CORRESPONDENCE

Comments on "How Much Does the Upward Advection of the Supergradient Component of Boundary Layer Wind Contribute to Tropical Cyclone Intensification and Maximum Intensity?"

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KEYWORD: Hurricanes

In a recent paper, Li et al. (2020) carried out an extensive ensemble of axisymmetric numerical simulations to examine "the importance of supergradient winds in TC [tropical cyclone, our insertion] intensification," claiming that this topic "is still under debate." In their introduction they state, "One view is that the spinup of the eyewall occurs by the upward advection of high tangential momentum associated with supergradient winds from the boundary layer. The other view argues that the upward advection of supergradient winds by eyewall updrafts results in an outward agradient force, leading to the formation of a shallow outflow layer immediately above the inflow boundary layer." As shown below, these are not "separate views," but rather part of the same picture that does not depend on the degree to which the ascending air is supergradient.

If the air that exits the boundary layer is supergradient, it must surely move outward. What other force would make the air move inward against the positive agradient force (which includes, of course, the radial pressure gradient force)? Li et al. (2020) recognize that the tangential wind component of air parcels moving radially outward while approximately conserving their absolute angular momentum will slow down. However, they appear to suggest that these air parcels adjust back to gradient balance *before* they ascend in the eyewall. On this basis and supported by their interpretations of their ensemble experiments, they argue that the upward advection of high tangential momentum associated with supergradient winds from the boundary layer "should *not* [our emphasis] be a dominant mechanism of TC intensification" as stated in their conclusions.

Since some readers of their paper may be puzzled by this remark, it would seem worth reiterating the argument of Schmidt and Smith (2016) and Montgomery and Smith (2017), which Li et al. (2020) seem to regard as debatable. In a cylindrical coordinate system (r, λ, z), with r the radius, λ the azimuth, and z the height, the tendency equation for the tangential velocity component v in an axisymmetric vortex may be written as

$$\frac{\partial v}{\partial t} = -(\zeta + f)u - w\frac{\partial v}{\partial z} + F_{\lambda}, \qquad (1)$$

where u and w are the radial and vertical velocity components, respectively, t is the time, f is the Coriolis parameter, ζ is the vertical component of relative vorticity, and F_{λ} represents the frictional and/or sub-grid-scale diffusion of tangential momentum. This equation is simply the azimuthal component of Newton's second law.[1] Assuming that, above the frictional boundary layer, F_{λ} can be neglected, the only way that v can increase locally in a cyclonic vortex $(\zeta + f > 0)$ when the radial flow is outward $(u > 0)$ is if the vertical advection of tangential momentum, $-w\partial v/\partial z$, is positive and exceeds the radial flux of absolute vorticity, $(\zeta + f)u$, in magnitude. This result seems so basic that it is hard to imagine why Li et al. (2020) consider it to be "still under debate." It is hard to imagine also why an ensemble of numerical experiments is required to investigate it further. If one is really interested to quantify the amount of cancellation between the two terms on the right-hand side of the equation for $\partial v/\partial t$ in footnote 1, one can do this with a single calculation. One could even calculate the contribution of the agradient wind to the vertical advection term rather easily.

If Li et al. (2020) are arguing that the vertical advection of tangential momentum *is not* a dominant mechanism for spinning up the eyewall in which the radial flow is outward, what is "the dominant mechanism" in their view? Surely, they cannot be invoking friction, the only other term in Eq. (1), that is neglected in the above argument. In a three-dimensional flow, of course, there would be additional eddy momentum contributions to the azimuthally averaged tangential momentum equation (e.g., Persing et al. 2013; Smith et al. 2017; Montgomery et al. 2020; Wang et al. 2020), but these are not present in Li et al.'s (2020) axisymmetric framework.

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[1] The inviscid form of the equation can be written alternatively as one for the conservation of absolute angular momentum, where $M = rv + (1/2)fr^2$; i.e.,

$$\frac{\partial v}{\partial t} = -\frac{u}{r}\frac{\partial M}{\partial r} - \frac{w}{r}\frac{\partial M}{\partial z}.$$

Writing the right-hand side in vector form shows that for v to increase locally, there must be a component of flow across the M surfaces toward low M.

