Boundary Layer Meteorology

Chapter 5

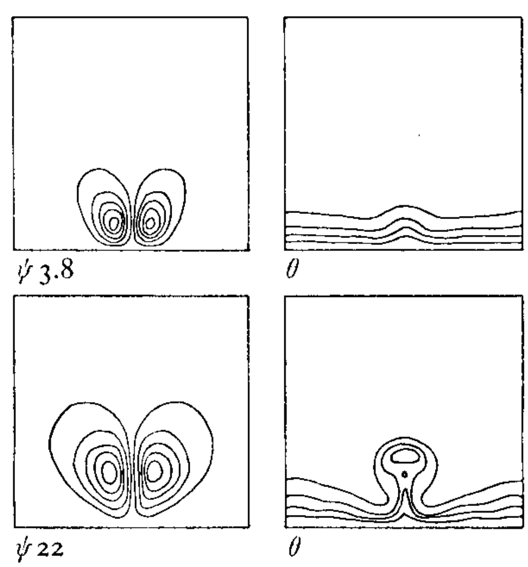
Stability concepts

- Unstable flows become or remain turbulent; stable flows become or remain laminar.
- If the net effect of all the destabilizing factors exceeds the net effect of the stabilizing factors, then turbulence will occur.
- In many cases, these factors can be interpreted in terms of TKE budget equations.
- We can compare one destabilizing factor with one stabilizing factor, expressed as a dimensionless ratio, e. g. Reynolds number, Richardson number, Froude number, and Rayleigh number.
- Static stability is not normally expressed non-dimensionally.

Static instability and convection

- Static instability is a measure of the capability for buoyant convection.
- “static” means “having no motion”
- Static instability occurs when less dense air underlies more dense air.
- The flow responds by supporting convective circulations (e.g. thermals) that allow the buoyant air to rise, thereby stabilizing the fluid.
- Thermals need some triggers to get them started.
- These are generally present in the atmosphere (e.g. hills, fields, trees, car parks, flow perturbations) and convection occurs.

Penetrative convection



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The formation of plumes or thermals rising from a heated surface

Higher heating rate

In the turbulent convection regime, the flux of heat from heated boundary is intermittent rather than steady and is accomplished by the formation of thermals

Local definitions of static instability

- **The traditional definition, that static instability is determined by the local lapse rate, is local in nature.**
- **It frequently fails in mixed layers, because the rise of thermals from near the surface or their descent from cloud tops depends on their excess buoyancy and not on the ambient lapse rate.**

Example

- **In the middle 50% of the convective mixed layer, the lapse rate is nearly adiabatic, causing an incorrect classification of stability on the basis of the local definition.**
- **We must clearly distinguish between "adiabatic lapse rate" and "neutral stability".**

- **An adiabatic lapse rate (in the virtual potential temperature sense) may be statically stable, neutral or unstable, depending on convection and the buoyancy flux.**
- **Neutral stability implies a very specific situation: adiabatic lapse rate and no convection.**
- **The two phrases should not be used interchangeably and the phrase "neutral lapse rate" should be avoided altogether.**

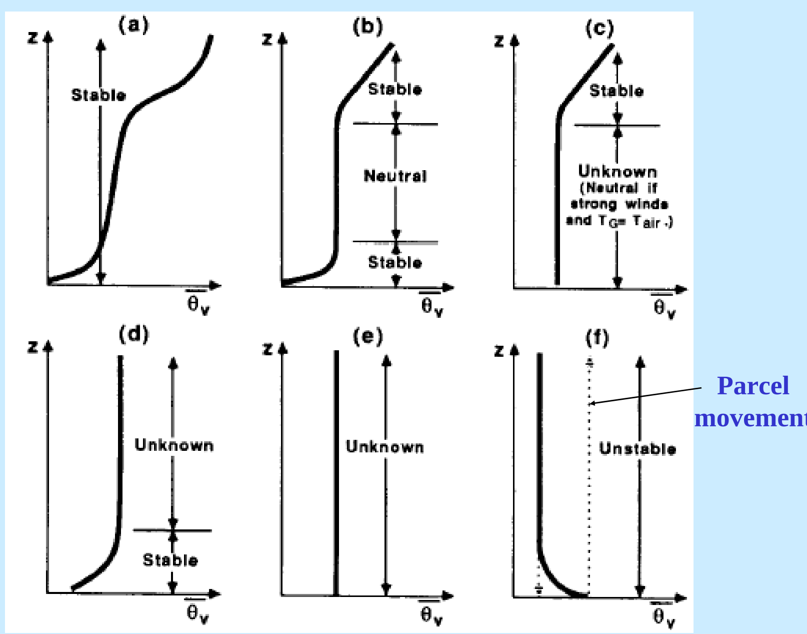
- **Conclusion: measurement of the local lapse rate alone is insufficient to determine the static (in)stability.**
- **Either knowledge of the whole $\overline{\theta_v}$ profile is needed, or measurement of the turbulent buoyancy flux must be made.**

