

On the symmetric circulation of a moving hurricane

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SUMMARY

The evolution of the symmetric circulation of a moving hurricane-scale vortex on a beta-plane is investigated and interpretations of this investigation are given in terms of vorticity fluxes. The study, which is fundamental to understanding vortex motion, is based largely on numerical integrations of the barotropic vorticity equation, using a finite-difference method. In the absence of any large-scale flow, the north-westward drift of an initially symmetric, cyclonic vortex in the northern hemisphere is accompanied by a decrease in the tangential circulation at most radii, and a consequent deceleration of the azimuthally averaged tangential velocity. This behaviour is not simply a consequence of the increase in the Coriolis parameter over the domain of the cyclone as is often supposed, but may be explained as the sum of three effects: the outwards radial flux of relative vorticity associated with the asymmetric component of flow, the corresponding flux of planetary vorticity, and the rate of change in planetary vorticity due to the meridional displacement of the vortex. In particular, the net rate of change in absolute vorticity, the sum of the last two effects, makes the largest contribution to the circulation changes. The flux of absolute vorticity is associated, *inter alia*, with the generation of Rossby waves by the vortex. The behaviour is different from that in recent calculations by Carr and Williams in which the flux of planetary vorticity was omitted.

It is pointed out that in the analytic theory for vortex motion developed by the first author and co-workers, an initially symmetric vortex with zero net relative circulation at large radial distances subsequently develops a finite negative relative circulation whose strength increases linearly with time; this is because the net flux of planetary vorticity at large radial distances is finite. In contrast, in the numerical model the flux is close to zero so that the symmetric circulation decays more rapidly with radius than the $1/(\text{radius})$ decay rate in the analytic model. At inner radii there is good qualitative agreement between the predictions of the two methods, even though there are quantitative differences between them. It is noted that the occurrence of the finite circulation in the analytic theory violates a theorem of Flierl *et al.* and that it represents a limitation of the long-term validity of the theory.

KEYWORDS: Barotropic Hurricane Modelling Vorticity

1. INTRODUCTION

In recent years there have been a number of studies of the motion of a barotropic vortex on a beta-plane because of the perceived relevance of this problem to tropical-cyclone motion (see, for example, Smith (1993) and references). The basic thought experiment is concerned with the subsequent motion of an initially symmetric vortex in the absence of any large-scale environmental flow. Most studies have assumed the flow to be nondivergent, notable exceptions being those of Shapiro and Ooyama (1990) and Evans *et al.* (1991). However, Shapiro and Ooyama *op. cit.* showed that for realistic tropical-cyclone-scale vortices, divergence effects are negligible in the shallow-water model. In broad terms, the subsequent behaviour of the flow is well understood. Consider the isolines of absolute vorticity in the initial state as depicted in Fig. 1. These are approximately circular in the inner-core region and lie approximately parallel to latitude circles at large distances from the vortex centre. Since absolute vorticity is materially conserved in the nondivergent model, the pattern of isolines is distorted by the vortex circulation, leading to an approximately east–west oriented dipole anomaly in the relative vorticity at early times. Subsequently, the asymmetry rotates cyclonically and increases in strength and scale (Smith *et al.* 1990, henceforth referred to as SUD; Shapiro and Ooyama 1990). The asymmetric flow associated with this vorticity asymmetry can be regarded as a secondary flow which advects the symmetric vortex, causing it to track towards the north-west (south-west) in the northern (southern) hemisphere.

Numerical calculations have shown that as the vortex moves polewards, the symmetric circulation becomes progressively more anticyclonic, especially at outer radii (e.g. Fiorino and Elsberry 1989, Fig. 6) and this has been supposed to be ‘... a consequence of conservation of absolute vorticity’ by Fiorino and Elsberry *op. cit.* or a consequence of ‘... conservation of angular momentum in the poleward moving vortex’ by Evans *et al.* (1991). The former argument is presumably that the general increase in planetary vorticity encompassed by the vortex circulation is accompanied by a commensurate decrease in relative vorticity. The Evans *et al.* argument begs a number of questions because it refers to a conversion of vortex angular momentum about the vortex axis into earth angular momentum which is about another axis. Such

