Boundary Layer Meteorology

Chapter 4

Contents

Prognostic equations for turbulent quantities

- So far we have obtained prediction equations for mean quantities in a turbulent flow.
- These equations involve covariances: e.g.

$$\frac{\partial \overline{u}}{\partial t} + \overline{u}_j \frac{\partial \overline{u}}{\partial x_j} = -\varepsilon_{ij3} f(\overline{v}_g - \overline{v}_j) - \frac{\partial \overline{(u_j' u')}}{\partial x_j}$$

- We now derive prediction equations for fluctuating quantities in a turbulent flow.

- Perturbation quantities represent turbulent fluctuations from their respective means.
- In theory, prognostic equations for these departures could be used to forecast each individual gust, given appropriate initial and boundary conditions.
- Unfortunately, the time span over which a forecast is likely to be accurate is proportional to the lifetime of the eddy itself: O(a few secs) for the smallest eddy to about 15 min for the larger thermals.
- Such durations are not useful in meteorological applications.
- Instead we derive prognostic equations as an intermediate step towards finding prognostic equations for variances and covariances of the variables.

Momentum equation

Recall that

$$\frac{\partial \bar{u}_i}{\partial t}+\frac{\partial u_i'}{\partial t}+\bar{u}_j\frac{\partial \bar{u}_i}{\partial x_j}+\bar{u}_j\frac{\partial u_i'}{\partial x_j}+u_j'\frac{\partial \bar{u}_i}{\partial x_j}+u_j'\frac{\partial u_i'}{\partial x_j}=\varepsilon_{ij3}f\bar{u}_j+\varepsilon_{ij3}fu_j'$$

$$-\frac{1}{\bar{\rho}}\frac{\partial \bar{p}}{\partial x_i}+-\frac{1}{\bar{\rho}}\frac{\partial p'}{\partial x_i}-\delta_{i3}\left[g-g\frac{\theta_v'}{\bar{\theta}_v}\right]+\nu\frac{\partial^2 \bar{u}_i}{\partial x_j^2}+\nu\frac{\partial^2 u_i'}{\partial x_j^2} \quad (1)$$

and

$$\frac{\partial \bar{u}_i}{\partial t}+\bar{u}_j\frac{\partial \bar{u}_i}{\partial x_j}=\varepsilon_{ij3}f\bar{u}_j-\frac{1}{\bar{\rho}}\frac{\partial \bar{p}}{\partial x_i}-\delta_{i3}g+\nu\frac{\partial^2 \bar{u}_i}{\partial x_j^2}-\frac{\overline{\partial(u_i'u_j')}}{\partial x_j} \quad (2)$$

Take 1 – 2

Momentum equation

$$\frac{\partial u_i'}{\partial t}+\bar{u}_j\frac{\partial u_i'}{\partial x_j}+u_j'\frac{\partial \bar{u}_i}{\partial x_j}+u_j'\frac{\partial u_i'}{\partial x_j}=$$

$$\varepsilon_{ij3}fu_j'-\frac{1}{\bar{\rho}}\frac{\partial p'}{\partial x_i}+\delta_{i3}g\frac{\theta_v'}{\bar{\theta}_v}+\nu\frac{\partial^2 u_i'}{\partial x_j^2}-\frac{\overline{\partial(u_i'u_j')}}{\partial x_j}$$

This is a prognostic equation for the turbulent gust u_i'.

Similarly for moisture

$$\frac{\partial q_T'}{\partial t}+\bar{u}_j\frac{\partial q_T'}{\partial x_j}+u_j'\frac{\partial \bar{q}_T}{\partial x_j}+u_j'\frac{\partial q_T'}{\partial x_j}=\nu_q\frac{\partial^2 q_T'}{\partial x_j^2}+\frac{\partial(\overline{u_j'q_T'})}{\partial x_j}$$

For heat

$$\frac{\partial\theta'}{\partial t}+\bar{u}_j\frac{\partial\theta'_i}{\partial x_j}+u'_j\frac{\partial\bar{\theta}}{\partial x_j}+u'_j\frac{\partial\theta'}{\partial x_j}=-\frac{1}{\bar{\rho}}\frac{\partial Q_j^{*\prime}}{\partial x_i}+\nu\frac{\partial^2\theta'}{\partial x_j^2}+\frac{\partial\overline{(u'_j\theta')}}{\partial x_j}$$

For a scalar quantity

$$\frac{\partial c'}{\partial t}+\bar{u}_j\frac{\partial c'}{\partial x_j}+u'_j\frac{\partial\bar{c}}{\partial x_j}+u'_j\frac{\partial c'}{\partial x_j}=\nu\frac{\partial^2 c'}{\partial x_j^2}+S_c\frac{\partial\overline{(u'_j c')}}{\partial x_j}$$

We can use these prognostic equations to obtain prognostic equations for the variances.

Free convection scaling variables

- **So far we have obtained prediction equations for mean quantities in a turbulent flow.**
- **These equations involve covariances: e.g.**

$$\frac{\partial\bar{u}}{\partial t}+\bar{u}_j\frac{\partial\bar{u}}{\partial x_j}=-\varepsilon_{ij3}f(\bar{v}_g-\bar{v}_j)-\frac{\partial\overline{(u'_j u')}}{\partial x_j}$$

- **We now derive prediction equations for fluctuating quantities in a turbulent flow.**
- **Before doing this we digress to see how experimental data are scaled, so that such data can be used for guidance.**

- We have learnt that turbulence can be produced by buoyant convective processes (i.e. thermals of warm air rising and cooler air subsiding) and by mechanical processes (i.e. wind shear).
- Sometimes one process dominates.
- When convective processes dominate, the BL is said to be in a state of free convection.
- When mechanical processes dominate, the BL is said to be in a state of forced convection.
- Free convection occurs over land on clear sunny days with light or calm winds.
- Here we focus on free convection scales (scales for forced convection were introduced earlier).

