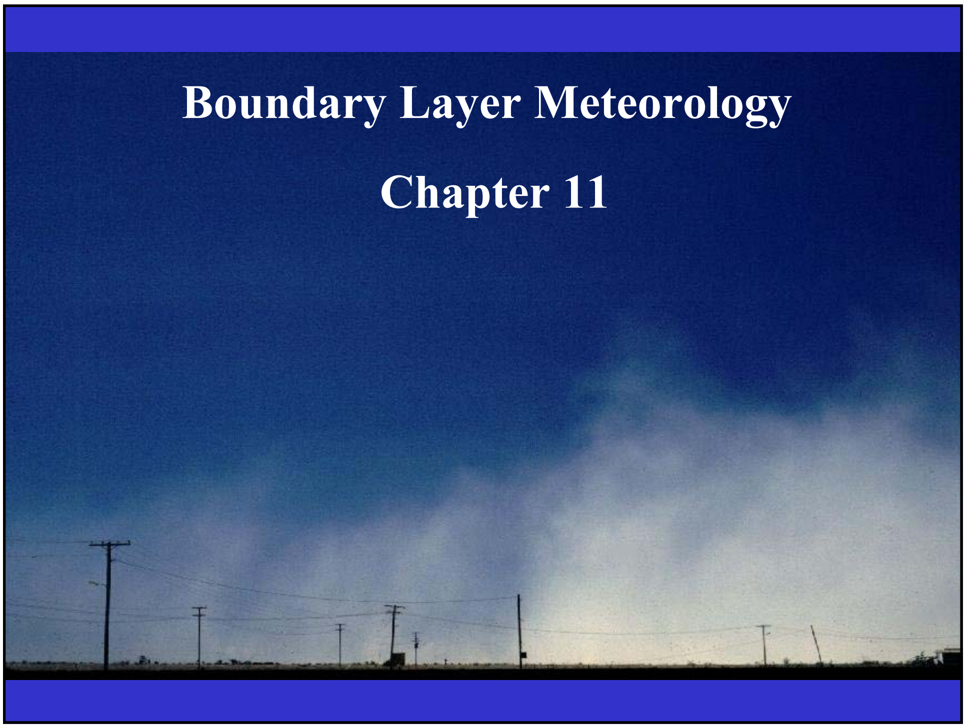


Boundary Layer Meteorology

Chapter 11

Convective mixed layer

- The mixed layer is so named because intense vertical mixing tends to leave conserved variables such as potential temperature and specific humidity nearly constant with height.
- Even wind speed and direction are nearly constant over the bulk of the mixed layer.
- The top of the whole convective mixed layer, z_i, is often defined as the level of most negative heat flux.
- This level is near the middle of the entrainment zone, often at the height where the capping inversion is strongest.
- Another measure of the depth is the level of neutral buoyancy for a surface lifted air parcel.

- **Mixing can be generated mechanically by shear or convectively by buoyancy.**
- **Buoyancy generated mixed layers tend to be more uniformly mixed than mechanically-driven ones, because anisotropy in convection favours vertical motions, while shear anisotropy favours horizontal motions.**

Equilibrium

- **The convective time scale,** t_***, is on the order** $10 - 20$ **min: the typical time for air to rise from the surface to the top of the mixed layer.**
- $\Rightarrow$ **changes in the surface fluxes can be communicated to the rest of the mixed layer in a relatively short time.**

Conservation equations

- **Let** $< >$ **denote an average of any quantity,** ξ **, over the mixed-layer depth:**

$$< \xi >= \frac{1}{z_i} \int_0^{z_i} \xi dz$$

$$z_i \frac{d\langle\overline{\theta}\rangle}{dt} = \overline{w'\theta'}\Big|_s - \overline{w'\theta'}\Big|_{z_i}$$

$$z_i \frac{d\langle\overline{q}\rangle}{dt} = \overline{w'q'}\Big|_s - \overline{w'q'}\Big|_{z_i}$$

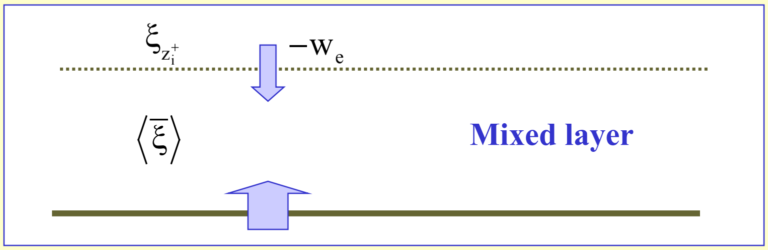
$$z_i \frac{d\langle\overline{u}\rangle}{dt} = \overline{w'u'}\Big|_s - \overline{w'u'}\Big|_{z_i} - f\left\langle\overline{v}_g - \overline{v}\right\rangle z_i$$

$$z_i \frac{d\langle\overline{v}\rangle}{dt} = \overline{w'v'}\Big|_s - \overline{w'v'}\Big|_{z_i} + f\left\langle\overline{u}_g - \overline{u}\right\rangle z_i$$

- The fluxes at the surface are usually specified as a boundary condition.
- The fluxes at the top of the mixed layer are usually found from:

$$\overline{w'\xi'}\Big|_{z_i} = -\underset{\uparrow}{w_e}(\xi_{z_i^+} - \langle\overline{\xi}\rangle)$$

Entrainment velocity



- We need also a continuity equation ⇒

Mixed layer evolution

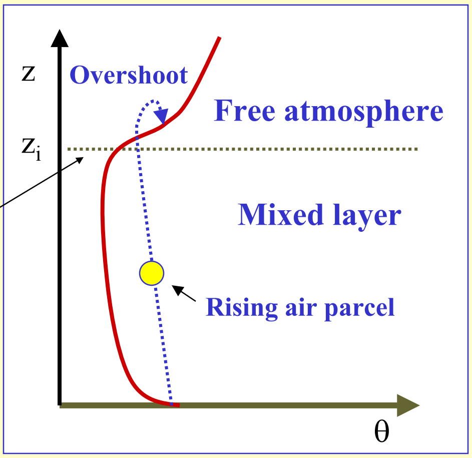
- A four phase process:
  1. Formation in the early morning of a shallow mixed layer, which slowly deepens (the burning off phase),
  2. Rapid mixed layer growth,
  3. Deep mixed layer of nearly constant thickness,
  4. Decay of turbulence.
- By late morning the top of the mixed layer reaches the base of the residual layer, after which rapid growth occurs.
- As the sun sets, the generation rate of convective turbulence decreases to the point that turbulence can no longer be maintained against dissipation.

Entrainment

- During free-convection, buoyant thermals gain momentum as they rise from the surface layer through the mixed layer.