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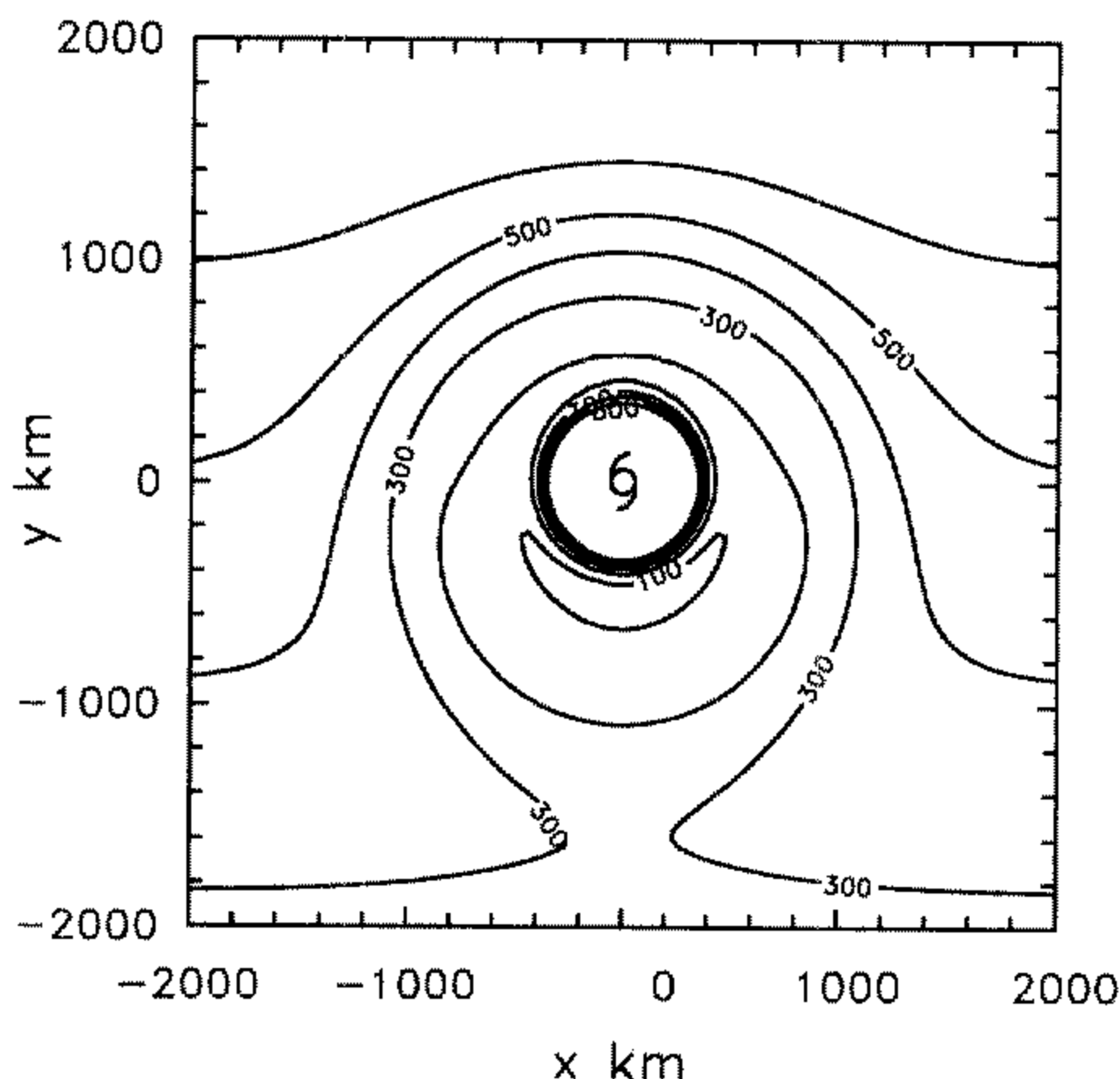


Figure 1. Absolute-vorticity contours for the broad vortex used by Smith *et al.* (1990) when initialized at the origin of a beta-plane centred at latitude 20°N in the absence of any large-scale flow. Contour interval is $1.0 \times 10^{-5}\text{s}^{-1}$.

conversions involve torques, and angular momentum is *not* conserved. An alternative interpretation is given here in terms of vorticity fluxes.