Non-local definitions of static stability

- **It is better to examine the stability of the whole layer:**
- **Example: if $\overline{w'\theta_v'}$ is positive at the earth's surface, or if displaced air parcels will rise from the ground or sink from cloud top as thermals travelling across a BL, then the whole BL is said to be unstable or convective.**
- **If $\overline{w'\theta_v'}$ is negative at the surface, or if displaced air parcels return to their starting points, then the BL is said to be stable.**
- **If when integrated over the depth of the BL, the mechanical production term in the TKE equation is much larger than the buoyancy term, or if the buoyancy term is near zero, then the BL is said to be neutral.**
- **In this case the BL is sometimes referred to as an Ekman BL.**

- During fair weather conditions over land, the BL touching the ground is rarely neutral.
- Neutral conditions are frequently found in the residual layer aloft.
- In overcast conditions with strong winds, but little little temperature difference between the air and the surface, the boundary layer is often close to neutral stability.
- In the absence of knowledge of convection or buoyancy flux, an alternative determination of static stability is possible if the $\overline{\theta}_v$ profile over the whole BL is known.

See figures



Static stability as a function of the θ_v profile.

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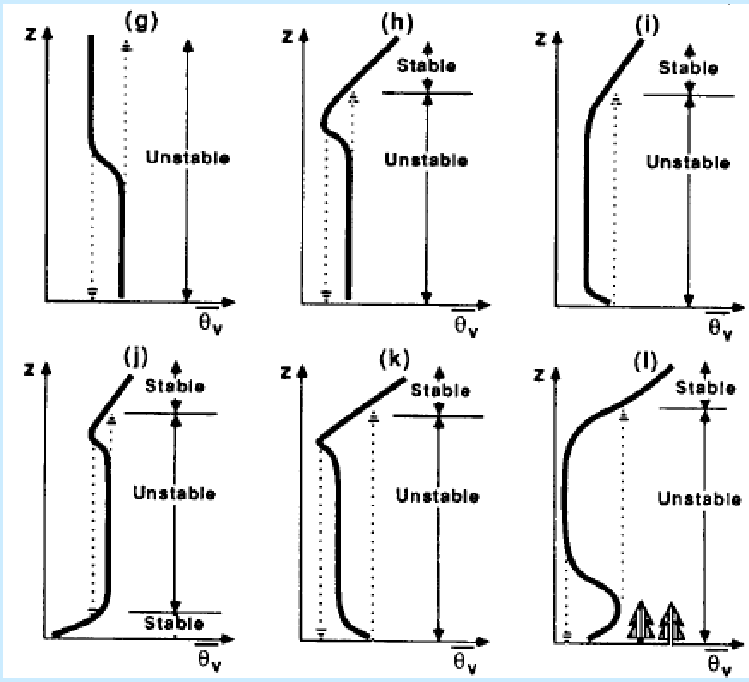
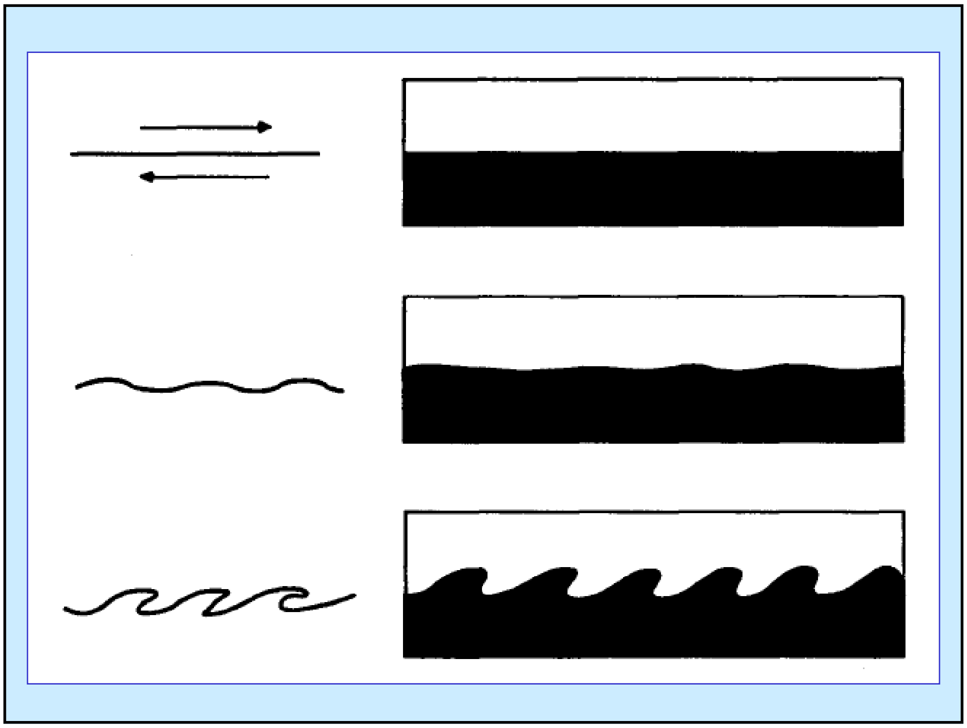
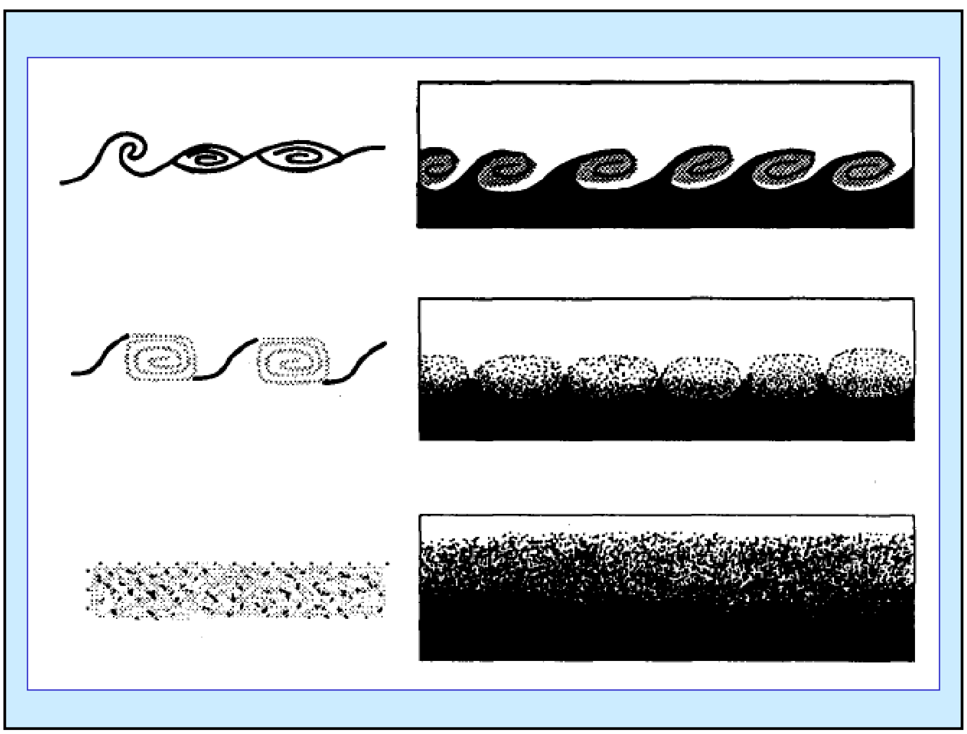