Penetrative convection ⇒

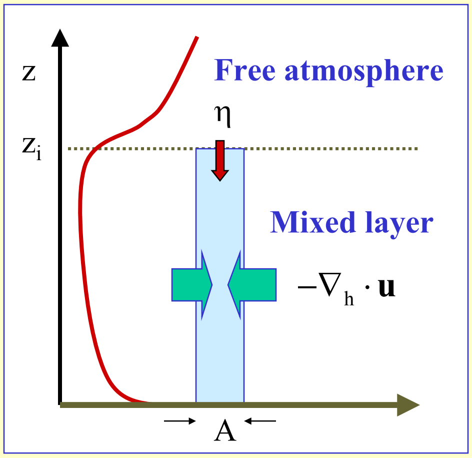
- Free atmosphere air is entrained into the mixed layer: less turbulent air is entrained into turbulent air.



Modelling entrainment:

$$A\frac{dz_i}{dt} = \eta$$

Volumetric rate of entrainment



$$\eta = w_e A - \int_0^{z_i} \int_A \nabla_h \cdot \mathbf{u} dxdy\, dz$$

$$\int_0^{z_i}\int_A \nabla_h \cdot \mathbf{u}\,dxdy\,dz = Aw_L$$

Large-scale vertical velocity at z_i

$$A\frac{dz_i}{dt} = \eta \quad \Rightarrow \quad \frac{dz_i}{dt} = w_e + w_L$$

–ve for subsidence

When active clouds are present that vent air out of the top of the mixed layer ⇒

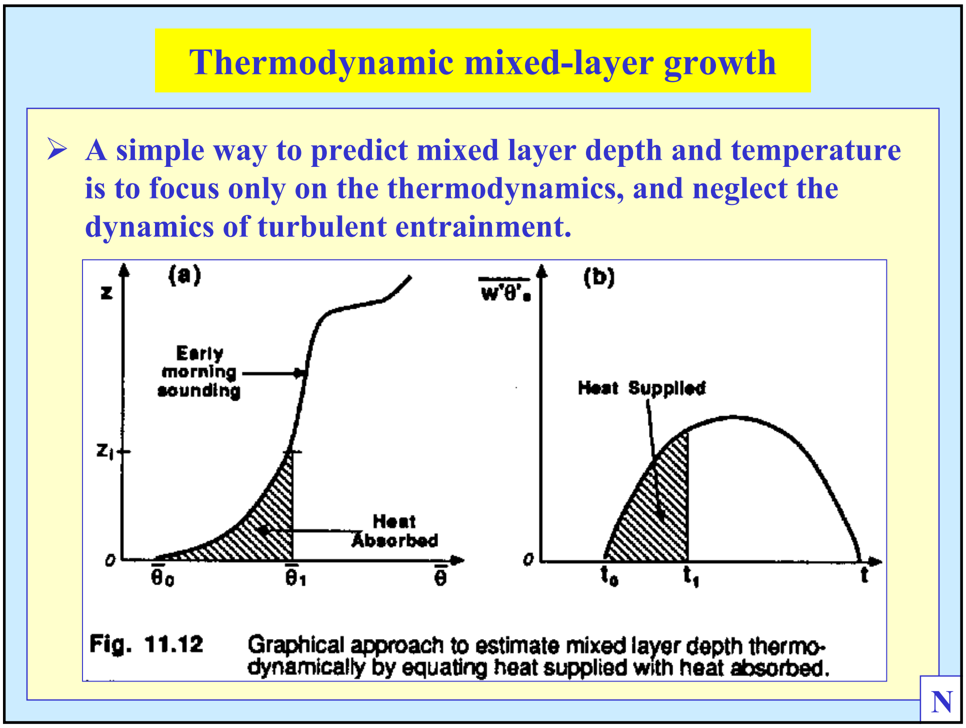
Fractional cloud cover

$$\frac{dz_i}{dt} = (1-\sigma_c)w_e - \sigma_c w_c + w_L$$

Mean updraft in clouds

$$\frac{dz_i}{dt} = (1-\cancel{\sigma}_c)w_e - \sigma_c\cancel{w}_c + \cancel{w}_L$$

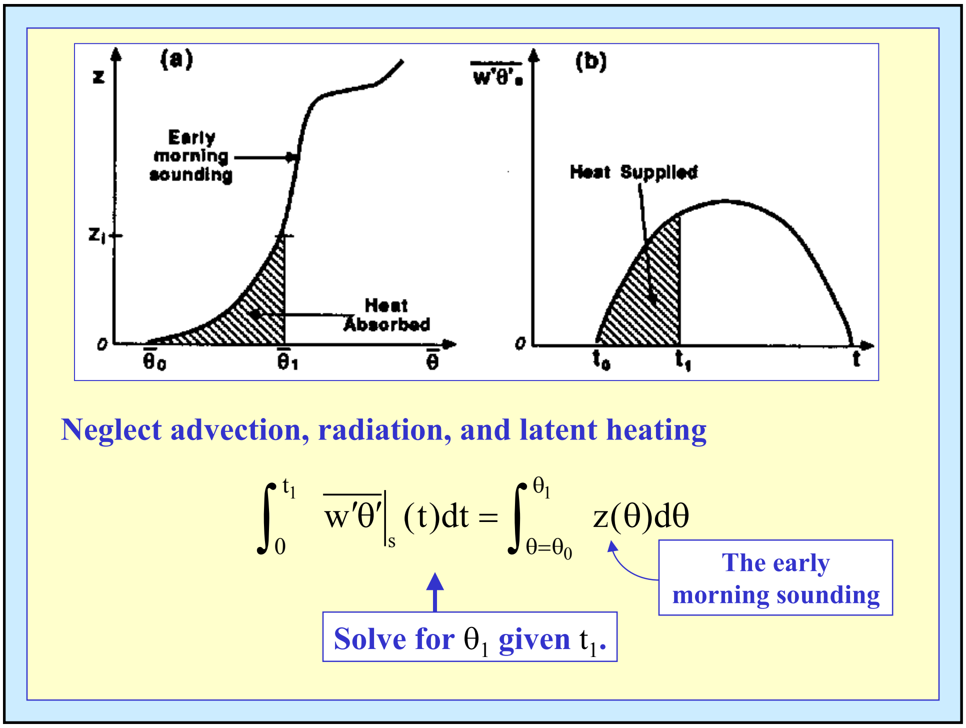
- **When there are no clouds and no subsidence, the mixed layer top grows at the rate w_e.**
- **Subsidence ($w_L < 0$) can reduce the rate of rise, or even lessen the mixed layer depth.**
- **Note that subsidence can never inject air into the mixed layer, because entrainment is controlled by w_e.**
- **In convergent situations, the mixed layer top can rise much faster than w_e.**
- **Note that horizontal advection may be important in deepening the mixed layer locally.**



Thermodynamic mixed-layer growth

- A simple way to predict mixed layer depth and temperature is to focus only on the thermodynamics, and neglect the dynamics of turbulent entrainment.

Fig. 11.12 Graphical approach to estimate mixed layer depth thermodynamically by equating heat supplied with heat absorbed.