Because of the relative importance of the outer circulation of a vortex on vortex motion (Fiorino and Elsberry 1989), the change in the symmetric circulation will feed back on the vorticity asymmetry and, therefore, it may have a significant effect on the long-term motion of the vortex. For this reason an understanding of the factors that determine the evolution of the symmetric circulation of a moving vortex are fundamental to understanding vortex motion itself. Some information on this question is provided by the analytic investigations of vortex stability to perturbations from axisymmetry by Carr and Williams (1989), by the numerical study of Shapiro and Ooyama (1990), and by the analytic theory of vortex motion developed in a series of papers by Smith and Ulrich (1990), Smith (1991, henceforth referred to as S91), Smith and Weber (1993) and Kraus *et al.* (1994, personal communication). We review the salient results of these studies and show that they raise other issues which we address in this note.

Carr and Williams (1989) calculated the change in symmetric wind speed with time as a function of radius and attributed this to '... a convergence of momentum flux associated with the β -induced asymmetry'. They used a Rankine vortex (tangential velocity inversely proportional to radius) and considered only an annular region between two finite radii at which the radial velocity was assumed to vanish. Accordingly, their basic vortex has zero relative vorticity everywhere in the annular domain, but has finite circulation at the inner radius. They showed that the β -induced asymmetries lead to an acceleration of the symmetric flow at inner radii, and to a deceleration thereof at outer radii. Moreover, the radius at which the changeover occurred is an increasing function of time (see their Fig. 6).

Shapiro and Ooyama (1990) investigated the evolution of the total relative angular momentum (RAM) in circular regions of different sizes (400 km, 1000 km and 3000 km) centred on the moving vortex. They showed that, in the inner two circles, the total RAM decreased steadily with time to about 96 hours after which it increased slightly in the 1000 km circle out to 120 hours (see their Fig. 7). However, in the 3000 km circle, the total RAM decreased sharply, became negative after about 32 hours, and reached a minimum at about 70 hours. Thereafter it increased sharply, recovering about half of its initial (positive) value by 120 hours. These large changes were shown to be associated with major changes in the integrated Coriolis torque as the flow evolved and, in turn, these were attributed to the development of successive anticyclonic and cyclonic gyres in the Rossby-wave wake induced by the vortex. However, Shapiro and Ooyama did not investigate the evolution of the symmetric vortex circulation as a function of radius.

A similar type of calculation to that of Carr and Williams *op. cit.* was carried out by Smith (1991, see Fig. 4) in the context of an approximate analytic theory for the motion of a barotropic vortex on a beta-plane. In this theory, the initial vortex is symmetric and spans the whole domain from $r = 0$ to infinity. Moreover, beyond the radius where it is a maximum, the tangential wind speed decays rapidly with radius r , specifically faster than $1/r$, whereupon the net relative circulation tends to zero as $r \rightarrow \infty$. Thus, according to Stokes's theorem, there is exactly as much positive vorticity as negative vorticity in an areally integrated sense. In contrast to Carr and Williams's calculation, the lowest-order correction to the symmetric circulation is everywhere anticyclonic, in line with the arguments given above. However, somewhat unexpectedly, the associated tangential velocity decays only like $1/r$ for large r , much more slowly indeed than the primary circulation. Therefore, as we shall show, there must be a finite and negative vorticity flux associated with the asymmetric perturbation. A further implication is that, beyond a certain radius, the correction to the symmetric flow exceeds the primary vortex flow, thereby invalidating the perturbation method of solution at large radii. This does not appear to be a severe limitation of the theory, at least for a day or two, since, as shown by the foregoing authors, for this time period excellent agreement is obtained between many predictions of the theory and corresponding results obtained from a full numerical solution of the problem. Nevertheless, it is an aspect of the theory that merits further exploration, even though the theory does not include the feedback effects associated with the changing circulation. This is particularly the case because a symmetric vortex whose tangential velocity decays like $1/r$ in an infinite domain has infinite kinetic energy and infinite relative angular momentum and even on a finite domain may not provide a very good approximation to reality. The foregoing considerations suggest that it would be of interest to know how the symmetric circulation behaves at large radii in the equivalent numerical calculation for a moving vortex because, notwithstanding the possible influence of boundaries, this might be regarded as the control calculation.