- In free convection, strong solar heating at the surface creates a pronounced diurnal cycle in turbulence and mixed layer depth.
- Earlier, profiles of heat and moisture fluxes were non-dimensionalized to remove these diurnal changes.
- The resulting heat flux profiles, for example, presented height in terms of a fraction of the mixed layer depth and flux values as fractions of surface flux values.
- A similar scheme to remove nonstationary effects is useful for determining the relative contributions of the various terms in the variance and flux equations.
- We consider now scalings for free convection conditions.

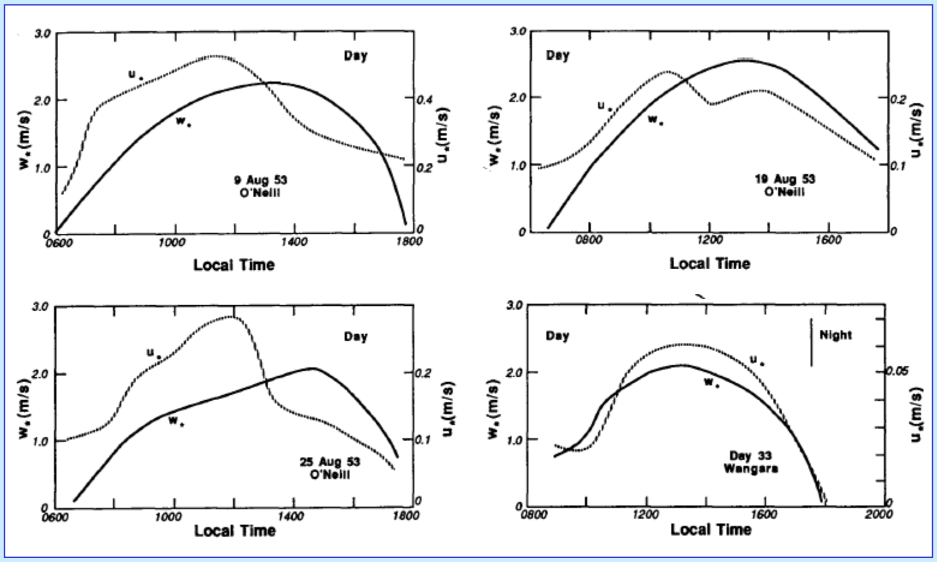
- **Length scale**: Thermals rise until they encounter the stable layer capping the mixed layer.
- Thermals are the dominant eddy in the convective boundary layer, and all smaller eddies feed on the thermals for energy.
- ⇒ would expect many turbulent processes to scale to the **mixed layer depth** (z_i) in convective situations.

- **Velocity scale**: The strong diurnal cycle in solar heating creates a strong heat flux into the air from the earth's surface.
- The buoyancy associated with this flux fuels the thermals.
- We can define a **buoyancy flux** as $\frac{g}{\overline{\theta}_v}\overline{w'\theta'_v}$.

- Although the surface buoyancy flux could be used directlly as a scaling variable, it is more convenient to generate a velocity scale instead, using the two variables we know to be important in free convection: **surface buoyancy flux**, and the **mixed layer depth**, z_i.
- Combining these variables gives the free convection scaling velocity, w^*, also called the **convective velocity scale**:

$$w^* = \left[\frac{g z_i}{\overline{\theta}_v}\left(\overline{w'\theta'_v}\right)_s\right]^{1/3}$$

- This scale appears to work quite well; for example the magnitude of the vertical velocity fluctuations in thermals is on the same order as w^*. For deep mixed layers with vigorous heating at the ground, w^* can be on the order of 1 to 2 m s^{-1}.



Sample variations of the friction velocity and the convective scaling velocity with time for the O'Neill (Nebraska) and Wangara (Australia) field programs.

- **Time scale: The velocity and length scales can be combined to give the free convection time scale, t_*:**

$$\frac{g}{\overline{\theta}_v}\overline{w'\theta_v'}$$

- **Velocity scale: The strong diurnal cycle in solar heating creates a strong heat flux into the air from the earth's surface.**
- **The buoyancy associated with this flux fuels the thermals.**
- **We can define a buoyancy flux as** $\frac{g}{\overline{\theta}_v}\overline{w'\theta_v'}$.

Prognostic equations for variances

- **Momentum variance**

$$\frac{\partial u_i'}{\partial t} + \bar{u}_j \frac{\partial u_i'}{\partial x_j} + u_j' \frac{\partial \bar{u}_i}{\partial x_j} + u_j' \frac{\partial u_i'}{\partial x_j} =$$

$$\varepsilon_{ij3} f u_j' - \frac{1}{\bar{\rho}} \frac{\partial p'}{\partial x_i} + \delta_{i3} g \frac{\theta_v'}{\bar{\theta}_v} + \nu \frac{\partial^2 u_i'}{\partial x_j^2} - \frac{\partial \overline{(u_i' u_j')}}{\partial x_j}$$

Multiply by $2u_i' \Rightarrow$

$$\frac{\partial u_i'^2}{\partial t} + \bar{u}_j \frac{\partial u_i'^2}{\partial x_j} + 2u_i'u_j' \frac{\partial \bar{u}_i}{\partial x_j} + 2u_i'u_j' \frac{\partial u_i'}{\partial x_j} =$$

$$\varepsilon_{ij3} 2 f u_i' u_j' - 2 \frac{u'}{\bar{\rho}} \frac{\partial p'}{\partial x_i} + 2\delta_{i3} g u_i' \frac{\theta_v'}{\bar{\theta}_v} + 2\nu u_i' \frac{\partial^2 u_i'}{\partial x_j^2} - 2u_i' \frac{\partial \overline{(u_i' u_j')}}{\partial x_j}$$