Neglect advection, radiation, and latent heating

$$\int_0^{t_1} \overline{w'\theta'}\Big|_s (t)dt = \int_{\theta=\theta_0}^{\theta_1} z(\theta)d\theta$$

The early morning sounding

Solve for θ_1 given t_1.

Alternative approach: use the local lapse rate of the morning sounding above the mixed layer:

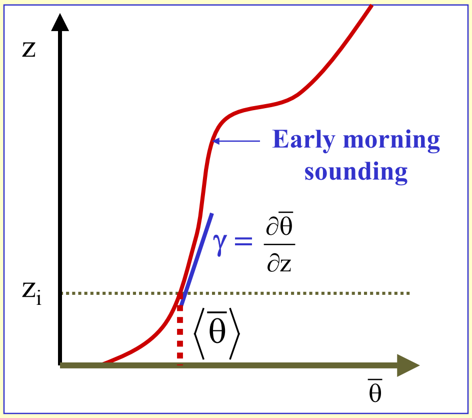
$$\frac{\partial z_i}{\partial t} = \frac{1}{\gamma}\frac{\partial \langle \overline{\theta} \rangle}{\partial t}$$

But

$$z_i \frac{d\langle \overline{\theta} \rangle}{dt} = \overline{w'\theta'}\Big|_s - \overline{w'\theta'}\Big|_{z_i}$$

⇓

$$\frac{\partial z_i}{\partial t} = \frac{\overline{w'\theta'}\Big|_s - \overline{w'\theta'}\Big|_{z_i}}{\gamma z_i}$$

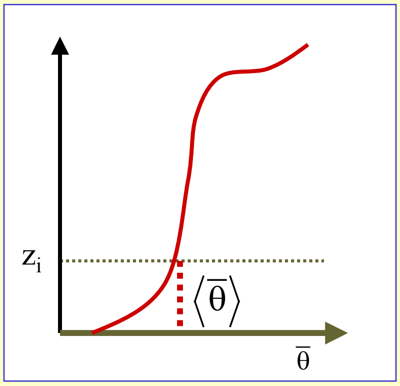


Assume: constant surface heat flux & constant lapse rate

$$z_i^2 - z_{i0}^2 = \frac{2}{\gamma}\left(\overline{w'\theta'}\Big|_s - \overline{w'\theta'}\Big|_{z_i}\right)(t - t_0)$$

Encroachment

- **The case where one assumes that the surface heat flux is the only heat source for the mixed layer is called encroachment, because the top of the mixed layer is never higher than the intercept of the mixed layer θ with the early morning sounding.**
- **The mixed layer encroaches, upwards only as the mixed layer warms.**
- **The thermodynamic approach explains 80-90 % of the observed variation in mixed-layer depth.**



Bulk model

- Assume that the mean variables are constant in the mixed layer, with a sharp discontinuity between the mixed layer and the free atmosphere.

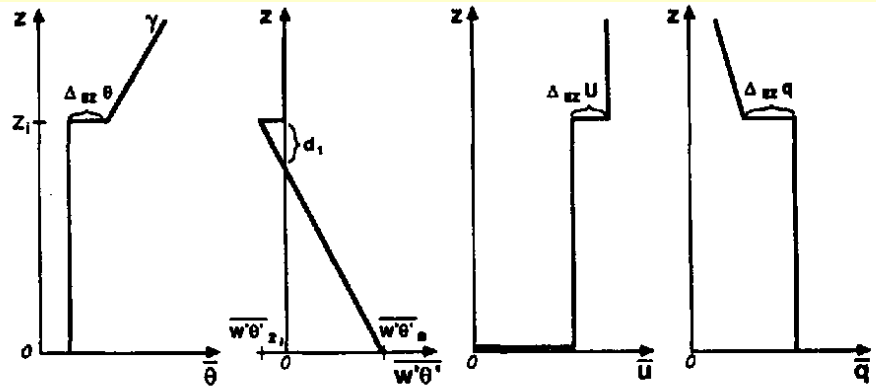


Fig. 11.13 Idealized slab mixed layer, with discontinuous jumps of variables at the mixed layer top.

Conservation equations

Potential temperature

$$z_i \frac{d\overline{\theta}}{dt} = \overline{w'\theta'}\Big|_s - \overline{w'\theta'}\Big|_{z_i}$$

x-**momentum**

$$z_i \frac{d\overline{u}}{dt} = \overline{w'u'}\Big|_s - \overline{w'u'}\Big|_{z_i} - f(\overline{v}_g - \overline{v})z_i$$

Similarly for moisture and y-**momentum**

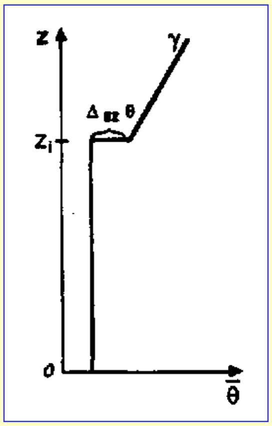
Mixed-layer depth

$$\frac{dz_i}{dt} = w_e + w_L$$

$$\overline{w'\theta'}\Big|_{z_i} = -w_e \underline{\Delta_{EZ}\overline{\theta}}$$

$$\Delta_{EZ}\overline{\theta} = \overline{\theta}_{z_i^+} - \overline{\theta}_{ML}$$

- Looking at the geometry of the idealized temperature profile, we expect the temperature jump to decrease as the mixed layer warms, and to increase as entrainment eats upward into the warmer air (Tennekes, 1973):



$$\underline{z_i}\frac{d\overline{\overline{\theta}}}{dt} = \underline{\overline{w'\theta'}\Big|_s} - \underline{\overline{w'\theta'}\Big|_{z_i}}$$

$$\frac{dz_i}{dt} = \underline{w_e} + \underline{w_L}$$

$$\overline{w'\theta'}\Big|_{z_i} = -w_e\underline{\Delta_{EZ}\overline{\theta}}$$

$$\frac{d\Delta_{EZ}\overline{\theta}}{dt} = \underline{\gamma} w_e - \frac{\partial\overline{\theta}}{\partial t}$$

Five unknowns: —

— Specified as boundary conditions

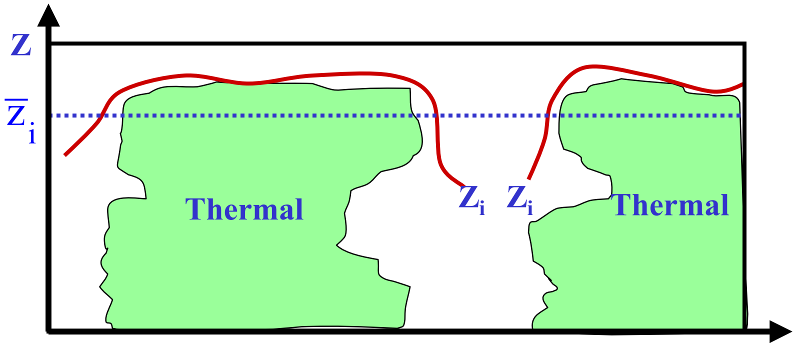
Closure assumption

- We require a closure assumption for one of the unknowns: two possibilities:
  - make an assumption for w_e
  - make an assumption for $\overline{w'\theta'}\Big|_{z_i}$
- Approach is categorized as a half-order closure, because the shapes of the mean profiles are fixed in advance and only one integrated value for each mixed layer variable is forecast.