The primary purpose of the present note is to investigate the evolution of the symmetric circulation in the numerical model for a moving vortex by SUD and to interpret this in terms of the vorticity budget. A subsidiary aim is to compare with this the behaviour of the circulation that occurs in the analytic model of S91. In particular, we seek to identify the reason for the evolution of the finite relative circulation in the analytic calculation.

2. VORTICITY BUDGET FOR A MOVING VORTEX

When expressed in a coordinate system moving with the centre of a vortex with velocity \mathbf{c} , the barotropic vorticity equation can be written in flux-form (see the appendix for its derivation) as

$$\partial(\zeta + f)/\partial T + \nabla \cdot [\mathbf{U}(\zeta + f)] = 0 \quad (1)$$

where \mathbf{U} is the wind vector in this frame, $\zeta = \mathbf{k} \cdot \nabla \wedge \mathbf{U}$ is the vertical component of relative vorticity, f is the Coriolis parameter, \mathbf{k} is the unit vector in the vertical and T is the time. Note that in the moving frame, f is a function of time as well as of the meridional coordinate Y , i.e. $f = f_0 + \beta(Y + \int_0^T c_2 dT)$ where f_0 and β are constants and $c_2 = \mathbf{c} \cdot \mathbf{j}$, \mathbf{j} being the unit vector in the meridional direction.

Consider any closed circuit C enclosing a surface S , which is fixed in the moving frame. Then

$$\frac{\partial}{\partial T} \int_S \zeta dS = - \int_S \nabla \cdot [\mathbf{U}(\zeta + f)] dS - \int_S \frac{\partial f}{\partial T} dS. \quad (2)$$

Using Stokes's theorem and the divergence theorem and noting that $\partial f/\partial T = \beta c_2$, where c_2 is spatially uniform, this equation may be rewritten as

$$\frac{\partial}{\partial T} \oint_C (\mathbf{U} \cdot \hat{\mathbf{t}}) dl = - \oint_C (\zeta + f)(\mathbf{U} \cdot \hat{\mathbf{n}}) dl - \beta c_2 A \quad (3)$$

where l measures distance anticyclonically around C in the tangential direction $\hat{\mathbf{t}}$, $\hat{\mathbf{n}}$ is the outward normal direction on C , and A is the area of S . It follows that the rate of change of the relative circulation about C , or equivalently the rate of change of total relative vorticity within C , is equal to the advective flux of absolute vorticity into the circuit (first term on the right-hand side of Eq. (3)) minus the rate of change of planetary vorticity due to the meridional displacement of the circuit on the beta-plane (last term on the right-hand side of Eq. (3)). Now let (r, θ) be polar coordinates with origin at the vortex centre, and let C be a circle of radius r centred at this point. We decompose any dependent variable into its azimuthal

average $\bar{\phi} = (1/2\pi) \int_0^{2\pi} \phi \, d\theta$ and a perturbation ϕ' . Then $dl = r \, d\theta$, $\hat{\mathbf{t}}$ and $\hat{\mathbf{n}}$ become the unit vectors $\hat{\boldsymbol{\theta}}$ and $\hat{\mathbf{r}}$ in polar coordinates, and $A = \pi r^2$. Writing $\mathbf{U} = u\hat{\mathbf{r}} + v\hat{\boldsymbol{\theta}}$, the azimuthal mean of Eq. (3) reduces to

$$\partial \bar{v} / \partial T = -\overline{u' \zeta'} - \beta r (\overline{u' \sin \theta} + \frac{1}{2} c_2). \quad (4)$$

If the Cartesian coordinates of \mathbf{c} in a fixed frame are (c_1, c_2) , then at large radial distances ($r \rightarrow \infty$) the radial flow associated with the vortex motion must be such that $u' \rightarrow -\mathbf{c} \cdot \hat{\mathbf{r}}$, and the last term in brackets in Eq. (4) should tend to zero.