Prognostic equations for variances

- **Now average and apply the Reynolds' averaging rules:**

$$\frac{\partial \overline{u_i'^2}}{\partial t} + \bar{u}_j \frac{\partial \overline{u_i'^2}_{\;i}}{\partial x_j} + 2\overline{u_i'u_j'} \frac{\partial \bar{u}_i}{\partial x_j} + 2\overline{u_i'u'}_j \frac{\partial u_i'}{\partial x_j} =$$

$$\varepsilon_{ij3} 2 f\, \overline{u_i' u_j'} - 2 \overline{\frac{u'}{\bar{\rho}} \frac{\partial p'}{\partial x_i}} + 2\delta_{i3} g \overline{u_i' \frac{\theta_v'}{\bar{\theta}_v}} + 2\nu \overline{u_i' \frac{\partial^2 u_i'}{\partial x_j^2}} - \cancel{2\overline{u_i' \frac{\partial \overline{(u_i' u_j')}}{\partial x_j}}}$$

because $\overline{u_i'}$

- **This general form of the prognostic equation for the variance of the wind speed is usually simplified further before being used for BL flows.**

Dissipation

Consider a term of the form $\partial^2(\overline{u_i'^2})/\partial x_j^2$. Using simple rules of calculus, we can rewrite it as:

$$\frac{\partial^2 \overline{u_i'^2}}{\partial x_j^2} = \frac{\partial}{\partial x_j}\left(\frac{\partial \overline{u_i'^2}}{\partial x_j}\right) = \frac{\partial}{\partial x_j}\left(\overline{2u'\frac{\partial u_i'}{\partial x_j}}\right) = 2\overline{\frac{\partial u_i'}{\partial x_j}\frac{\partial u_i'}{\partial x_j}} + \overline{2u_i'\frac{\partial^2 u_i'}{\partial x_j^2}} =$$

$$2\overline{\left(\frac{\partial u_i'}{\partial x_j}\right)^2} + \overline{2u_i'\frac{\partial^2 u_i'}{\partial x_j^2}}$$

If we multiply the last term above by ν, then it would be identical to the last term in (4.3.1a). Thus, we can write the last term in (4.3.1a) as

$$2\nu\overline{u_i'\frac{\partial^2 u_i'}{\partial x_j^2}} = \nu\frac{\partial^2(\overline{u_i'^2})}{\partial x_j^2} - 2\nu\overline{\left(\frac{\partial u_i'}{\partial x_j}\right)^2}$$

Dissipation 2

$$2\nu\overline{u_i'\frac{\partial^2 u_i'}{\partial x_j^2}} = \nu\frac{\partial^2(\overline{u_i'^2})}{\partial x_j^2} - 2\nu\overline{\left(\frac{\partial u_i'}{\partial x_j}\right)^2} \qquad (4.3.1b)$$

The first term on the right, which physically represents the molecular diffusion of velocity variance, contains the curvature of a variance. The variance changes fairly smoothly with distance within the boundary layer, its curvature being on the order of 10^{-6} s^{-2} in the ML to 10^{-2} s^{-2} in the SL. When multiplied by ν, the first term ranges in magnitude between 10^{-11} and 10^{-7} $m^2 s^{-3}$.

The last term on the right can be much larger. For example, if the eddy velocity changes by only 0.1 m/s across a very small size eddy (for example, 1 cm in diameter), then the instantaneous shear across that eddy is 10 s^{-1}. For smaller size eddies, the shear is larger. When this value is squared, averaged, and multiplied by 2ν, the magnitudes observed in the turbulent boundary layer range between about 10^{-6} and 10^{-2} $m^2 s^{-3}$. Typical values in the ML are on the order of 10^{-4} to 10^{-3} $m^2 s^{-3}$, while in the surface layer, values on the order of 10^{-2} $m^2 s^{-3}$ can be found. Thus, we can neglect the first term on the right and use:

Dissipation 3

We can neglect the first term on the right and use:

$$2\nu \overline{u_i' \frac{\partial^2 u_i'}{\partial x_j^2}} = \nu \cancel{\frac{\partial^2 (\overline{u_i'^2})}{\partial x_j^2}} - 2\nu \overline{\left(\frac{\partial u_i'}{\partial x_j}\right)^2}$$

$$\boxed{2\nu \overline{u_i' \frac{\partial^2 u_i'}{\partial x_j^2}} \equiv -2\nu \overline{\left(\frac{\partial u_i'}{\partial x_j}\right)^2}} \qquad (4.3.1c)$$

The ***viscous dissipation***, ε, is defined as:

$$\varepsilon = +\nu \overline{\left(\frac{\partial u_i'}{\partial x_j}\right)^2}$$

It is obvious that this term is always positive, because it is a squared quantity. Therefore, when used in (4.3.1a) with the negative sign as required by (4.3.1c), it is always causing a decrease in the variance with time. That is, ***it is always a loss term***. In addition, it becomes larger in magnitude as the eddy size becomes smaller. For these small eddies, the eddy motions are rapidly damped by viscosity and irreversibly converted into heat. [This heating rate is so small, however, that it has been neglected in the heat conservation equation (3.4.5b).]

Not so in a hurricane BL!