Higher-order local closure models

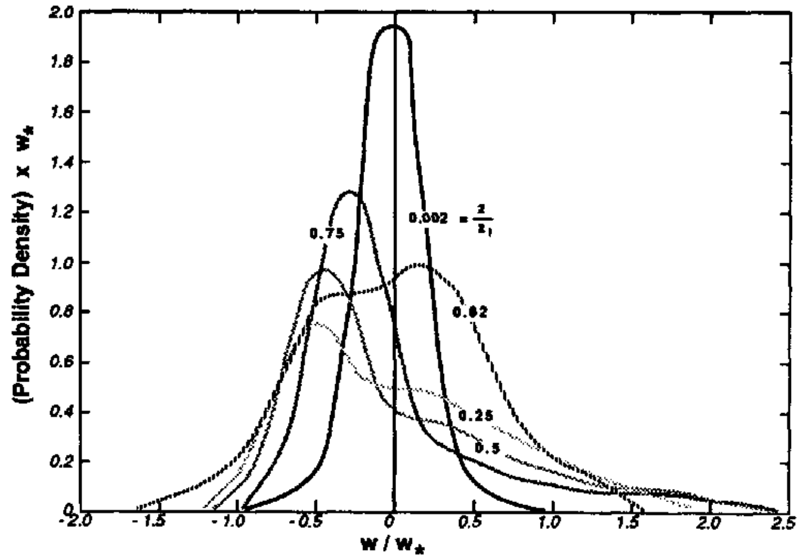
- **Higher-order local closure** models (one-and-a-half through to third order) have been very successful.
- Advantages:
  - Ability to forecast TKE, variances, and fluxes (if using second and higher order models).
- **Top down/bottom up diffusion**
  - K-theory (first-order local closure) has difficulties in the mixed layer.
  - An infinite eddy diffusivity is required to maintain fluxes in the absence of mean gradients.
  - One can try to use first-order closure and treat upward and downward mixing separately (see Stull, pp458-459).

Thermals

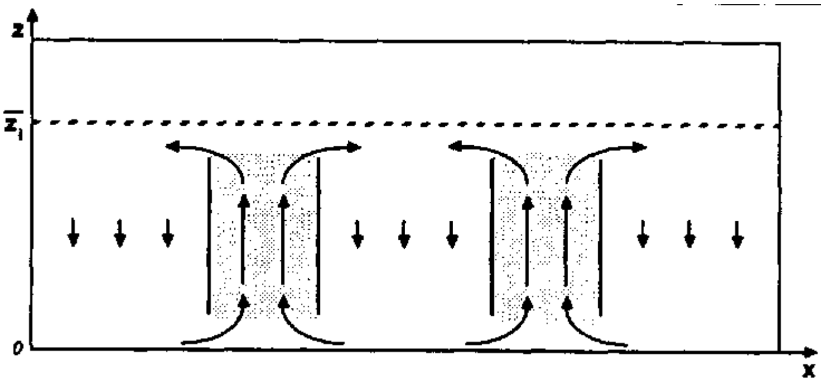


Idealized vertical cross-section showing thermals.

N



Probability density of vertical velocity at five levels of a convective mixed layer.



Exaggerated idealization showing thermals with strong updraughts covering a relatively small fractional area, with weak downdraughts in between.

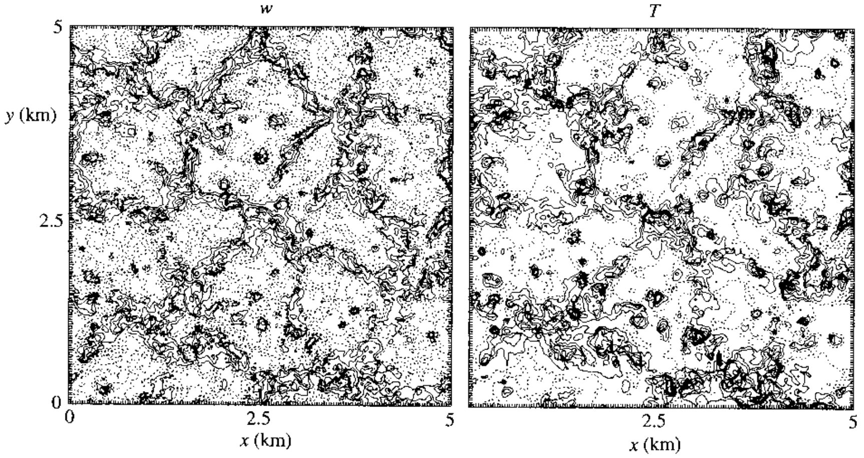
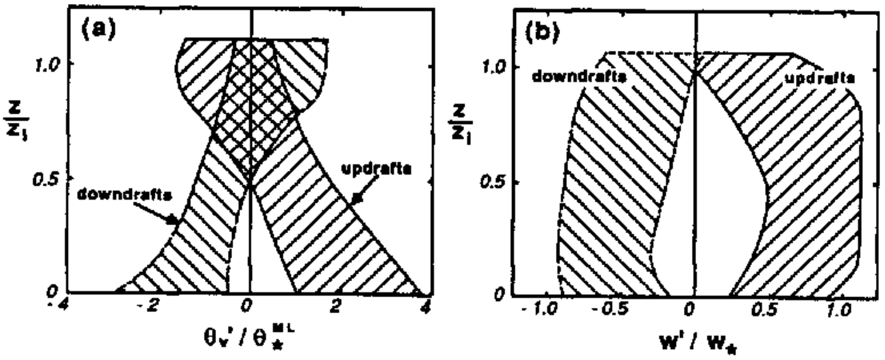


Fig. 6.5 Large-eddy model simulation of convection in the unstable ABL. The left and right panels show contour plots of normalized vertical velocity and normalized temperature fluctuations respectively. Each panel has a 5 km side, and the fields are shown for $z/h = 0.25$. The pecked curves correspond to negative contour values (i.e. cool, subsiding air). Thus, the patterns demonstrate an irregular polygon structure with warm, rising motion confined mostly to the "thin" walls of the 1–2 km-wide columns. From Schmidt and Schumann (1989).



Conditionally sampled values of (a) temperature, and (b) vertical velocity in updraughts (thermals) and downdraughts.

N

- Part of the difficulty in defining thermal boundaries is that although thermals start rising in the bottom third of the boundary layer as elements that are warmer than their environment, they are found to be cooler than their environment in the entrainment zone region.
- Some thermals gain most of the buoyancy from their moisture content, allowing the top half to be cooler than the environment even though the middle third might still be positively buoyant (using virtual potential temperature).