3. EVOLUTION OF THE SYMMETRIC CIRCULATION

We now investigate the evolution of the symmetric circulation and, in particular, the relative vorticity for a moving vortex in the numerical calculation of SUD. We examine also the extent to which this evolution is captured by the analytic theory. Because of the approximations made in the analytic theory, one might expect the numerical calculation to be the most exact, provided that the grid resolution is adequate and the computational domain is large enough. Inevitably, the presence of boundaries at a finite distance in the numerical model will influence the calculated far-field behaviour of the circulation. To minimize this influence we used a domain size of 5000 km \times 5000 km, two and a half times that used by SUD. The grid size was 10 km, one half of that used by SUD.

Figure 2 shows, *inter alia*, the radial profiles of azimuthally averaged perturbation relative vorticity obtained from the numerical integration and from the analytic theory at 24 h intervals to 72 hours. Perturbation quantities are defined relative to the initial symmetric profiles. We focus attention first on the numerically calculated profiles. These show a relatively large negative perturbation in an inner core region which expands radially with time as the mean perturbation amplitude increases. While the magnitude fluctuates rapidly with radius in this region, the mean value is close to the value $-\beta y_c$, where y_c is the poleward displacement of the vortex (this value is indicated in panels (a) to (c) in Fig. 2 by a dashed line). Shapiro and Ooyama (1990, p. 181) show that this behaviour is associated with the homogenization of the absolute vorticity in the inner core region and argue that it is '... a simple consequence of the conservation of angular momentum'. However, it is clear from the structure of the vorticity perturbation outside this inner core that such an explanation is not valid at larger radii.

The behaviour in the inner core is captured by the analytic calculation also, but the absolute value of the minimum is less than in the numerical calculation. This is because the magnitude of y_c , which determines the magnitude of $-\beta y_c$ at the rotation axis, is based upon the zero-order solution in the theory and is not accurate (see Smith and Ulrich (1990, Fig. 5)). As in the numerical calculation, the inner core region expands with time, and its size is well predicted also. Note that the analytic curves are smoother in the inner region, suggesting that the rapid oscillations in the numerical calculation are due to inadequate resolution in the core region.

In both the numerical and analytic calculations, the local maximum neighbouring the inner core ultimately assumes positive values, and its radial position drifts to a radius of about 500 km in both cases. However, a significant difference between the two profiles is apparent at outer radii, beyond about 1000 km, where the numerical profile is positive but the analytic perturbation tends to zero from below. Although the positive values of vorticity are small compared with those at inner radii, they extend over a much larger area and allow for the possibility that when integrated over a large radius, they give a net relative circulation that is zero. This is in contrast to the situation in the analytic calculation, where it is apparent that the relative circulation is negative. Note that to 48 hours and at radii less than 2500 km, the symmetric velocity perturbation in both models is anticyclonic, in contrast to that in the calculation by Carr and Williams (1989, see their Fig. 6). The reason for this difference appears to be that Carr and Williams calculate only the effect of the relative-vorticity flux on the evolving tangential circulation, i.e. the first term on the right-hand side of Eq. (4)—cf. their Eq. (35).

One may interpret the changes in the mean tangential flow associated with the vortex motion in terms of the kinetic-energy budget (Carr and Williams 1989, Section 4(a)), the relative angular momentum budget (Shapiro and Ooyama 1990), or in terms of the vorticity budget, the method chosen here.