Dynamic instability & Kelvin-Helmholtz Instability

- **The word "dynamic" refers to motion; hence dynamic instability depends in part on the winds.**
- **Even if the air is statically stable, wind shears may be able to generate turbulence dynamically.**
- **The mechanism is called Kelvin-Helmholtz instability.**





KH-instability in the atmosphere

The Richardson number

Flux Richardson number

- In a statically stable environment, the vertical component of turbulent motion acts against the restoring force of gravity $\Rightarrow$
- Buoyancy tends to suppress turbulence, while the wind shear tends to generate turbulence mechanically.
- The buoyancy production term of the TKE equation is negative, while the mechanical production term is positive:

$$\frac{\partial \bar{e}}{\partial t} + \bar{u}_j \frac{\partial \bar{e}}{\partial x_j} = \underbrace{\frac{g}{\bar{\theta}_v}\overline{w'\theta'_v}}_{<0} \underbrace{- \overline{u'_i u'_j}\frac{\partial \bar{u}_i}{\partial x_j}}_{>0} - \frac{\partial(\overline{u'_j e})}{\partial x_j} - \frac{1}{\bar{\rho}}\frac{\partial \overline{u'_i p'}}{\partial x_i} - \varepsilon$$

The Flux Richardson number

$$\frac{\partial \bar{e}}{\partial t} + \bar{u}_j \frac{\partial \bar{e}}{\partial x_j} = \frac{g}{\bar{\theta}_v}\overline{w'\theta'_v} - \overline{u'_i u'_j}\frac{\partial \bar{u}_i}{\partial x_j} - \frac{\partial(\overline{u'_j e})}{\partial x_j} - \frac{1}{\bar{\rho}}\frac{\partial \overline{u'_i p'}}{\partial x_i} - \varepsilon$$

Define the **flux Richardson number**, Ri_f:

$$Ri_f = \frac{\frac{g}{\bar{\theta}_v}\overline{w'\theta'_v}}{\overline{u'_i u'_j}\frac{\partial \bar{u}_i}{\partial x_j}}$$

dimensionless

9 terms

- The negative term in term IV is dropped by convention.
- If we assume horizontal homogeneity and neglect subsidence, the above equation reduces to the more common form of the flux Richardson number:

$$Ri_f = \frac{\frac{g}{\overline{\theta}_v}\overline{w'\theta'_v}}{\overline{u'w'}\frac{\partial \overline{u}}{\partial z} + \overline{v'w'}\frac{\partial \overline{v}}{\partial z}}$$

- **For statically unstable flows, Ri_f is usually negative (because of the denominator). For neutral flows it is zero. For stable flows it is positive.**
- **Richardson proposed that $Ri_f = +1$ is a critical value, because the mechanical production rate balances the buoyant consumption of TKE.**
- **At any value of $R_f < +1$, static stability is insufficiently strong to prevent the mechanical generation of turbulence.**
- **For $R_f < 0$, the numerator even contributes to the generation of turbulence.**

- **Therefore, Richardson expected that**
  - **The flow is turbulent (dynamically unstable) when** $R_f < +1$
  - **The flow becomes laminar (dynamically stable) when** $R_f > +1$.
- **We recognize that statically unstable flow is, by definition, always dynamically unstable.**

The gradient Richardson number

- A problem: we can calculate R_f only for turbulent flow because it contains factors involving turbulent correlations $\Rightarrow$ we can use it only to determine whether turbulent flow will become laminar, but not whether laminar flow will become turbulent.
- How can we proceed?

Let us assume that $$-\overline{w'\theta_v'} \propto \frac{\partial \overline{\theta}_v}{\partial z}$$

Similarly $$-\overline{u'w'} \propto \frac{\partial \overline{u}}{\partial z} \qquad -\overline{v'w'} \propto \frac{\partial \overline{v}}{\partial z}$$

- This is the basis of K-theory, or eddy-diffusivity theory to be discussed later. For the moment we assume that it is possible.