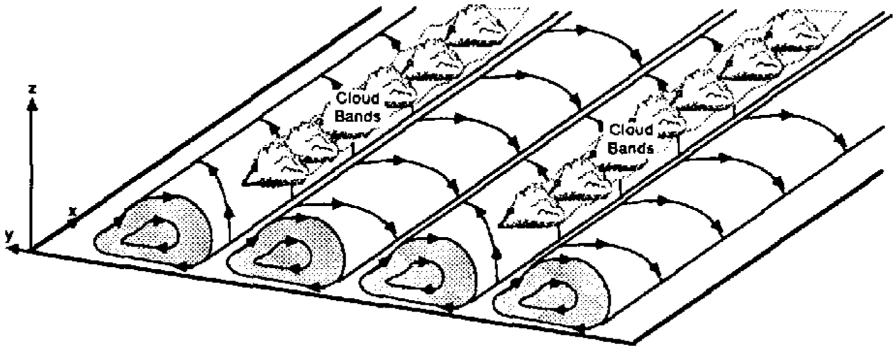
- Almost all of the observations indicate that the thermals are not like bubbles, but are more like finite length columns that persist for some time.
- This suggests that the best model might be the “wurst” model - namely, the idealized thermal shape is like that of a sausage or wurst.
- Real thermals are not perfect columns of rising air, but twist and meander horizontally and bifurcate and merge as they rise.
- Nevertheless, thermals are anisotropic, with most of their energy in the vertical.

- **When hot spots exist on land surfaces, thermals predominantly form there (Smolarkiezicz and Clark, 1985).**
- **Glider pilots look for such hot spots as indicators of persistent lift (Reichmann, 1978).**
- **Over moist or vegetated surfaces, thermals are often moister than their environment.**
- **They are also usually more turbulent.**
- **Thermals are observed over oceans as well as land surfaces, suggesting that a surface hot spot is not necessarily needed as a triggering mechanism.**
- **In the absence of hot spots, thermals can be swept into lines or rings by weak mesoscale motions (secondary circulations).**
- **These patterns are visible from satellite by the cloud streets and open/closed cells.**

Horizontal roll vortices & mesoscale cellular convection

- **During conditions of combined surface heating and strong winds, weak horizontal helical circulations can form in the boundary layer.**
- **These circulations are called horizontal roll vortices or rolls, and consist of clockwise and counterclockwise pairs of helices with their major axis aligned almost parallel with the mean wind.**
- **Some studies suggest that the roll axis should be roughly** 18° **to the left of the geostrophic wind direction for nearly neutral conditions, and that the angle decreases as the mixed layer becomes more statically unstable.**
- **The depth of these rolls equals the ML depth, and the ratio of lateral to vertical dimensions for a roll pair is about** 3:1**.**

N



Typical horizontal roll vortices in the planetary boundary layer.

N

- Rolls are frequently observed during cold-air advection over warmer bodies of water, and are strongly associated with air-mass modification.
- Rolls are also common in the low-level jet ahead of cold fronts, and can occur between pairs of closed isobars of warm season anticyclones.
- Forest fires are also modulated by rolls, as is evident by long rows of unburned tree crowns in the middle of burned areas (Haines, 1982).

- Theories for roll formation include thermal instabilities and inertial instabilities.
- Thermally, we would expect there to be less friction on the rising thermals if they align in rows, because thermals would have neighbours that are also updrafts.
- Thus, alignment into rows can be buoyantly more efficient, and the alignment provides protection from the ambient wind shear.
- Other studies have suggested that secondary circulations such as rolls develop whenever an inflection point occurs in the mean wind profile.
- For example, the Ekman spiral solution always has an inflection point near the top of the Ekman layer.

- Over the ocean, roll patterns and cloud streets gradually change to cellular patterns further downwind in a cold air advection situation.
- These are seen as honeycomb cloud patterns visible by satellite.
- This pattern is caused by **mesoscale cellular convection.**
- Open cells consists of hexagonal rings of updraft and clouds around clear central areas of downdraft, while closed cells are clear rings of descending air around mesoscale cloud clusters and updrafts.
- Cells typically have diameters on the order of 10 to 100 km, but are nevertheless boundary layer phenomena. Cells have depths on the order of 2 to 3 km, yielding aspect ratios of about 10:1 to 30:1.

Example 1

Given a cloud free mixed layer with constant $w_e = 0.1$ m s^{-1}, and a constant divergence of 5×10^{-5} s^{-1}, find the mixed layer depth versus time. Initially $z_i = 0$ at $t = t_0 = 0$.

Solution: Integrate the continuity equation to find w_L at z_i.

$$w_L\big|_{z_i} = -\mathrm{Div} \times z_i$$

$$\frac{dz_i}{dt} = w_e + w_L \quad \Rightarrow \quad \frac{dz_i}{dt} = w_e - \mathrm{Div} \times z_i$$

$$\Rightarrow \quad \frac{dz_i}{w_e - \mathrm{Div} \times z_i} = dt$$

Integrate

$$\Rightarrow \quad z_i = \frac{w_e}{\mathrm{Div}} - \left(\frac{w_e}{\mathrm{Div}} - z_i(t_0)\right) e^{-\mathrm{Div}(t-t_0)}$$

The resulting values for the mixed layer depths are:

t (h)	0	1	2	3	4	5	10	20
z_i (m)	0	329	605	834	1026	1187	1669	1945

➢ **Discussion:**

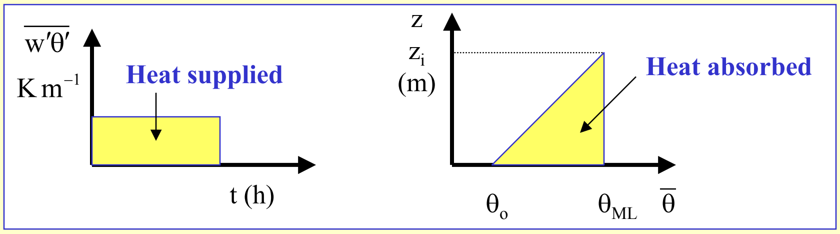
- **With constant divergence, w_L increases with height making mixed layer growth more difficult.**
- **The top cannot grow past 2 km, because at that height $|w_L| > |w_e|$.**
- **Actually $|w_e|$ becomes smaller as the mixed layer deepens.**
- **These two factors combine to limit the mixed layer depth.**
- **There have been observed cases where $|w_L| > |w_e|$ and z_i decreases with time.**

Example 2

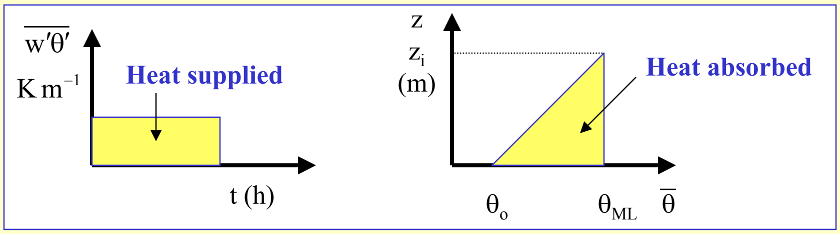
Suppose that initially $\theta = \theta_0 + \gamma z$ where $\theta_0 = 300$ K and $\gamma = 0.01$ K m^{-1}. Assume also that the surface heat flux is constant with time: $\left.\overline{w'\theta'}\right|_s = 0.2 \text{ K m}^{-1}$

If $z_i = 0$ at $t = 0$, use the thermodynamic method to find z_i at $t = 4$ h.