Figure 3 shows the radial profiles of the relative-vorticity flux ($\text{RVF} = 2\pi r u' \zeta'$) and absolute-vorticity flux ($\text{AVF} = 2\pi \beta r^2 (\overline{u' \sin \theta} + \frac{1}{2} c_2)$) in the numerical model at 24 and 48 hours. These contribute to the circulation tendency at radius r since multiplication of Eq. (4) by $2\pi r$ gives

$$\partial(2\pi r \bar{v}) / \partial T = -\text{RVF} - \text{AVF}. \quad (5)$$

Positive values of RVF and AVF correspond with outward vorticity fluxes through the circuit of radius r and thereby lead to a reduction of relative vorticity inside this circuit. By Stokes's theorem this is

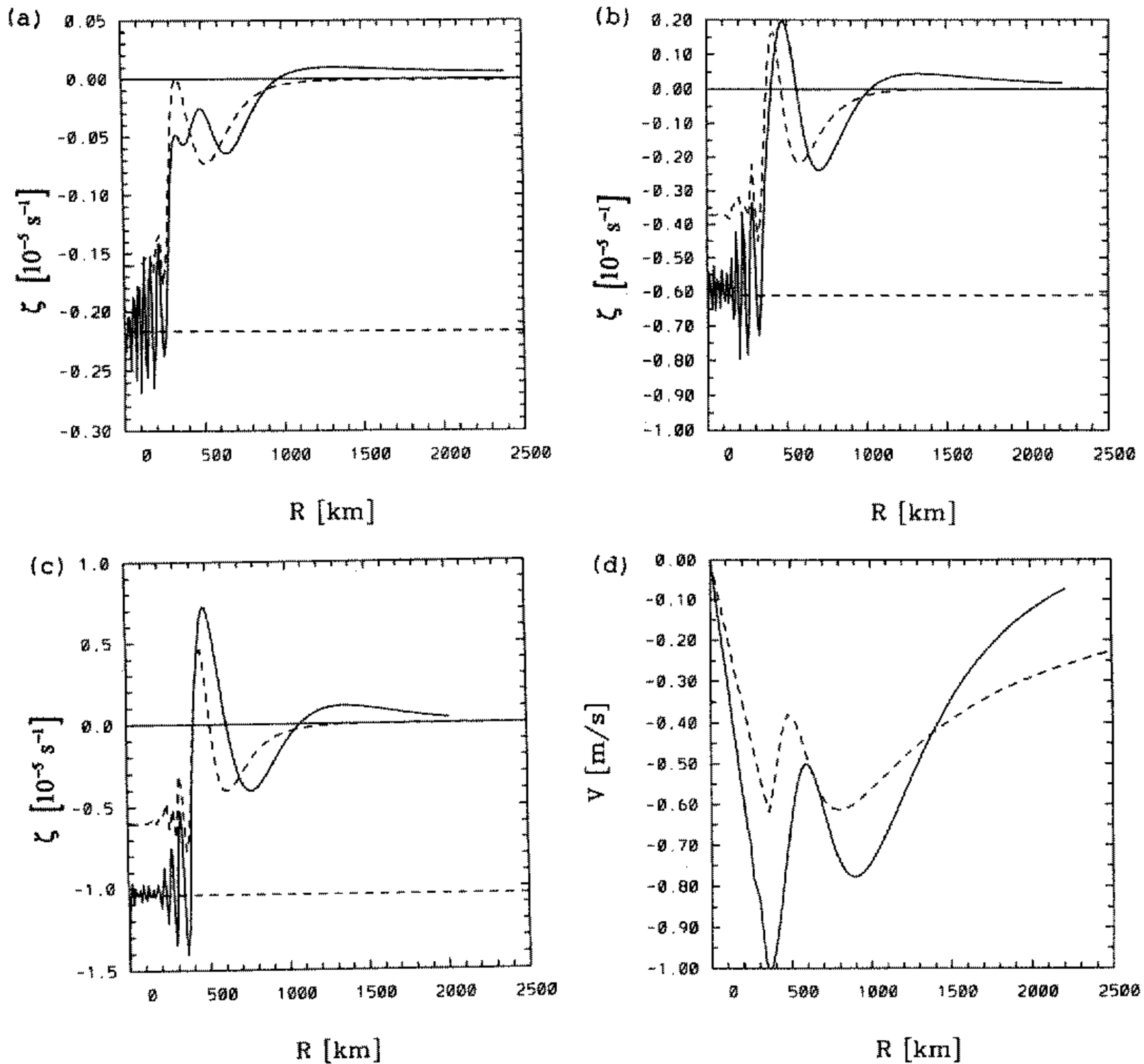


Figure 2. Radial profiles of azimuthally averaged perturbation relative vorticity calculated from the numerical model (solid lines) and from the analytic theory (dashed lines) at (a) 24 hours, (b) 48 hours, and (c) 72 hours.