Pressure perturbations

Using the product rule of calculus again, the pressure term $-2\,\overline{(u_i'/\bar{\rho})\,\partial p'/\partial x_i}$ in (4.3.1a) can be rewritten as

$$-2\overline{\left(\frac{u_i'}{\bar{\rho}}\right)\frac{\partial p'}{\partial x_i}} = -\left(\frac{2}{\bar{\rho}}\right)\frac{\partial(\overline{u_i'p'})}{\partial x_i} + 2\overline{\left(\frac{p'}{\bar{\rho}}\right)\left[\frac{\partial u_i'}{\partial x_i}\right]}$$

The last term is called the ***pressure redistribution term***. The factor in square brackets consists of the sum of three terms: $\partial u'/\partial x$, $\partial v'/\partial y$, and $\partial w'/\partial z$. These terms sum to zero because of the turbulence continuity equation (3.4.2c); hence, the last term in the equation above does not change the total variance (by total variance we mean the sum of all three variance components). But it does tend to take energy out of the components having the most energy and put it into components with less energy. Thus it makes the turbulence more isotropic, and is also known as the ***return-to-isotropy term***.

Terms like $\partial u'/\partial x$ are larger for the smaller size eddies. Thus, we would expect that smaller size eddies are more isotropic than larger ones. As we shall see later, this is indeed the case in the boundary layer.

Pressure perturbations 2, Coriolis term

The end result of this analysis is that:

$$-2\overline{\left(\frac{u_i'}{\bar{\rho}}\right)\frac{\partial p'}{\partial x_i}} \equiv -\left(\frac{2}{\bar{\rho}}\right)\frac{\partial\left(\overline{u_i'p'}\right)}{\partial x_i} \qquad (4.3.1e)$$

Coriolis Term. The Coriolis term $2f_c\varepsilon_{ij3}\ \overline{u_i'u_j'}$ is identically zero for velocity variances, as can be seen by performing the sums implied by the repeated indices:

$$2f_c\varepsilon_{ij3}\ \overline{u_i'u_j'} = 2f_c\varepsilon_{213}\ \overline{u_2'u_1'} + 2f_c\varepsilon_{123}\ \overline{u_1'u_2'}$$

$$= -2f_c\overline{u_2'u_1'} + 2f_c\overline{u_1'u_2'}$$

$$= 0 \qquad (4.3.1f)$$

because $\overline{u_1'u_2'} = \overline{u_2'u_1'}$ (see section 2.9.2). Many of the terms in the above sum were not written out because the alternating unit tensor forced them to zero.

Coriolis term 2

Physically, this means that Coriolis force can not generate turbulent kinetic energy. Kinetic energy enters the picture because the variance $\overline{u_i'^2}$ is nothing more than twice the turbulence kinetic energy per unit mass. The Coriolis term merely redistributes energy from one horizontal direction to another. Furthermore, the magnitude of the redistribution term $2f_c\ \overline{u_1'u_2'}$ is about three orders of magnitude smaller than the other terms in (4.3.1a). For that reason, the Coriolis terms are usually neglected in the turbulence variance and covariance equations, even for the cases where they are not identically zero.

Simplified velocity variance budget equations

$$\frac{\partial \overline{u_i'^2}}{\partial t} + \overline{u}_j \frac{\partial \overline{u_i'^2}}{\partial x_j} = 2\delta_{i3} g \frac{\overline{u_i'\theta_v'}}{\overline{\theta}_v} - 2\overline{u_i'u_j'}\frac{\partial \overline{u}_i}{\partial x_j} - \frac{\partial \overline{u_i'^2 u_j'}}{\partial x_j} - \frac{2}{\overline{\rho}}\frac{\partial \overline{u_i'p'}}{\partial x_i} - 2\varepsilon$$

I II III IV V VI VII

Term I represents the rate-of-change of variance
Term II is the advection of variance by the mean wind
Term III is the production or loss term, depending on the sign of the buoyancy flux
Term IV is a production term. The momentum flux is usually negative in the BL because momentum is lost to the surface; thus it results in a positive contribution to the variance when multiplied by the negative sign.

Terms V - VII

Simplified velocity variance budget equations

$$\frac{\partial \overline{u_i'^2}}{\partial t} + \overline{u}_j \frac{\partial \overline{u_i'^2}}{\partial x_j} = 2\delta_{i3} g \frac{\overline{u_i'\theta_v'}}{\overline{\theta}_v} - 2\overline{u_i'u_j'}\frac{\partial \overline{u}_i}{\partial x_j} - \frac{\partial \overline{u_i'^2 u_j'}}{\partial x_j} - \frac{2}{\overline{\rho}}\frac{\partial \overline{u_i'p'}}{\partial x_i} - 2\varepsilon$$

I II III IV V VI VII

Term V is a turbulent transport term. It describes how the variance is moved around by the turbulent eddies.
Term IV describes how variance is redistributed by pressure perturbations. It is often associated with gravity waves.
Term VII represents the viscous dissipation of velocity variance.

Prognostic equations for each component separately

- **We can examine also the prognostic equations for each individual component of the velocity variance if we relax slightly the summation requirement associated with repeated indices: e. g. put i = 2 for an equation for $\overline{v'^2}$.**
- **Any other repeated indices, such as j, continue to imply a sum. We must remember to reinsert the terms that were omitted by assuming anisotropy.**

The full set of equations is

Prognostic equations for each component separately

$$\frac{\partial \overline{u'^2}}{\partial t} + \bar{u}_j \frac{\partial \overline{u'^2}}{\partial x_j} = -2\overline{u'u'_j}\frac{\partial \bar{u}_i}{\partial x_j} - \frac{\partial \overline{u'^2 u'_j}}{\partial x_j} - \frac{2}{\bar{\rho}}\frac{\partial \overline{u'p'}}{\partial x} + \overline{\frac{2p'}{\bar{\rho}}\frac{\partial u'}{\partial x}} - 2\nu\overline{\left(\frac{\partial u'}{\partial x_j}\right)^2}$$