- Substitution gives the gradient Richardson number, Ri:

$$Ri_f = \frac{\dfrac{g}{\overline{\theta}_v}\dfrac{\partial \overline{\theta}_v}{\partial z}}{\left(\dfrac{\partial \overline{u}}{\partial z}\right)^2 + \left(\dfrac{\partial \overline{v}}{\partial z}\right)^2}$$

- When only the Richardson number is referred to without specifying which one, usually the gradient Richardson number is meant.
- Theoretical and laboratory research suggest that laminar flow becomes unstable to KH-wave formation and ONSET of turbulence when Ri is smaller than the critical Richardson number, Ri_c.
- Another value, Ri_T, indicates the termination of turbulence.

Richardson number criteria

- The dynamic stability criteria can be stated as follows:
- Laminar flow becomes turbulent when $Ri < Ri_c$.
- Turbulent flow becomes laminar when $Ri > Ri_T$.
- Although there is some debate on the correct values of Ri_c and Ri_T, it appears that $Ri_c = 0.21$ to 0.25 and $Ri_T = 1.0$ work fairly well.
- Thus, there appears to be a **hysteresis** effect because Ri_T is greater than Ri_c.
- One hypothesis for apparent hysteresis is as follows:
- Two conditions are needed for turbulence: instability, and some trigger mechanism.
- Suppose that dynamic instability occurs whenever $Ri < Ri_T$.

Hysteresis effect

- If one trigger mechanism is existing turbulence in or adjacent to the unstable fluid, then turbulence can continue as long as $Ri < Ri_T$ because of the presence of both the instability and the trigger.
- If KH waves are another trigger mechanism, then, in the absence of existing turbulence, one finds that Ri must fall well below Ri_T before KH waves can form.
- Laboratory and theoretical have shown that the criterion for KH wave formation is $Ri < Ri_c$.
- This leads to the apparent hysteresis, because the Ri of nonturbulent flow must be lowered to Ri_c before turbulence will start, but once turbulent, the turbulence can continue until Ri is raised above Ri_T.

The bulk Richardson number

- Theoretical work yielding $Ri_c = 0.25$ is based on local measurements of the wind shear and temperature gradient.
- We rarely know the actual local gradients, but can approximate these using observations at a series of discrete height intervals, setting

$$\frac{\partial\overline{\theta}_v}{\partial z} \approx \frac{\Delta\overline{\theta}_v}{\Delta z} \quad \frac{\partial\overline{u}}{\partial z} \approx \frac{\Delta\overline{u}}{\Delta z} \quad \frac{\Delta\overline{v}}{\Delta z} \approx \frac{\partial\overline{v}}{\partial z}$$

- Define the bulk Richardson number, Ri_b:

$$Ri_b = \frac{g}{\overline{\theta}_v} \frac{\Delta\overline{\theta}_v \partial z}{\left[\left(\Delta\overline{u}\right)^2 + \left(\Delta\overline{v}\right)^2\right]}$$

- This is the most frequently used form.