Solution: This problem can be represented graphically.



Heat supplied = 0.2 **K m/s** × 4 **h** × 3600 **s/h** = 2880 **K m**
Heat absorbed = 0.5 × **base** × **height** = $0.5\,(\theta_{ML} - \theta_o) \times z_i = 0.5\gamma z_i^2$



Heat supplied = 0.2 **K m/s** × 4 **h** × 3600 **s/h** = 2880 **K m**

Heat absorbed = 0.5 × **base** × **height** = $0.5\ (\theta_{ML} - \theta_o) \times z_i = 0.5\gamma z_i^2$

Equating the two heats gives:

Mixed layer depth $z_i = \text{sqrt}(2 \times 2880/0.01) = 759$ **m.**

Mixed layer potential temperature = $\theta_{ML} = \theta_o + \gamma z_i$
$= 300 + 0.01 \times 759 = 307.6$ **K**

- **Discussion: In reality, the heat flux usually varies with time, and the initial sounding is more complex.**

Entrainment velocity and its parameterization

Flux-ratio method

- **For free convection, the turbulence causing entrainment is directly related to the buoyancy flux at the surface.**
- **This constrains the buoyancy flux at the top of the mixed layer to be a nearly constant fraction of the surface flux (Ball, 1960):**

$$\frac{-\overline{w'\theta'_v}\Big|_{z_i}}{\overline{w'\theta'_v}\Big|_s} = A_R$$

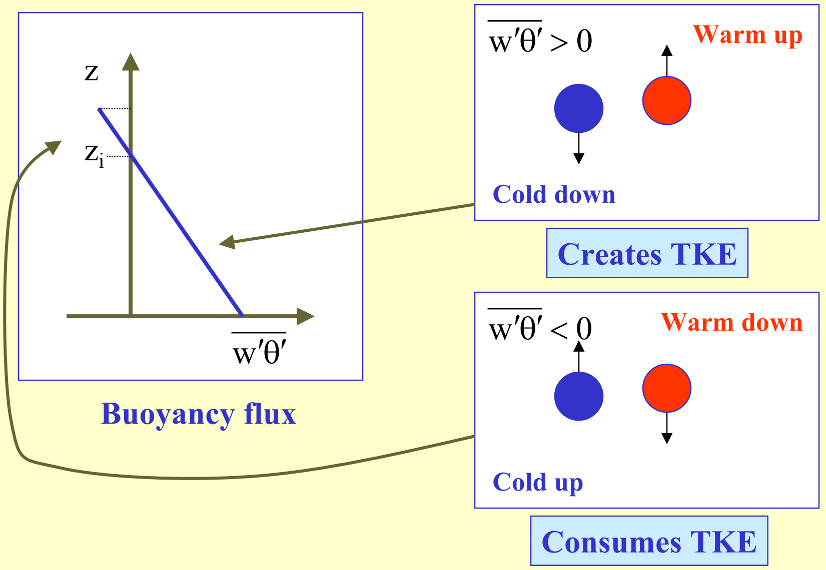
Typically $0.1 < A_R < 0.3$. $A_R = 0.2$ **is a good value to assume.**

$$\overline{w'\theta'}\Big|_{z_i} = -w_e \Delta_{EZ}\overline{\theta} \quad \Longrightarrow \quad w_e = \frac{A_R\ \overline{w'\theta'_v}\Big|_s}{\Delta_{EZ}\overline{\theta}_v}$$

Energetics method

- The flux-ratio method fails when wind shear generates the turbulence.
- One class of closure uses the TKE equation.
- For (potentially) warm air to be entrained into the cooler mixed layer, it must be forced down against the restoring force of gravity.
- By lowering this buoyant air, the (available) potential energy (PE) of the mixed layer/free atmosphere system is increased.
- The rate-of-change of PE with time equals the integral over height of the negative portions of buoyancy flux.
- Some of the mixed-layer TKE is used to do the work necessary to bring the entrained air down.

Buoyancy flux and TKE production



Buoyancy flux

$\overline{w'\theta'} > 0$

Warm up

Cold down

Creates TKE

$\overline{w'\theta'} < 0$

Warm down

Cold up

Consumes TKE

Energetics method (continued)

- The TKE equation is:

small

$$\frac{\partial \bar{e}}{\partial t} = \frac{g}{\bar{\theta}_v}\overline{w'\theta_v'} - \overline{u'w'}\frac{\partial \bar{u}}{\partial z} - \frac{\partial(\overline{w'e})}{\partial z} - \cancel{\frac{1}{\bar{\rho}}\frac{\partial \overline{w'p'}}{\partial z}} - \varepsilon$$

Integrates to zero

- Integrate over the total BL depth:

$$\frac{\partial}{\partial t}\int \bar{e}dz = \underbrace{\frac{g}{\bar{\theta}_v}\int \overline{w'\theta_v'}dz}_{B} - \int \overline{u'w'}\frac{\partial \bar{u}}{\partial z}dz - \frac{1}{\bar{\rho}}\overline{w'p'}\Big|_{z_i} - \int \varepsilon dz$$

Write $\overline{w'\theta_v'} = \overline{w'\theta_v'}\Big|_{\text{production}} + \overline{w'\theta_v'}\Big|_{\text{consumption}}$

$$B = \underbrace{\frac{g}{\bar{\theta}_v}\int \overline{w'\theta_v'}\Big|_{\text{production}} dz}_{\boxed{B_P}} + \underbrace{\frac{g}{\bar{\theta}_v}\int \overline{w'\theta_v'}\Big|_{\text{consumption}} dz}_{\boxed{B_N}}$$

$$B = \underbrace{\frac{g}{\bar{\theta}_v}\int \overline{w'\theta_v'}\Big|_{\text{production}} dz}_{\boxed{B_P}} + \underbrace{\frac{g}{\bar{\theta}_v}\int \overline{w'\theta_v'}\Big|_{\text{consumption}} dz}_{\boxed{B_N}}$$

B_P represents the production of TKE via buoyancy

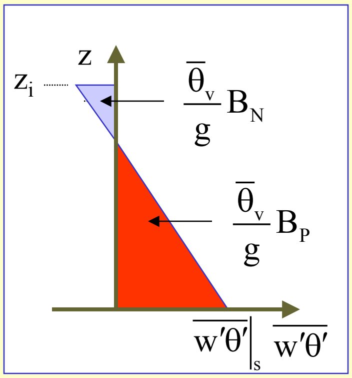
B_N represents the consumption of TKE via buoyancy as it is converted into PE