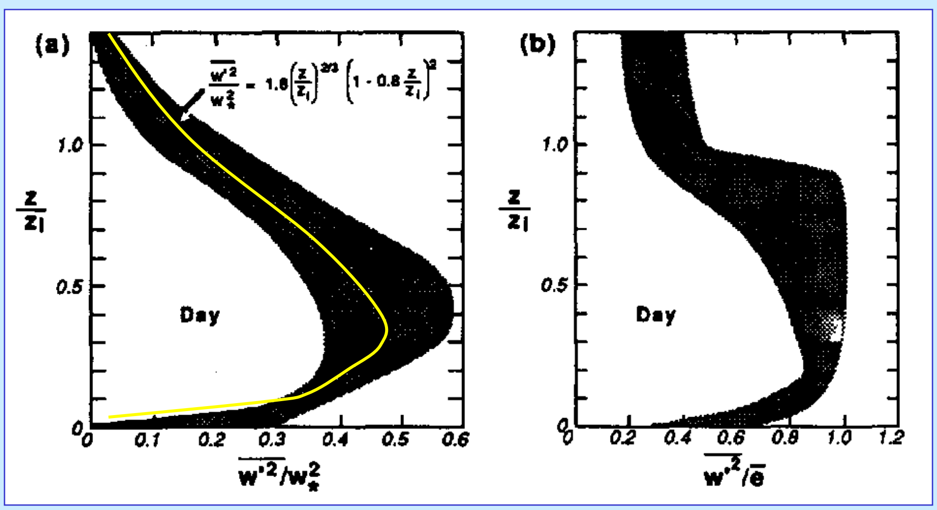
$$\frac{\partial \overline{v'^2}}{\partial t} + \bar{u}_j \frac{\partial \overline{v'^2}}{\partial x_j} = -2\overline{v'u'_j}\frac{\partial \bar{u}_i}{\partial x_j} - \frac{\partial \overline{v'^2 u'_j}}{\partial x_j} - \frac{2}{\bar{\rho}}\frac{\partial \overline{v'p'}}{\partial y} + \overline{\frac{2p'}{\bar{\rho}}\frac{\partial v'}{\partial y}} - 2\nu\overline{\left(\frac{\partial v'}{\partial x_j}\right)^2}$$

$$\frac{\partial \overline{w'^2}}{\partial t} + \bar{u}_j \frac{\partial \overline{w'^2}}{\partial x_j} = 2g\frac{\overline{w'\theta'_v}}{\bar{\theta}_v} - 2\overline{w'u'_j}\frac{\partial \bar{u}_i}{\partial x_j} - \frac{\partial \overline{w'^2 u'_j}}{\partial x_j} - \frac{2}{\bar{\rho}}\frac{\partial \overline{w'p'}}{\partial z}$$
$$+ \underline{\overline{\frac{2p'}{\bar{\rho}}\frac{\partial w'}{\partial z}}} - 2\nu\overline{\left(\frac{\partial w'}{\partial x_j}\right)^2}$$

Most terms have the same meaning as before. represents pressure redistribution, associated with the return to isotropy.

Budget studies

- **Budget study** is the name given to an evaluation of the contributions of each term in prognostic equations.
- Some terms are very difficult to measure in field experiments, which is why computer simulations are carried out.
- In the budget studies to be described, field data and numerical simulations are combined.
- In most cases, field data have significantly more scatter than the simulations.



(a) Variation of vertical velocity variance with height, z during daytime. Range of measured and modelled values are shaded. (b) Range of the ratio of the vertical velocity variance to the eddy kinetic energy.

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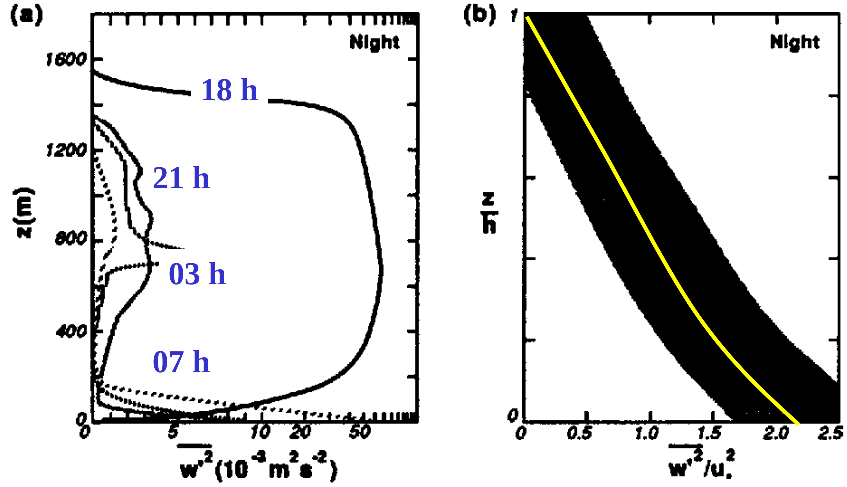

(a) Modelled profiles of vertical velocity variance during Night 33-34 of Wangara. Abscissa changes from the linear to logarithmic at 10. (b) Range of vertical velocity variance, normalized by a measure of stable boundary layer depth, h.

