Boundary Layer Meteorology

Chapter 09

Similarity theory

- **For a number of boundary layer situations, our knowledge of the governing physics is insufficient to derive laws based on first principles.**
- **However, BL observations often show repeatable characteristics and we can derive empirical relationships for the variables of interest.**
- **Similarity theory provides a way to organize and group variables to our maximum advantage, and provides guidelines on how to design experiments to gain the maximum information.**
- **The idea is to organize variables into dimensionless groups.**
- **A dimensional-analysis procedure to aid us:**

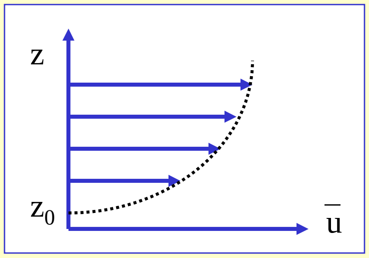
Buckingham Pi theory

- **This theory aids us in forming dimensionless groups from selected variables.**
- **The hope is that the proper choice of groups will lead to empirical relationships between these groups that are universal – i.e. they work everywhere and all the time for a particular situation.**
- **There are four steps in developing a similarity theory:**
  1) **Select (guess) which variables are relevant to the situation,**
  2) **Organize variables into dimensionless groups**
  3) **Use experimental data to determine the values of the groups**
  4) **Fit an empirical curve to the data**

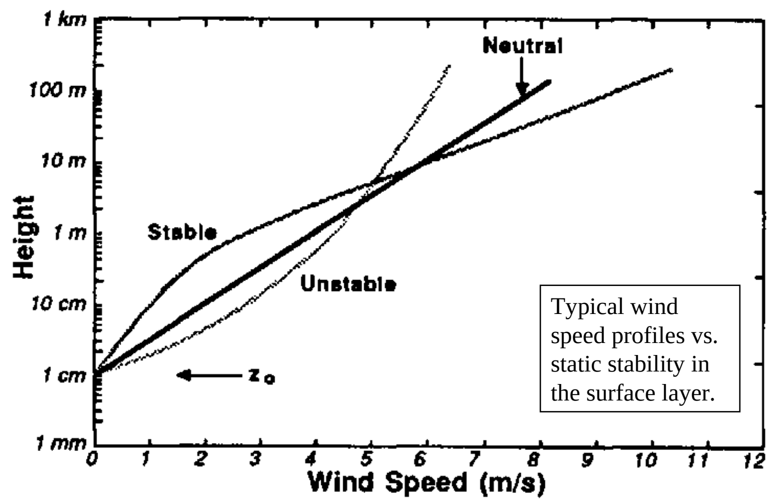
See Stull, Ch. 9

The log wind profile

- **One important application of similarity theory is to the mean wind profile in the surface layer.**
- **The nature of this profile is relevant to the structure of buildings, bridges, snow fences, wind breaks, pollutant dispersion, and wind turbines, for example.**
- **The surface layer wind profile has been studied extensively because of its accessibility to surface measurements.**
- **The wind speed usually varies approximately logarithmically with height in the surface layer. Frictional drag causes the mean wind speed to become zero close to the ground.**



- When plotted on semi-log graph paper, a logarithmic relationship such as the wind profile in statically neutral conditions appears as a straight line.



Typical wind speed profiles vs. static stability in the surface layer.

Wind profile in statically neutral conditions

- We want a relationship for the mean wind $\overline{U}(z)$
- We speculate that the relevant variables are:
- **Surface stress**, represented by u_*, and **surface roughness**, represented by the aerodynamic roughness length, z_o.
- Buckingham Pi theory indicates the two dimensionless groups: $\overline{U}/u_*$, and z/z_o.

From the graph
$$\frac{\overline{U}}{u_*} = \frac{1}{k}\ln\left(\frac{z}{z_o}\right)$$

Von Kármán constant

- The precise value is not agreed upon, but it is about 0.35-0.4.