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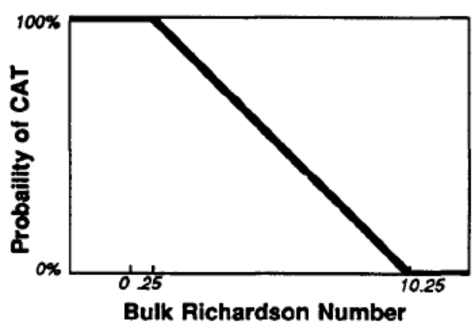
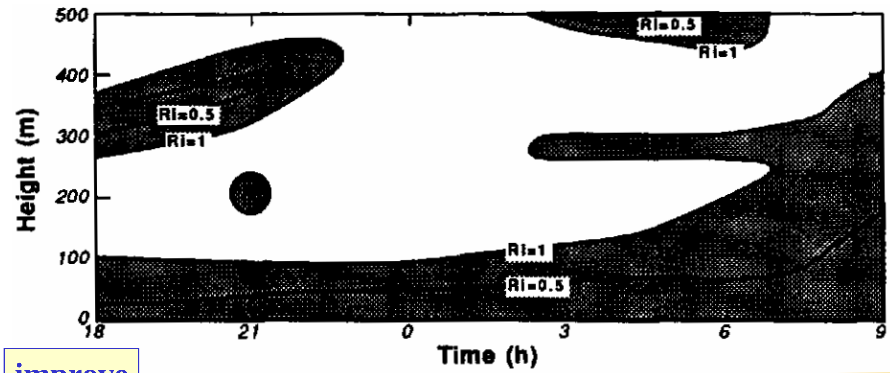
- Unfortunately, the critical value of 0.25 applies only for local gradients, not for finite differences across thick layers.
- In fact, the thicker the layer is, the more likely we are to average out large gradients that occur within small sub regions of the layer of interest.
- The net result is:
  1) we introduce uncertainty into our prediction of the occurrence of turbulence, and
  2) we must use an artificially large (theoretically unjustified) value of the critical Richardson number that gives reasonable results using our smoothed gradients.
- The thinner the layer, the closer the critical Richardson number will likely be to 0.25.

- Since data points in soundings are sometimes spaced far apart in the vertical, approximations such as shown in the graph and table in **the next figure** can be used to estimate the probability and intensity of turbulence (Lee et al., 1979).

Fig. 5.19
(a) Relationship between the bulk Richardson number, Ri, over a layer and the probability of turbulence within that layer. This curve was developed empirically. (b) Empirically derived relationship between turbulence intensity and the wind speed and shear. "N" indicates none, "L" is light, "M" is moderate, "S" is severe, and "X" is extreme turbulence.



Wind Speed m/s (kt)	Vector wind shear s^{-1} (kt/1000 feet) .0084-.0118 (5-7)	.0118-.0169 (7-10)	.0169-.0338 (10-20)	.0338-.0506 (20-30)	.0506-.0844 (30-50)	.0844+ (50+)
20-30 (40-60)	N	L	L-M	M	M-S	S
30-60 (60-120)	L	L-M	M	M-S	S	S-X
60+ (120+)	L	L-M	M	M-S	S	X



improve

Example of the evolution of Richardson number with height and time during one night. Regions with Ri < 1 are shaded and likely to be turbulent.

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The Obukhov length

- The Obukhov length (L) is a scaling parameter that is useful in the surface layer.
- To show how this parameter is related to the TKE equation, first recall that one definition of the surface layer is that region where turbulent fluxes vary by less than 10% of their magnitude with height.
- By making the constant flux (with height) approximation, one can use surface values of heat and momentum flux to define turbulence scales and nondimensionalize the TKE equation.

The Obukhov length

- Start with the TKE equation:

$$\frac{\partial \bar{e}}{\partial t} + \bar{u}_j \frac{\partial \bar{e}}{\partial x_j} = \frac{g}{\bar{\theta}_v} \overline{w_i'\theta_v'} - \overline{u_i'u_j'} \frac{\partial \bar{u}_i}{\partial x_j} - \frac{\partial (\overline{u_j'e})}{\partial x_j} - \frac{1}{\bar{\rho}} \frac{\partial \overline{u_i'p'}}{\partial x_i} - \varepsilon$$

- Multiply the whole equation by $-kz/u_*^3$, assume all turbulent fluxes equal their respective surface values, and focus on just terms III, IV and VII:

$$\ldots = -\frac{kzg\left(\overline{w_i'\theta_v'}\right)_s}{\bar{\theta}_v u_*^3} - \frac{kz\left(\overline{u_i'u_j'}\right)_s}{u_*^3} \frac{\partial \bar{u}_i}{\partial x_j} \ldots - \frac{kz\varepsilon_s}{u_*^3}$$

III IV VII

- Term III is usually assigned the symbol, ζ, and is further defined as $\zeta \equiv z/L$, where L is the Obukhov length.