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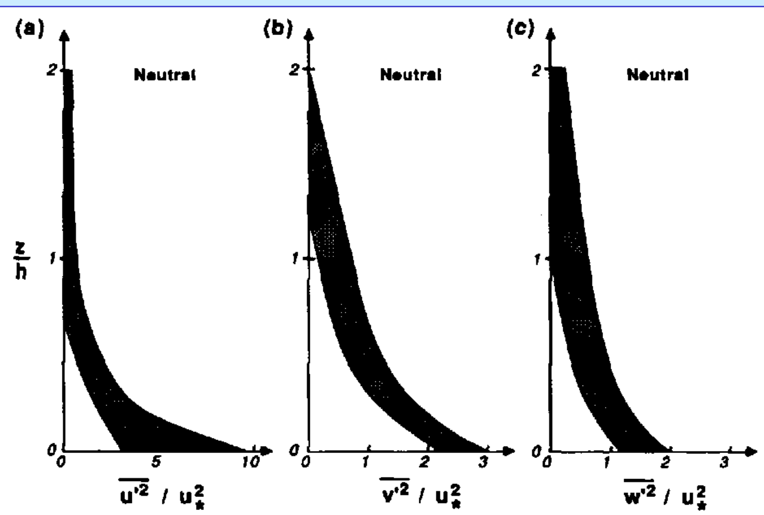

Normalized velocity variance verses height in statically neutral conditions, where h (≈ 2 km) is the height where $\bar{v}$ is zero. Based on a large-eddy simulation by Mason and Thomson (1987) using u_g = 10 m s^{-1}, v_g = 0, and u_* = 0.4 m s^{-1}.

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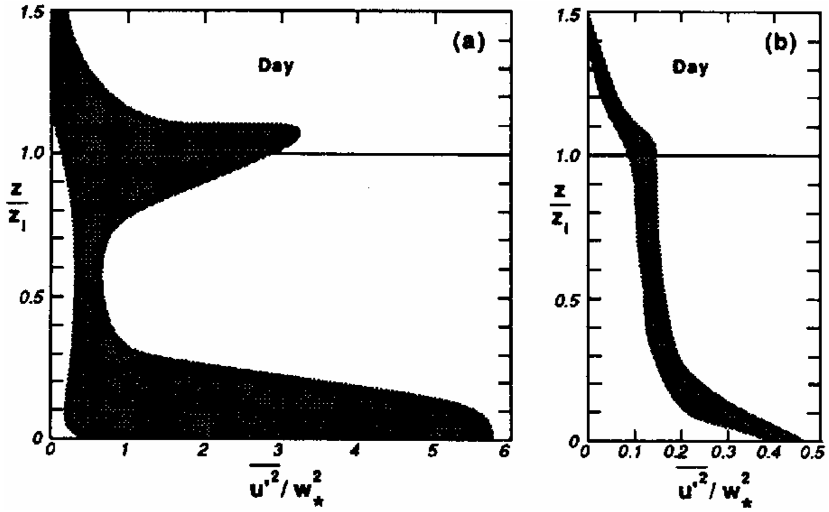

(a) Range of horizontal velocity variance, normalized by the convective velocity scale w_*^2, versus dimensionless height z/z_i, for typical conditions with combined convection and wind shear. (b) Idealized range for free convection with no mean shear.

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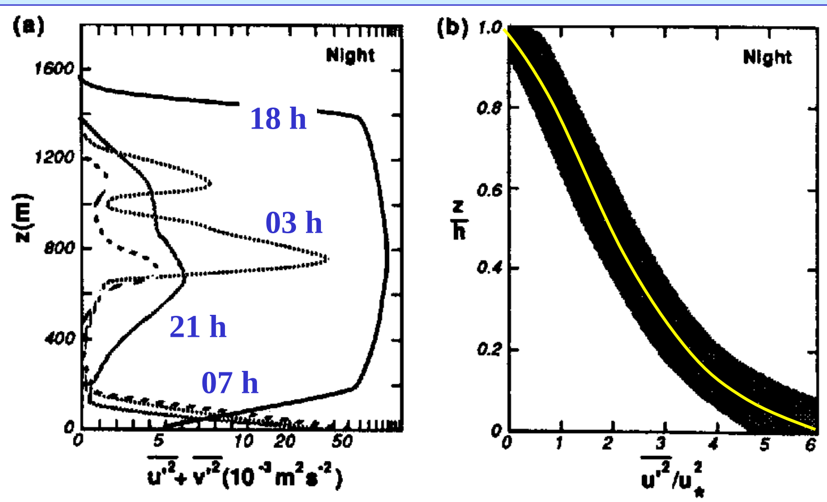

(a) Modelled profiles of horizontal eddy kinetic energy during Night 33-34 of Wangara. Abscissa changes from the linear to logarithmic at 10. (b) Range of vertical velocity variance, normalized by a measure of stable boundary layer depth, h.

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Moisture variance

Consider only the vapour part of the specific humidity:

$$2q' \times \frac{\partial q'}{\partial t} + \ldots = \ldots$$

$$\frac{\partial q'^2}{\partial t} + \bar{u}_j \frac{\partial q'^2}{\partial x_j} + 2q'u'_j \frac{\partial \bar{q}}{\partial x_j} + u'_j \frac{\partial q'^2}{\partial x_j} = 2q'\nu_q \frac{\partial^2 q'}{\partial x_j^2} + 2q' \frac{\partial \overline{u'_j q'}}{\partial x_j}$$

Next, average and apply the Reynolds averaging rules:

$$\frac{\partial \overline{q'^2}}{\partial t} + \bar{u}_j \frac{\partial \overline{q'^2}}{\partial x_j} + 2\overline{q'u'_j} \frac{\partial \bar{q}}{\partial x_j} + \overline{u'_j \frac{\partial q'^2}{\partial x_j}} = \overline{2q'\nu_q \frac{\partial^2 q'}{\partial x_j^2}}$$

To change this into flux form, add the averaged turbulent continuity equation multiplied by q'^2 **(i.e. add** $\overline{q'^2 \partial u'_j / \partial x_j} = 0$ **) and rearrange slightly.**

$$\frac{\partial \overline{q'^2}}{\partial t} + \bar{u}_j \frac{\partial \overline{q'^2}}{\partial x_j} = -2\overline{q'u'_j} \frac{\partial \bar{q}}{\partial x_j} - \frac{\partial \overline{u'_j q'^2}}{\partial x_j} + 2\nu_q \overline{q' \frac{\partial^2 q'}{\partial x_j^2}}$$