- Ways to partition the buoyancy profile into its production and consumption components are discussed by Stull (Section 11.4.3: pp479-483). See also Randall, JAS, 1984: Buoyant production and consumption of turbulence kinetic energy in cloud-topped mixed layers, p402.
- To obtain an expression for the entrainment velocity we need to make a few more assumptions.
- First assume a steady state.

$$\underbrace{-\frac{g}{\overline{\theta}_v}\int \overline{w'\theta_v'}\Big|_{\text{consumption}} dz}_{\mathbf{B_P}} = \underbrace{\frac{g}{\overline{\theta}_v}\int \overline{w'\theta_v'}\Big|_{\text{production}} dz}_{\mathbf{B_N}}$$

$$-\int \overline{u'w'}\frac{\partial \overline{u}}{\partial z}dz - \frac{1}{\rho}\cancel{\overline{w'p'}\Big|_{z_i}} - \int \varepsilon dz$$

MP **Gravity waves small** **Dis**



Parameterized in terms of the surface stress, u*, or the shear ΔU across the top of the mixed layer.

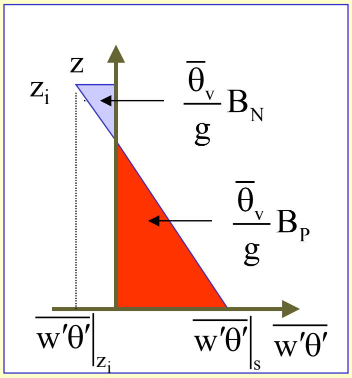
Assume Dis **proportional to total production rate** $\Rightarrow \propto B_P - MP$

(80% to 99% of TKE produced is dissipated)

$$-B_N = B_P - MP - Dis$$

Related to $\overline{w'\theta'}\big|_{z_i} = -w_e \Delta_{EZ}\overline{\theta}_v$

$$w_e = -\frac{\overline{w'\theta'}\big|_{z_i}}{\Delta_{EZ}\overline{\theta}_v}$$



- **The advantage of this production/consumption approach is that it can be applied to a variety of situations: free convection, forced convection, mixed convection, and cloud-topped mixed layers.**
- **When clouds are present, the production area should include the effects of radiation divergence.**

- Example: when the energetics method is applied to a cloud-free mixed layer with shear at both the surface and at the top of the mixed layer, one can derive the following relationship assuming an Eulerian partitioning geometry as before (Stull, 1976):

$$w_e = \frac{2\,\overline{\theta}_v}{g\,d_1\,\Delta_{EZ}\overline{\theta}_v}\left[\underline{c_1\,w_*^3} + \underline{c_2\,u_*^3} + \underline{c_3(\Delta_{EZ}\overline{U})^3}\right]$$

where $c_1 = 0.0167$, $c_2 = 0.5$, and $c_3 = 0.0006$.

▬ buoyant production

▬ surface mechanical production

▬ mechanical production at the mixed-layer top

d_1 = distance between the mixed-layer top and the height where the heat flux is zero ⇒

$$\frac{d_1}{z_i} = \frac{-\overline{w'\theta_v'}_{z_i}}{-\overline{w'\theta_v'}_{z_i} + \overline{w'\theta_v'}_s}$$

- It can be easily shown that this energetics parameterization reduces to the flux-ratio parameterization for the special case of free convection.

Other methods

- Many other methods have appeared in the literature.
- Most are variations of the above methods.
- Some, such as proposed by Deardorff (1979), offer generalized approaches that do not depend on a jump or slab type of model.
- Most approaches have been tested against observed data and give realistic results within the uncertainties of the data.
- Contributing to some of the uncertainty is subsidence, which is not easily measurable and can be as large as the entrainment velocity.

Other methods

- There were some additional methods to forecast mixed-layer depth that were tested, but which did not work well.
- One method assumed that the mixed-layer depth was proportional to the Ekman layer depth.
- Another assumed that a bulk Richardson number could be formed using the wind and temperature difference across the whole mixed-layer depth, and z_i was found using the requirement that the bulk Richardson number equal some constant critical value.

Example

Given: $\overline{w'\theta_v'} = 0.2$ K m/s, $u_* = 0.2$ m/s, $g/\overline{\theta_v} = 0.0333$ m s^{-2} K^{-1},

$z_i = 1$ km, $\Delta_{EZ}\overline{\theta_v} = 2$ K.

Find w_e **using**

a) The flux method,

b) The energetics method (special case equation XX)

Solution:

a) The flux method,

$$w_e = A_R \frac{\overline{w'\theta_v'}_s}{\Delta_{EZ}\overline{\theta_v}} = \frac{0.2 \cdot 0.2}{2} = 0.0200 \text{ m/s}$$

b) The energetics method (special case equation XX)

$$w_e = \frac{2}{(g/\overline{\theta_v})\, d_1\, \Delta_{EZ}\overline{\theta_v}} \left[c_1\, w_*^3 + c_2\, u_*^3 \right] = \frac{30.03}{d_1}\left[0.1112 + 0.004\right] = \frac{3.46}{d_1}$$

Example

But

$$d_1 = 1000 \left[\frac{2\, w_e}{0.2 + 2\, w_e} \right]$$

Combining these equations gives

$$w_e^2 = 0.00173\, [\, 2\, w_e + 0.2\,]$$

which can be solved to give $w_e = 0.0204$ m s^{-1}.

Discussion: We see that the addition of small values of surface stress have little effect on the entrainment rate in a free convection situation, and that the flux ratio method gives essentially the same answer with much fewer computations. For forced convection, however, the flux ratio method fails completely, but the energetics method can be used in the form (neglecting shear at the ML top, and using $d_1 = z_i$):

$$w_e = \frac{2\, c_2\, u_*^3}{(g/\overline{\theta_v})\, z_i\, \Delta_{EZ}\overline{\theta_v}}$$

Advection

- Even with the large vertical fluxes and vigorous turbulence in a convective mixed layer, horizontal advection of state characteristics by the mean wind can have as large an effect as turbulence.
- Neglect of advection is unwarranted for most simulations of the real boundary layer.
- One measure of the relative importance of turbulence vs. the mean wind is the dimensionless convective distance X^{ML} (Willis and Deardorff, 1976):

$$X^{ML} = \frac{x}{z_i}\frac{w_e}{U}$$

Advection

$$X^{ML} = \frac{x}{z_i}\frac{w_e}{U}$$

- This can be interpreted as the ratio of measured horizontal distance to the theoretical distance travelled during one convective circulation (updraft and downdraft).
- For large X^{ML} (>>1) turbulent mixing dominates over the mean wind, for small X^{ML} (<<1) the turbulent mixing is less important.