Alternative derivation

- **The momentum flux in the surface layer is:**

$$\overline{u'w'} = -k^2 z^2 \left|\frac{\partial \bar{U}}{\partial z}\right| \frac{\partial \bar{U}}{\partial z}$$

- **But momentum flux in approximately constant in the surface layer**

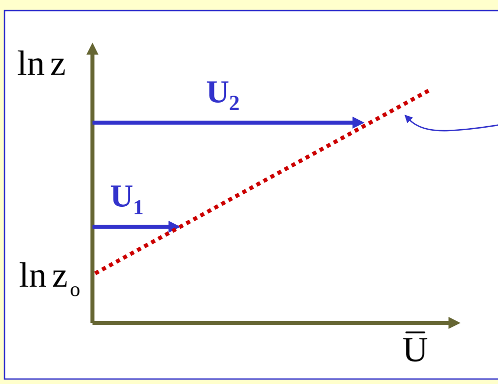
$$\overline{u'w'}(z) = \overline{u'w'}(0) = u_*^2$$

Substitution gives $\frac{\partial \bar{U}}{\partial z} = \frac{u_*}{kz}$

- **Integrate with respect to z from z_o gives** $\frac{\bar{U}}{u_*} = \frac{1}{k}\ln\left(\frac{z}{z_o}\right)$.

Aerodynamic roughness length

- **The aerodynamic roughness length, z_o, is defined as the height where the wind speed becomes zero.**
- **Given observations of wind speed at two or more heights, it is easy to solve for z_o and u_*:**



$$\frac{\bar{U}}{u_*} = \frac{1}{k}(\ln z - \ln z_o)$$

Interpretation

Aerodynamic roughness length

- The roughness length is **not** equal to the height of the individual **roughness elements** on the ground, but there **is** a one-to-one correspondence between those roughness elements and the aerodynamic roughness length.
- In other words, once the aerodynamic roughness length is determined for a particular surface, it does not change with wind speed, stability, or stress.
- It can change if the roughness elements on the surface change, such as caused by changes in the height and coverage of vegetation, erection of fences, construction of houses, deforestation or lumbering, etc.

Roughness length over the sea

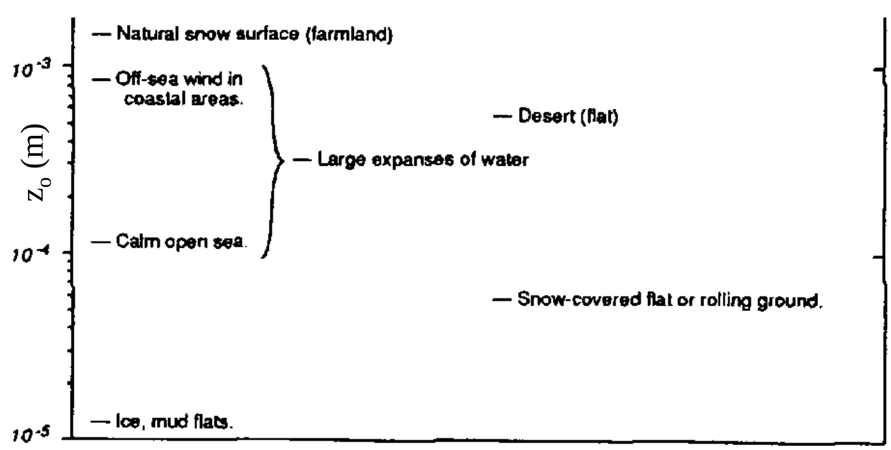
- Charnock's relationship for the roughness length of the sea surface

$$z_o = \frac{\alpha_c u_*^2}{g}$$

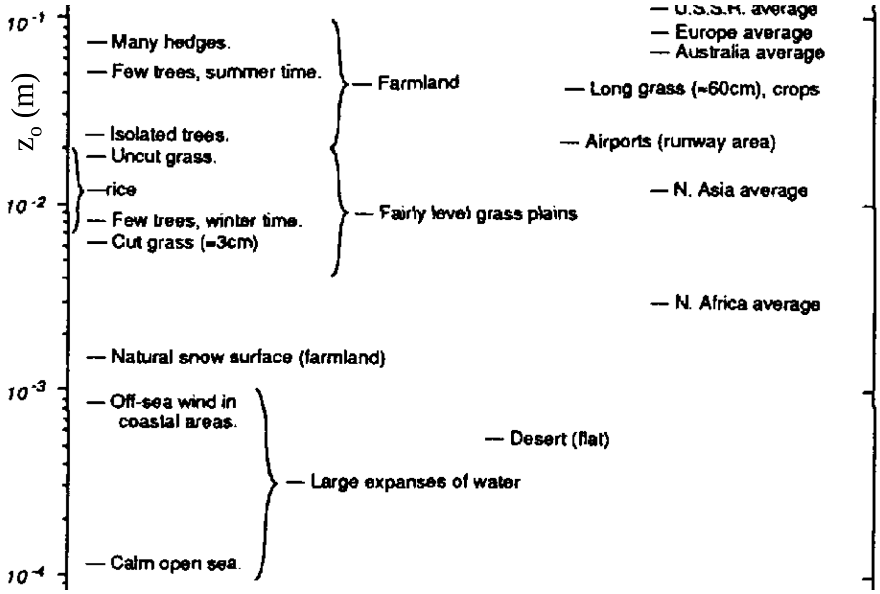
- For the sea, $\alpha_c = 0.016$.
- This relationship can be applied also to blowing snow with appropriate change in parameter, α_c (Chamberlain, 1983).