As was done for momentum, the last term is split into two parts, one of which (the molecular diffusion of specific humidity variance) is small enough to be neglected. The remaining part is defined as twice the molecular diffusion term, ε_p, by analogy with momentum:

$$\varepsilon_q = \nu_q \overline{\left(\frac{\partial q'}{\partial x_j}\right)^2}$$

The prognostic equation for specific humidity variance is

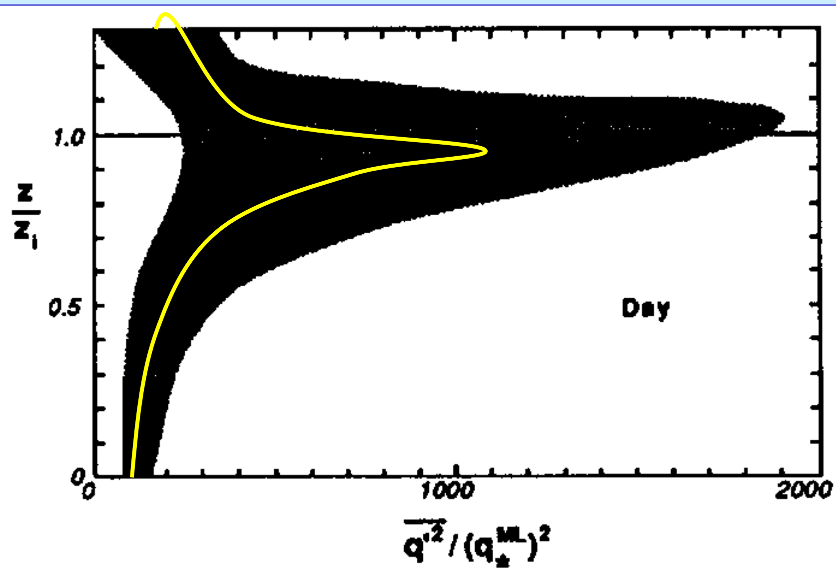
$$\frac{\partial \overline{q'^2}}{\partial t} + \bar{u}_j \frac{\partial \overline{q'^2}}{\partial x_j} = -2\overline{q'u'_j} \frac{\partial \bar{q}}{\partial x_j} - \frac{\partial \overline{u'_j q'^2}}{\partial x_j} - 2\varepsilon_q$$

Interpretation

$$\frac{\partial \overline{q'^2}}{\partial t} + \bar{u}_j \frac{\partial \overline{q'^2}}{\partial x_j} = -2\overline{q'u'_j}\frac{\partial \bar{q}}{\partial x_j} - \frac{\partial \overline{u'_j q'^2}}{\partial x_j} - 2\varepsilon_q$$

I II IV V VII

Term I **represents the rate-of-change of humidity variance**
Term II **is the advection of humidity variance by the mean wind**
Term IV **is the production term, associated with turbulent motions occurring within a mean moisture gradient**
Term V **represents the turbulent transport of humidity variance.**



Modelled vertical profiles of dimensionless specific humidity variance for Wangara Day 33.

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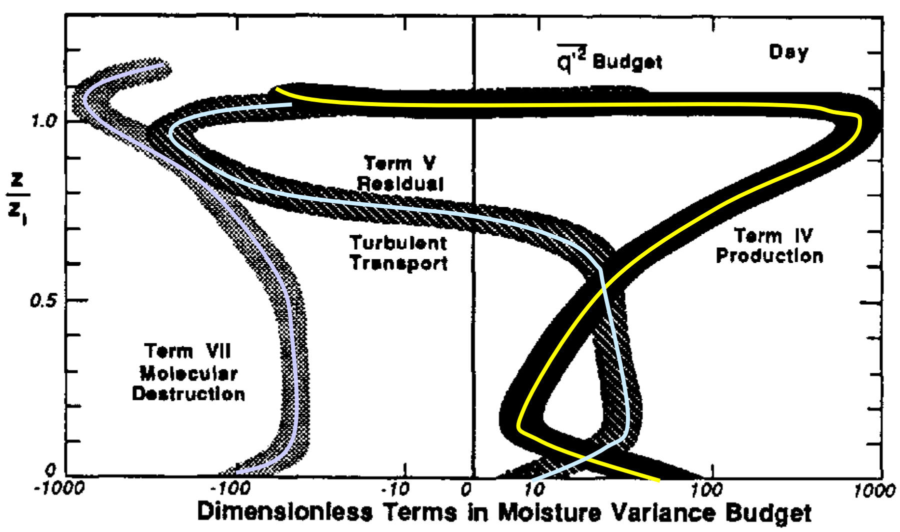

Modelled vertical profiles of terms in the specific humidity variance equation for Wangara Day 33 at hour 14.1.