Shown in (d) are the corresponding tangential velocity profiles for the two calculations at 48 hours.

equivalent to a decrease in the circulation, $2\pi r\bar{v}$, at radius r as expressed by Eq. (5). At 24 hours the RVF term is predominantly negative inside a radius of 1000 km and, if dominant, would lead to an acceleration of the tangential velocity in this region. At 48 hours this negative flux has increased significantly in amplitude and leads to a cyclonic acceleration of the symmetric flow at radii near 500 km where its magnitude exceeds the AVF term (see Fig. 3(b)). The AVF term is dominant in magnitude at most other radii and to 48 hours and inside 2500 km radius it is mainly positive so that the symmetric flow is mostly decelerated (Fig. 3(b)). This term decreases rapidly with radius beyond about 750 km, the radius where it attains its maximum value, and at 48 hours it goes slightly negative before the calculation has to be terminated at 2200 km because of the domain boundary. The influx of absolute vorticity at large radii and the consequent acceleration of the mean tangential flow can be attributed to the far-field generation of Rossby waves by the vortex. In a nondivergent model with an infinite domain, there is no bound to the phase speed of such waves, although in the numerical model the maximum phase speed is determined by the domain size. In the analytic theory these waves are not properly represented because, at any order in the small parameter expansion, changes in relative vorticity associated with the beta-term do not feed back to influence the advection of vorticity at that order. Presumably, this accounts for the breakdown of the analytic theory at large radii. In this theory one can show that the planetary vorticity flux at large radii decreases monotonically to some negative asymptotic value.