Aerodynamic roughness length

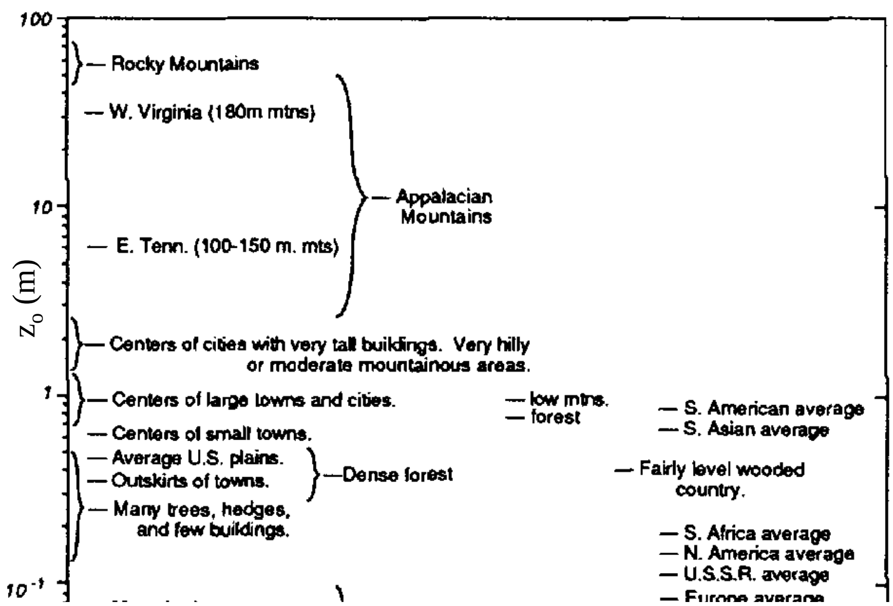
- **For many large-scale numerical weather-forecast models the lowest grid-points (at height z_1 above the surface) are so high that the surface layer is not resolved.**
- **Nevertheless, it is important to account for varying roughness in the model forecast.**
- **André and Blondin (1986) suggested that the effective roughness length (z_{oeff}) to be used in the model decreases as the altitude of the lowest grid point increases.**
- **In particular, the ratio $z_{o\,eff}/h^*$ decreases from about 0.1 to 0.01 as z_1 increases from 0.1 km to 1 km.**
- **Taylor (1987), however, suggests that $z_{o\,eff}$ is independent of z_1.**



Aerodynamic roughness lengths for typical terrain types.



z_o (m)
10^{-1}
10^{-2}
10^{-3}
10^{-4}
— Many hedges.
— Few trees, summer time.
— Isolated trees.
— Uncut grass.
—rice
— Few trees, winter time.
— Cut grass (≈3cm)
— Farmland
— Fairly level grass plains
— Natural snow surface (farmland)
— Off-sea wind in coastal areas.
— Calm open sea.
— Large expanses of water
— Desert (flat)
— U.S.S.R. average
— Europe average
— Australia average
— Long grass (≈60cm), crops
— Airports (runway area)
— N. Asia average
— N. Africa average



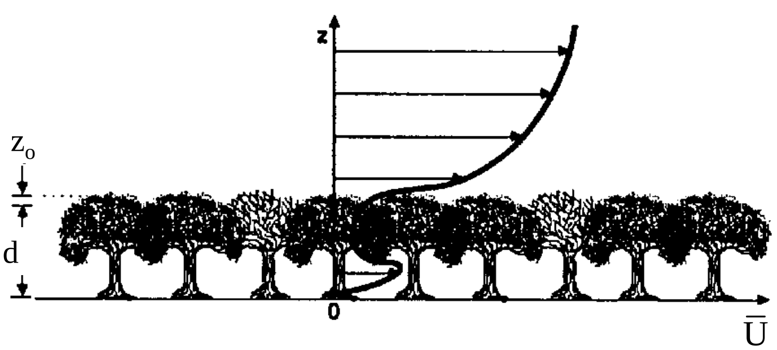
100
10
1
10^{-1}
z_o (m)
— Rocky Mountains
— W. Virginia (180m mtns)
— E. Tenn. (100-150 m. mts)
— Appalacian Mountains
— Centers of cities with very tall buildings. Very hilly or moderate mountainous areas.
— Centers of large towns and cities.
— low mtns.
— forest
— S. American average
— S. Asian average
— Centers of small towns.
— Average U.S. plains.
— Outskirts of towns.
— Many trees, hedges, and few buildings.
—Dense forest
— Fairly level wooded country.
— S. Africa average
— N. America average
— U.S.S.R. average