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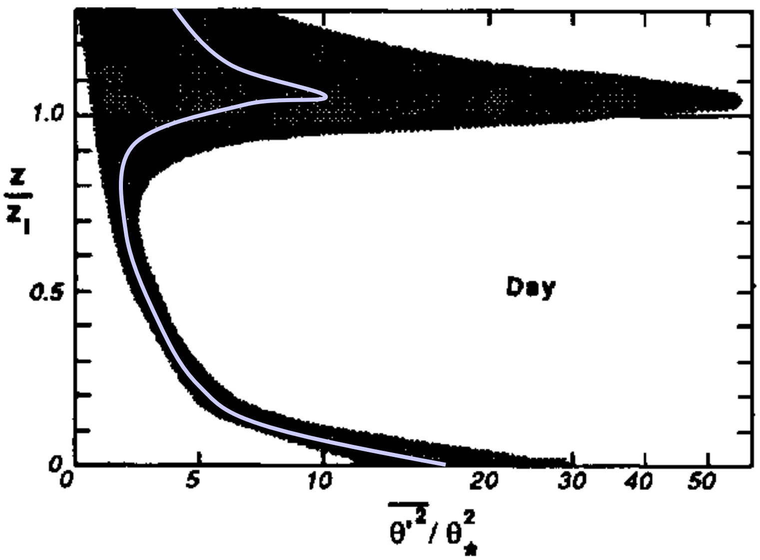
Heat (potential temperature) variance

Budget Equations. As was done with the moisture equation, start with (4.1.3), multiply by $2\theta'$, use the product rule of calculus, Reynolds average, put into flux form, neglect molecular diffusion but retain the molecular dissipation, and rearrange to yield:

$$\frac{\partial\overline{\theta'^2}}{\partial t}+\overline{u}_j\frac{\partial\overline{\theta'^2}}{\partial x_j}=-2\overline{\theta'u'_j}\frac{\partial\overline{q}}{\partial x_j}-\frac{\partial\overline{u'_j\theta'^2}}{\partial x_j}-2\varepsilon_\theta-\frac{2}{\overline{\rho}c_p}\overline{\theta'\frac{\partial Q'_j}{\partial x_j}}$$

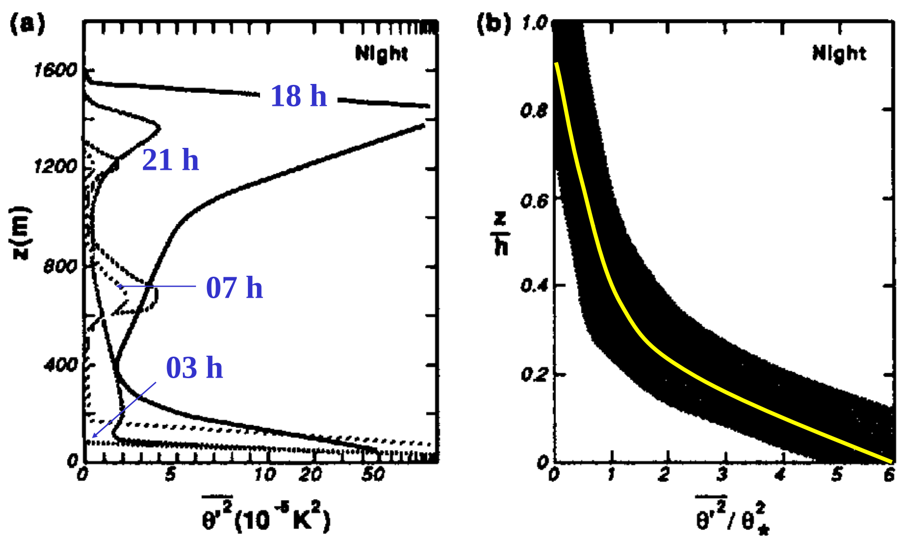
I **II** **IV** **V** **VII** **VIII**

The terms above have physical representations analogous to those in (4.3.2). Term VIII is the radiation destruction term (sometimes given the symbol ε_R). It is difficult to measure this term directly, but sometimes it is modeled as $\varepsilon_R \equiv (0.036\text{ m/s})\cdot\varepsilon\,\overline{\theta'^2}\,/\,\bar{e}^{\,3/2}$, where ε_R is about 1% to 10% of ε_θ (Coantic and Simonin, 1984).



Modelled vertical profiles of dimensionless virtual potential temperature variance for Wangara Day 33.

N



Modelled profiles of virtual potential temperature variance during the night 33-34 of Wangara.

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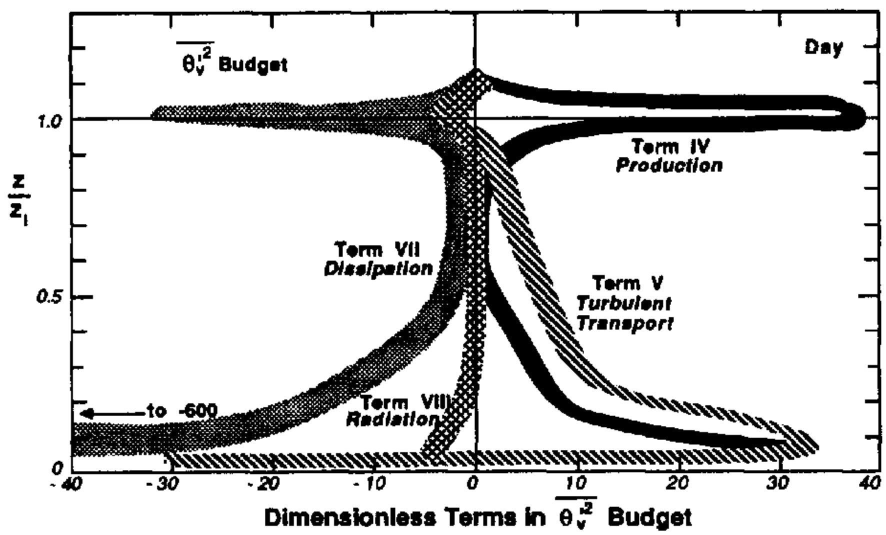

Modelled vertical profiles in the virtual potential temperature variance budget equation.

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Summary

- Prognostic equations for turbulent quantities
- Free convection scaling variables
- Prognostic equations for variances
- Dissipation
- Pressure perturbations
- Coriolis term
- Simplified velocity variance budget equations
- Prognostic equations for each component
- Budget studies
- Moisture and heat variance


End