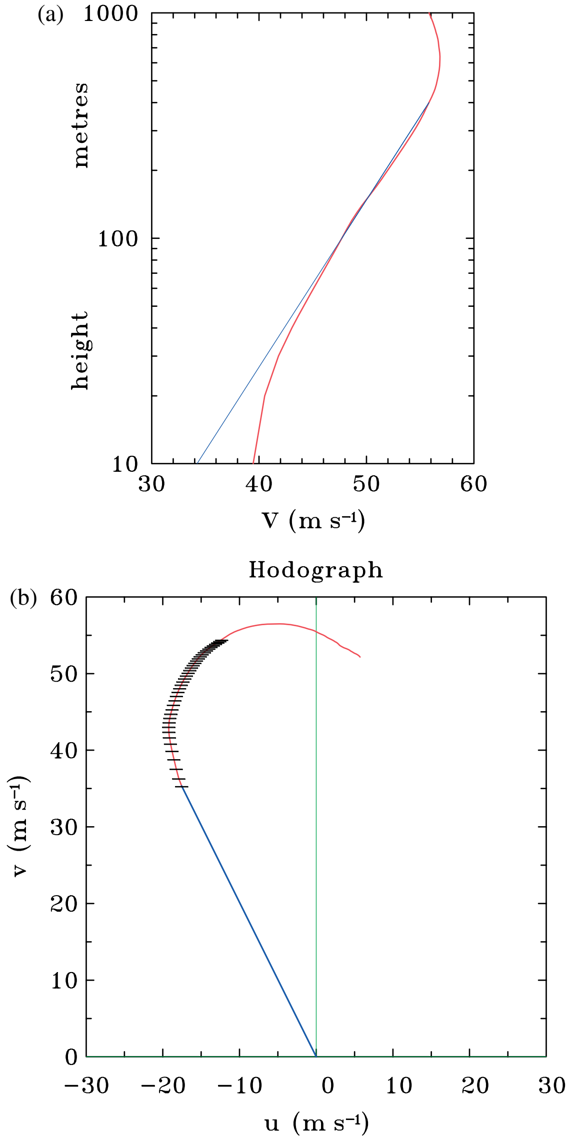

Figure 8. (a) Vertical profiles of the total wind, V, speed corresponding with the tangential and radial wind components shown in Figure 7, but plotted on a logarithmic vertical scale. (b) Hodograph of the wind components plotted in Figure 7. Horizontal tick marks indicate heights from 10 to 400 m at 10 m intervals. This figure is available in colour online at wileyonlinelibrary.com/journal/qj

5. Conclusions

We have questioned the validity of the traditional surface-based logarithmic layer in the inner core of hurricanes. Definitive observational evidence for its existence in previous studies is tenuous and is based on data that have a significant amount of scatter. Indeed, many individual eyewall soundings and a composite comprising thirteen Atlantic hurricanes do not support its existence. There are theoretical reasons why the logarithmic layer may be violated in the inner core of hurricanes: this is because the inward-directed effective pressure gradient force is largest at the surface, where the tangential wind is reduced the most from its gradient value aloft. The existence of this cross-stream pressure-gradient force raises the possibility that the largest inflow occurs at, or very close to, the surface, which would imply that the horizontal shear-stress vector is *not* unidirectional near the surface and that the magnitude of the transverse wind component decreases with height.

Both of these properties are inconsistent with a traditional log layer. We have presented both numerical model results and observational analyses in support of these ideas. We noted that deviations from a logarithmic layer in the inner core of hurricanes described herein must affect the ability to infer the surface drag coefficient from dropwindsonde wind profiles using methods that assume a logarithmic layer from the outset. Finally, we drew attention to a study examining a range of boundary-layer schemes demonstrating that a recently articulated boundary-layer spin-up mechanism transcends the presence of a logarithmic layer.

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References

Bell MM, Montgomery MT. 2008. Observed structure, evolution, and potential intensity of category 5 hurricane *Isabel* (2003) from 12 to 14 September. *Mon. Weather Rev.* **65**: 2025–2046.

Bell MM, Montgomery MT, Emanuel KA. 2012. Air–sea enthalpy and momentum exchange at major hurricane wind speeds observed during CBLAST. *J. Atmos. Sci.* **69**: 3197–3222.

Bister M, Emanuel KA. 1998. Dissipative heating and hurricane intensity. *Meteorol. Atmos. Phys.* **52**: 233–240.

Blackadar AK, Tennekes H. 1968. Asymptotic similarity in neutral barotropic atmospheric boundary layers. *J. Atmos. Sci.* **25**: 1015–1020.

Braun SA, Tao W-K. 2000. Sensitivity of high-resolution simulations of hurricane *Bob* (1991) to planetary boundary-layer parameterizations. *Mon. Weather Rev.* **128**: 3941–3961.

Brown RA. 1974. *Analytical methods in planetary boundary-layer modelling*. John Wiley and Sons: New York.

Bryan GH. 2012. Effects of surface exchange coefficients and turbulence length scales on the intensity and structure of numerically simulated hurricanes. *Mon. Weather Rev.* **140**: 1125–1143.

Carrier GF. 1971a. Swirling flow boundary layers. *J. Fluid Mech.* **49**: 133–144.

Carrier GF. 1971b. The intensification of hurricanes. *J. Fluid Mech.* **49**: 145–158.

Carrier GF, Hammond AL, George OD. 1971. A model of the mature hurricane. *J. Fluid Mech.* **47**: 145–170.

Csanady GT. 1967. On the resistence law of a turbulent Ekman layer. *J. Atmos. Sci.* **24**: 467–471.

Emanuel KA. 1986. An air–sea interaction theory for tropical cyclones. Part I: Steady state maintenance. *J. Atmos. Sci.* **43**: 585–604.