N

Displacement distance

- Over land, if the individual roughness elements are packed very closely together, then the top of those elements begins to act like a displaced surface.
- For example, in some forest canopies the trees are close enough together to make a solid-looking mass of leaves, when viewed from the air.
- In some cities the houses are packed close enough together to give a similar effect; namely, the average roof-top level begins to act on the flow like a displaced surface.

Displacement distance



Flow over a forest canopy showing wind speed as a function of height. The thick canopy acts like a surface displaced a distance, d, above the true surface. z_o = roughness length.

- Above the canopy top, the wind profile increases logarithmically with height.
- We can define both a displacement distance, d, and a roughness length, z_o, such that:

$$\bar{U} = \frac{u_*}{k} \ln\left(\frac{z-d}{z_o}\right)$$

for statically neutral conditions

- We now define $\bar{U} = 0$ at $z = d + z_o$.
- Given wind speed observations in statically neutral conditions at three or more heights, it is easy to use computerized non-linear regression algorithms such as the Marquardt Method or the Gauss-Newton Method to solve for the three parameters, u_*, $\mathbf{z_o}$, and d.

Nondimensional wind shear

$$\frac{\partial \bar{U}}{\partial z} = \frac{u_*}{kz}$$

- The nondimensional wind shear is:

$$\phi_M = \frac{kz}{u_*}\frac{\partial \bar{U}}{\partial z} = 1$$

Wind profile in non-neutral conditions

$\frac{\bar{U}}{u_*} = \frac{1}{k}\ln\left(\frac{z}{z_o}\right)$ relates the momentum flux, as represented by u_*^2, to the vertical velocity profile.

- Such formulae are called **flux-profile relationships**.
- These relationships can be extended to non-neutral surface layers.
- In non-neutral conditions we expect the buoyancy parameter and surface heat flux to be additional relevant variables.
- Buckingham Pi analysis gives three dimensionless groups (neglecting the displacement distance):

$$\frac{\bar{U}}{u_*} \quad \frac{z}{z_o} \quad \frac{z}{L}$$

L = Obukhov length

- Alternatively, if we consider the shear instead of the speed, we get two dimensionless groups: ϕ_M and z/L.
- Based on field data, Businger et al. (1971) and Dyer (1974) independently estimated the functional form to be:

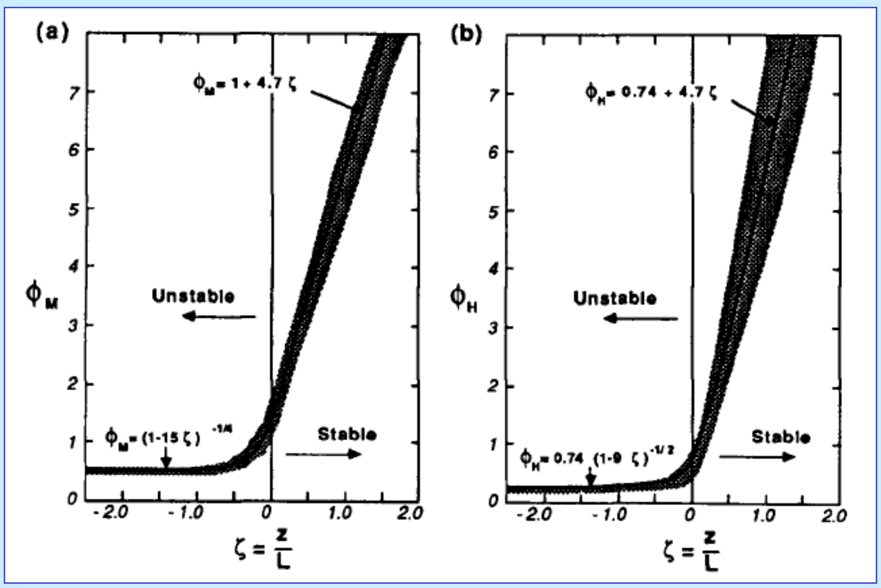
$$\phi_M = \begin{cases} 1 + 4.7z/L & \text{for } z/L > 0 \quad \text{(stable)} \\ 1 & \text{for } z/L = 0 \quad \text{(neutral)} \\ [1-(15z/L)]^{-1/4} & \text{for } z/L > 0 \quad \text{(unstable)} \end{cases}$$