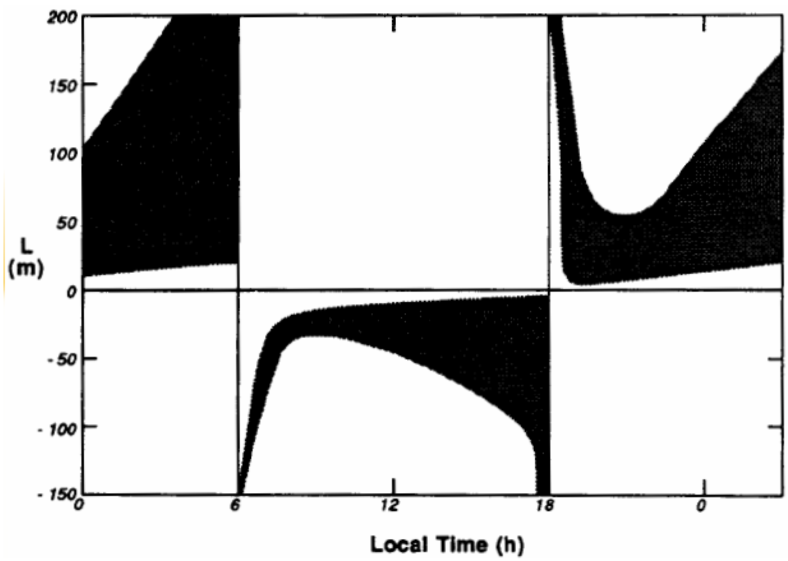
Thus

$$\zeta = \frac{z}{L} = \frac{-kzg\left(\overline{w'\theta_v'}\right)_s}{\bar{\theta}_v u_*^3}$$

$$L = \frac{-\bar{\theta}_v u_*^3}{kg\left(\overline{w'\theta_v'}\right)_s}$$

is called the Obukhov length

- One physical interpretation of the Obukhov length is that it is proportional to the height above the surface at which buoyant factors first dominate over mechanical (shear) production of turbulence.
- For convective situations, buoyant and shear production terms are approximately equal at $z = -0.5L$.



Typical range of variations of the Obukhov length in fair weather conditions over land.

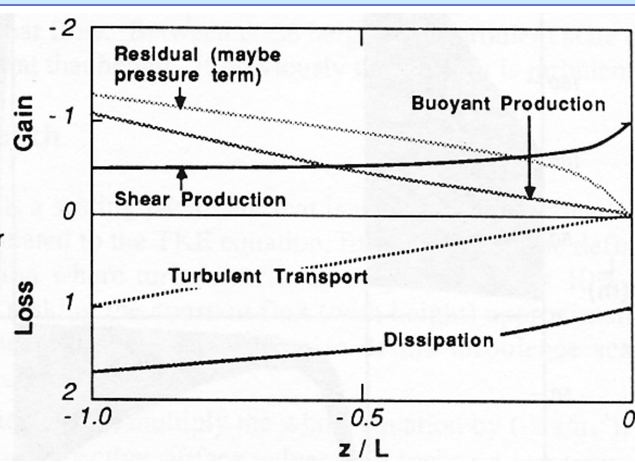
- **The parameter ζ turns out to be very important for scaling and similarly arguments of the surface layer.**
- **It is sometimes called a stability parameter, although its magnitude is not directly related to static or dynamic stability.**
- **Only its sign relates to static stability: negative implies unstable, positive implies statically stable.**
- **A better description of ζ is "a surface-layer scaling parameter".**

- We can write an alternative form for ζ by employing the definition of w_*:

$$\zeta = \frac{z}{L} = \frac{-kzw_*^3}{z_i u_*^3}$$

- The next figure shows the variation of TKE budget terms with ζ, as ζ varies between 0 (statically neutral) and −1 (slightly unstable).

Fig. 5.22
Behavior of the terms (made dimensionless with kz/u_*^3) in the unstable surface-layer turbulence kinetic energy budget. After Wyngaard (1973).



- The decrease in importance of shear and increase of buoyancy as ζ decreases from 0 to −1 is particularly obvious.

End