- **In addition to horizontal advection of momentum, moisture, heat and pollutants, we must be concerned about advection of z_i.**
- **In essence, the latter is a measure of the advection of volume within the mixed layer.**
- **For example, a slowly rising, shallow mixed layer over a moist irrigated region can grow rapidly if a deeper mixed layer advects into the area.**
- **The local change of z_i is described by:**

$$\frac{\partial z_i}{\partial t} = -\overline{u}_j \frac{\partial z_i}{\partial x_j} + w_e + w_L$$

- **The slope of z_i can be significant, and its neglect can cause forecast errors in mixed-layer depth tendency as great the magnitude of the entrainment velocity (Lenschow, 1973).**
- **For strong winds and abrupt changes in surface conditions, advection can dominate to prevent growth of the local mixed-layer with time.**
- **The resulting thermal internal boundary layer, which is a function of downwind distance x, is discussed in Chapter 14 of Stull.**

Subsidence and divergence

- Mean vertical velocities ranging from $-0.22\ \mathrm{m\ s^{-1}}$ (subsidence) to $0.27\ \mathrm{m\ s^{-1}}$ (upward motion) have been observed in a limited case study based on BLX83 field experiment data (Vachalek, et al., 1988).
- These magnitudes are very large compared to the entrainment velocity, and can not be neglected.
- Unfortunately, subsidence at the top of the mixed layer is very difficult to measure.
- Vertical velocity measurements from aircraft often have a mean bias that is greater than the true subsidence, and therefore cannot be used.

Subsidence and divergence

- The mean vertical velocity magnitudes are also below the noise level of most Doppler remote sensors.
- The downward movement of elevated smoke, moisture, or stable layers can be tracked, but usual horizontal advection affects this movement and is frequently not known.
- Also, the tracking of elevated layers can be applied only if the layers are in the free atmosphere above the mixed layer, so as not to be influenced by mixed-layer growth.
- Alternately, mean vertical motion at z_i can be estimated if divergence is known as a function of height within the ML, using:

$$w_L(z_i) = -\int_0^{z_i} \nabla \cdot \mathbf{u}\, dz$$

- For the special case of constant divergence with height:

$$w_L(z_i) = -(\nabla \cdot \mathbf{u})z_i$$

- This expression is frequently used for lack of better data.
- Horizontal divergence is not trivial to measure.
- Theoretically, we must measure the normal velocities out of a specified horizontal area, and we must make these measurements at every point on the perimeter of the area.
- Some remote sensors, such as Doppler radar and Doppler lidar, come closest to satisfying this requirement by measuring radial velocity at any specified range as a function of azimuth.
- A plot of this data is called a **velocity-azimuth display** (VAD) - unfortunately, ground clutter for the radar can contaminate the velocity statistics.

- Surface meso-network stations placed close together along the perimeter of the area can be used to estimate divergence near the surface, assuming that surface terrain features do not contaminate the velocities.
- As stations are spaced further apart along the perimeter, divergence accuracy drops.
- If the diameter of the network is too small, then the horizontal velocity difference across the network could be too small to resolve.

- A network of rawinsonde launch sites can be used to find divergence using the **Bellamy method**; however, sonde accuracy, large site separations, and sonde-to-sonde calibration errors can contaminate divergence calculations.
- Vachalek, et al. (1988) found that the rawinsonde divergence integrated over the mixed layer depth, and surface divergence from mesonetwork stations yielded the best results.
- Divergence fluctuations on a wide range of horizontal and temporal scales are usually superimposed on each other.

- Smaller diameter features appear to have greater magnitudes (by factors of up to 100) and shorter durations than the large diameter features.
- For example, a region of diameter 5 km was found to have peak divergence magnitudes in the range of 10^{-4} to 10^{-5} s^{-1}, while regions of diameter 100 km had peak divergences of 10^{-5} to 10^{-7} s^{-1}.
- A comparison of the relative frequency of events of different divergence magnitude and horizontal scale is presented in Stull, Fig 11.34.

- The short duration (1 h) divergence events occur about 10 times more frequently than long duration events, and about 95% of all divergence events had durations shorter than 8 h.
- This implies that divergence and subsidence estimated from a large-diameter network will not show large-magnitude short-period variations, and thus nay be useless for estimating subsidence over a fixed point at any specific time.

End

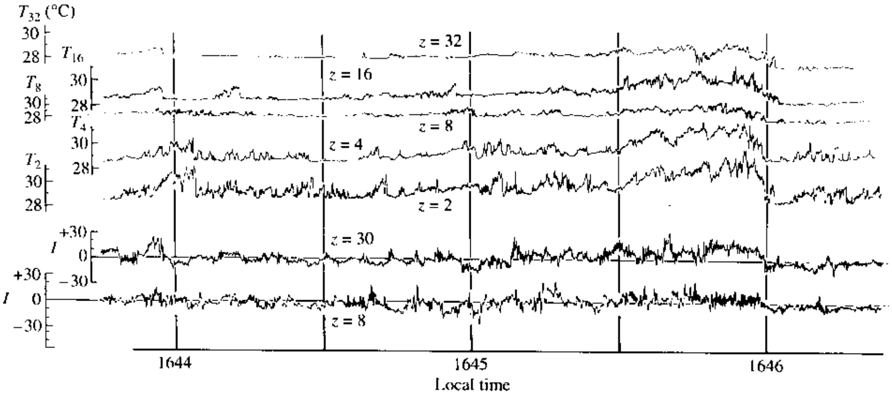


Fig. 6.3 Examples of recorded fluctuations of temperature T (in °C) and inclination angle I over flat, dry grassland (at Hay, Australia) at heights from 2 m to 32 m as shown. The mean wind speed at 2 m was 4.5 $\mathrm{m\,s^{-1}}$ with $L = -18.5$ m. The traces illustrate the characteristic ramp structure for temperature associated with passing thermals (indicated also by positive inclination, or upwards motion). From Webb (1977).

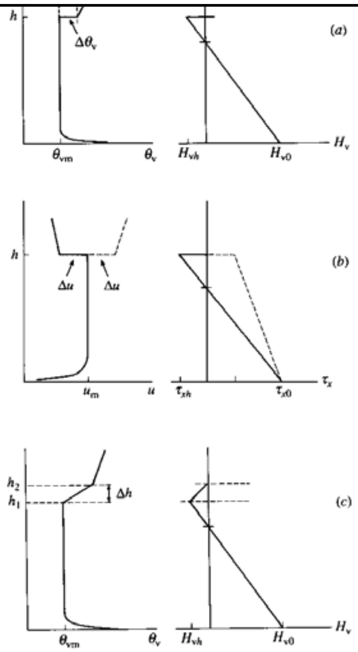


Fig. 6.6 Schematic representation of the vertical structure of the CBL, for wind and temperature. In (*a*) and (*b*) the inversion layer is assumed to be infinitesimally thin, and represented by zero-order jumps in properties. The CBL is assumed to be well mixed,