- Businger et al. (1971) suggested that $k = 0.35$ from their data set.
- Similar expressions have been estimated for the heat flux versus the virtual potential temperature profile.

$$\phi_H = \begin{cases} \dfrac{K_M}{K_H} + \dfrac{4.7z}{L} & \text{for } \dfrac{z}{L} > 0 \quad \text{(stable)} \\ \dfrac{K_M}{K_H} & \text{for } \dfrac{z}{L} = 0 \quad \text{(neutral)} \\ \dfrac{K_M}{K_H}\left(1 - \dfrac{15z}{L}\right)^{-1/4} & \text{for } \dfrac{z}{L} > 0 \quad \text{(unstable)} \end{cases}$$

Where K_M/K_H is the ratio of eddy diffusivities of momentum and heat. This ratio equals 0.74 in neutral conditions.

- **It is often assumed that the flux profile relationships for moisture or pollutants are equal to those for heat.**



Range of dimensionless (a) wind shear observations and (b) temperature gradient observations in the surface layer, plotted with interpolation formulae.

Diabatic wind profile

- The Businger-Dyer relationships can be integrated with height to yield the wind speed profiles:

$$\frac{\bar{U}}{u_*} = \frac{1}{k}\left[\ln\left(\frac{z}{z_o}\right) + \psi_M\left(\frac{z}{L}\right)\right]$$

where $\psi_M\left(\frac{z}{L}\right) = \frac{4.7z}{L}$ for stable conditions $\frac{z}{L} > 0$,

and $\psi_M\left(\frac{z}{L}\right) = -2\ln\left(\frac{1+x}{2}\right) - \ln\left(\frac{1+x^2}{2}\right) + 2\tan^{-1}(x) - \frac{\pi}{2}$

for unstable conditions, $\frac{z}{L} < 0$ where $x = \left(1 - \frac{15z}{L}\right)^{1/4}$

- Both expressions reduce to the log wind profile when $z/L = 0$.

Inertial subrange

- There are many situations where middle size turbulent eddies "feel" neither the effects of viscosity, nor the generation of TKE.
- These eddies get their energy inertially from the larger-size eddies, and lose their energy the same way to smaller-size eddies.
- For a steady-state turbulent flow, the cascade rate of energy down the spectrum must balance the dissipation rate at the smallest eddy sizes.
- Hence, there are only three variables relevant to the flow: S, κ, and ε. S = spectral energy density.
- This similarity approach was pioneered by Kolmogorov (1941) and Obukhov (1941).

Inertial subrange

- By performing a Buckingham Pi dimensional analysis, we can make only dimensionless group from these three variables.

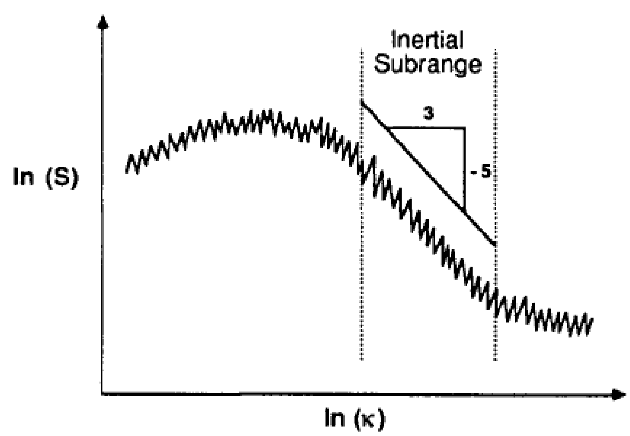
$$\pi_1 = \frac{S^3 \kappa^5}{\varepsilon^2}$$

- We know that Pi group must be equal to a constant, because there are no other Pi groups for it to be a function of.
- Solving the above equation for S yields:

$$S(\kappa) = \alpha_k \, \varepsilon^{2/3} \, \kappa^{-5/3}$$

  where the α_k is known as the **Kolmogorov constant.**

- The value of α_k has yet to be pinned down (Gossard, et al., 1982), but it is in the range of $\alpha_k = 1.53$ to 1.68.



To determine whether any measured spectrum has an inertial subrange one can plot the spectrum (S vs. κ) on a log-graph. The inertial subrange portion should appear as a straight line with a −5/3 slope.