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In the ensemble experiments that are carried out to support the idea that it is not the upward advection of tangential momentum (including the supergradient component) by the eyewall updraft that is the main mechanism for spinning up the eyewall, Li et al. (2020) suppress the upward advection of the supergradient part of the tangential momentum ascending out of the boundary layer and show that the storm and eyewall still spin up.[2] However, they do not appear to have noticed that by suppressing the upward advection of the supergradient component of the tangential momentum as air ascends out of the boundary layer, they are, in effect, introducing a ring of negative impulsive torque to the tangential momentum equation. This torque would appear as an extra term on the right-hand side of Eq. (1), but would be of the wrong sign to be considered as the mysterious "main mechanism" for the spinup of the eyewall in axisymmetric hurricanes. While Li et al.'s (2020) description of their calculations is consistent with what one might anticipate such a torque would do, it is difficult to see what one can learn about the real world by such a thought experiment, since air ascending in real storms does not experience such a ring of negative torque as it exits the boundary layer.

As a final remark, we draw attention to the results of several studies showing that the tangential wind in the eyewall is supergradient through the depth of troposphere (Zhang et al. 2001, their Fig. 7b; Montgomery et al. 2020, their Figs. 4a,b; Wang et al. 2020, their Fig. 5b). All these studies showed that the agradient force is positive throughout most of the eyewall and the assumption that the supergradient winds adjust rapidly back to gradient wind balance just as the air exits the top of the boundary layer during storm spinup and maturity is not correct.

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REFERENCES

Li, Y., Y. Wang, and Y. Lin, 2020: How much does the upward advection of the supergradient component of boundary layer wind contribute to tropical cyclone intensification and maximum intensity? *J. Atmos. Sci.*, **77**, 2649–2664, https://doi.org/10.1175/JAS-D-19-0350.1.

Montgomery, M. T., and R. K. Smith, 2017: Recent developments in the fluid dynamics of tropical cyclones. *Annu. Rev. Fluid Mech.*, **49**, 541–574, https://doi.org/10.1146/annurev-fluid-010816-060022.

——, G. Kilroy, R. K. Smith, and N. Črnivec, 2020: Contribution of mean and eddy momentum processes to tropical cyclone intensification. *Quart. J. Roy. Meteor. Soc.*, **146**, https://doi.org/10.1002/qj.3837, in press.

Persing, J., M. T. Montgomery, J. C. McWilliams, and R. K. Smith, 2013: Asymmetric and axisymmetric dynamics of tropical cyclones. *Atmos. Chem. Phys.*, **13**, 12 299–12 341, https://doi.org/10.5194/acp-13-12299-2013.

Schmidt, C. J., and R. K. Smith, 2016: Tropical cyclone evolution in a minimal axisymmetric model revisited. *Quart. J. Roy. Meteor. Soc.*, **142**, 1505–1516, https://doi.org/10.1002/qj.2753.

Smith, R. K., J. A. Zhang, and M. T. Montgomery, 2017: The dynamics of intensification in a Hurricane Weather Research and Forecasting simulation of Hurricane Earl (2010). *Quart. J. Roy. Meteor. Soc.*, **143**, 293–308, https://doi.org/10.1002/qj.2922.

Wang, S., R. K. Smith, and M. T. Montgomery, 2020: Upper-tropospheric inflow layers in tropical cyclones. *Quart. J. Roy. Meteor. Soc.*, **146**, https://doi.org/10.1002/qj.3856, in press.

Zhang, D.-L., Y. Liu, and M. K. Yau, 2001: A multiscale numerical study of Hurricane Andrew (1992). Part IV: Unbalanced flows. *Mon. Wea. Rev.*, **129**, 92–107, https://doi.org/10.1175/1520-0493(2001)129<0092:AMNSOH>2.0.CO;2.

[2] Li et al.'s (2020) calculations appear to have been motivated by a misinterpretation of the argument of Schmidt and Smith (2016) and Montgomery and Smith (2017), who did not argue that it was the vertical advection of the supergradient part of the tangential momentum alone that spins up the eyewall.