Emanuel KA, Rotunno R. 2011. Self-stratification of tropical cyclone outflow. Part I: Implications for storm structure. *J. Atmos. Sci.* **68**: 2236–2249.

Garratt JR. 1992. *The atmospheric boundary layer*. Cambridge University Press: Cambridge, UK.

Gill AE. 1968. Similarity theory and geostrophic adjustment. *Q. J. R. Meteorol. Soc.* **94**: 586–588.

Gill AE. 1982. *Atmospheric–ocean dynamics*. Academic Press: New York.

Holthuijsen LH, Powell MD, Pietrzak JD. 2012. Wind and waves in extreme hurricanes. *J. Geophys. Res.* **117**: 1–15.

Kepert JD. 2001. The dynamics of boundary-layer jets within the tropical cyclone core. Part I: Linear theory. *J. Atmos. Sci.* **58**: 2469–2484.

Kepert JD. 2006a. Observed boundary-layer wind structure and balance in the hurricane core. Part I. Hurricane *Georges*. *J. Atmos. Sci.* **63**: 2169–2193.

Kepert JD. 2006b. Observed boundary-layer wind structure and balance in the hurricane core. Part II. Hurricane *Mitch*. *J. Atmos. Sci.* **63**: 2194–2211.

Kepert JD. 2012. Choosing a boundary-layer parameterisation for tropical cyclone modelling. *Mon. Weather Rev.* **140**: 1427–1445.

McWilliams JC. 2006. *Fundamentals of geophysical fluid dynamics*. Cambridge University Press: Cambridge, UK.

Montgomery MT, Bell MM, Aberson SD, Black ML. 2006. Hurricane *Isabel* (2003): New insights into the physics of intense storms. Part I – Mean vortex structure and maximum intensity estimates. *Bull. Amer. Meteorol. Soc.* **87**: 1335–1347.

Nolan DS, Zhang JA, Stern DP. 2009a. Evaluation of planetary boundary-layer parameterizations in tropical cyclones by comparison of *in situ* observations and high-resolution simulations of hurricane *Isabel* (2003). Part I: Initialization, maximum winds, and the outer core boundary layer. *Mon. Weather Rev.* **137**: 3651–3674.

Nolan DS, Zhang JA, Stern DP. 2009b. Evaluation of planetary boundary-layer parameterizations in tropical cyclones by comparison of in-situ observations and high-resolution simulations of hurricane *Isabel* (2003). Part II: Inner core boundary layer and eyewall structure. *Mon. Weather Rev.* **137**: 3675–3698.

Ooyama KV. 1969. Numerical simulation of the life cycle of tropical cyclones. *J. Atmos. Sci.* **26**: 3–40.

Panofsky HA, Dutton JA. 1984. *Atmospheric Turbulence*. John Wiley and Sons: New York.

Powell MD, Vickery PJ, Reinhold TA. 2003. Reduced drag coefficient for high wind speeds in tropical cyclones. *Nature* **422**: 279–283.

Rotunno R, Bryan GH. 2012. Effects of parameterized diffusion on simulated hurricanes. *J. Atmos. Sci.* **69**: 2284–2299.

Sanger NT, Montgomery MT, Smith RK, Bell MM. 2012. An observational study of tropical-cyclone spin-up in supertyphoon *Jangmi* (2008) from 24 to 27 September. *Mon. Weather Rev.* submitted.

Schlichting H. 1979. *Boundary-layer theory*. McGraw-Hill: New York.

Smith RK. 1968. The surface boundary layer of a hurricane. *Tellus* **20**: 473–484.

Smith RK. 2003. A simple model of the hurricane boundary layer. *Q. J. R. Meteorol. Soc.* **129**: 1007–1027.

Smith RK, Montgomery MT. 2010. Hurricane boundary-layer theory. *Q. J. R. Meteorol. Soc.* **136**: 1665–1670.

Smith RK, Thomsen GL. 2010. Dependence of tropical-cyclone intensification on the boundary-layer representation in a numerical model. *Q. J. R. Meteorol. Soc.* **136**: 1671–1685.

Smith RK, Vogl S. 2008. A simple model of the hurricane boundary layer revisited. *Q. J. R. Meteorol. Soc.* **134**: 337–351.

Smith RK, Montgomery MT, Vogl S. 2008. A critique of Emanuel's hurricane model and potential intensity theory. *Q. J. R. Meteorol. Soc.* **134**: 551–561.

Smith RK, Montgomery MT, Nguyen SV. 2009. Tropical cyclone spin-up revisited. *Q. J. R. Meteorol. Soc.* **135**: 1321–1335.

Stull RB. 1988. *An introduction to boundary-layer meteorology*. Kluwer: Dordrecht, the Netherlands.

Tennekes H. 1973. The logarithmic wind profile. *J. Atmos. Sci.* **30**: 234–238.

Vogl S, Smith RK. 2009. Limitations of a linear model for the hurricane boundary layer. *Q. J. R. Meteorol. Soc.* **135**: 839–850.

Von Kármán T. 1921. Über laminare und turbulente Reibung. *Z. Angew. Math. Mech.* **1**: 233–252.

Zhang JA, Montgomery MT. 2012. Observational estimates of the horizontal eddy diffusivity and mixing length in the low-level region of intense hurricanes. *J. Atmos. Sci.* **69**: 1306–1316.

Zhang JA, Rogers RF, Nolan DS, Marks FD. 2011a. On the characteristic height scales of the hurricane boundary layer. *Mon. Weather Rev.* **139**: 2523–2535.

Zhang JA, Marks FD, Montgomery MT, Lorsolo S. 2011b. Estimation of turbulence characteristics of eyewall boundary layer of hurricane *Hugo* (1989). *Mon. Weather Rev.* **139**: 1447–1462.