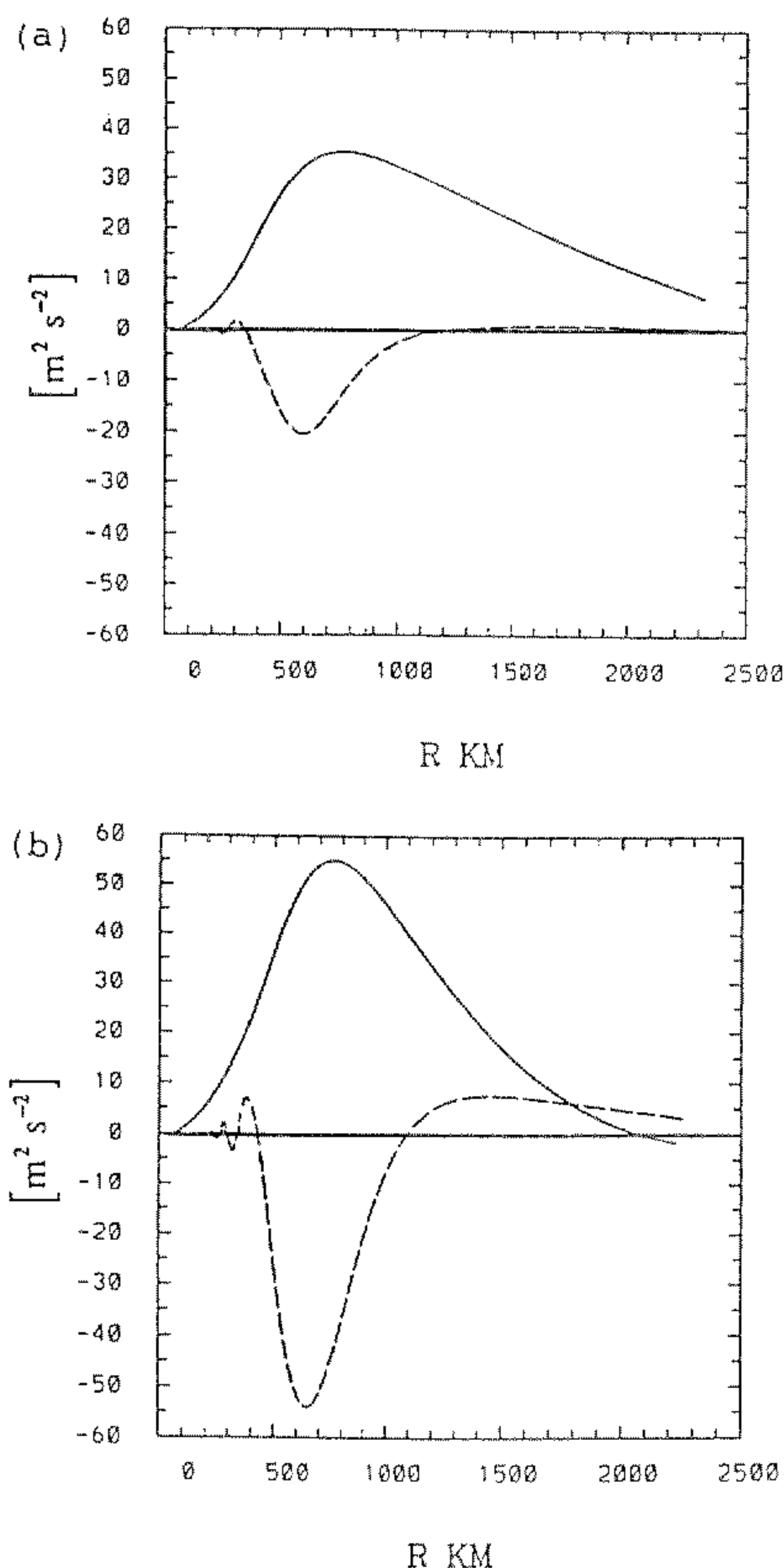


Figure 3. Radial profile of the radial flux of absolute vorticity $AVF = 2\pi\beta r^2(\overline{u' \sin \theta} + \frac{1}{2}c_2)$ (solid line) and relative vorticity $RVF = 2\pi r \overline{u' \zeta'}$ (dashed line) for the numerical model integration at (a) 24 hours and (b) 48 hours.

4. CONCLUSIONS

We have calculated the evolution of the symmetric circulation of a moving, hurricane-scale, barotropic vortex on a beta-plane in the absence of any large-scale flow using the numerical model described by Smith *et al.* (1990). We have shown that the north-westward drift of an initially symmetric, cyclonic vortex in the northern hemisphere is accompanied by a decrease in the tangential circulation at most radii, and a consequent deceleration of the azimuthally averaged tangential velocity. This decrease can be attributed to the sum of three effects: the radial flux of relative vorticity associated with the asymmetric component

of flow, the corresponding flux of planetary vorticity, and the rate of change in planetary vorticity due to the meridional displacement of the vortex. The net rate of change in planetary vorticity, the sum of the last two effects, makes the largest contribution to the circulation changes. The vorticity fluxes at large radii are influenced by the generation of long Rossby waves. These interpretations are analogous to those given by Shapiro and Ooyama (1990) in terms of integrated relative angular-momentum budgets. An advantage of considering vorticity budgets rather than angular-momentum budgets is that, unlike the latter, they do not give undue prominence to fluxes at large distances from the rotation axis.

The results presented in this note are different from those of Carr and Williams (1989) who found an acceleration of the symmetric flow at inner radii and a deceleration further from the axis of rotation. This is because Carr and Williams omit the direct effect of the absolute-vorticity flux from their calculations.

The predictions of the numerical model have been compared with those based on the analytic theory developed by the first author and co-workers. The circulation changes calculated using the analytic theory are qualitatively similar to those obtained from the numerical model, but they are a fraction too weak in the inner core region and, although small at large radial distances, they have the wrong sign there. In fact the analytic theory predicts a finite net outflux of planetary vorticity at large radii which leads to the development of a net anticyclonic circulation there. Accordingly, the azimuthally averaged tangential velocity decays only inversely with radius, irrespective of the initial decay rate, and this is inconsistent with the asymptotic theory of Flierl *et al.* (1983). This represents a limitation of the long-term validity of the analytic theory which we attribute to the fact that it does not properly represent the far-field Rossby waves. In the numerical model, the decay in the azimuthally averaged tangential velocity is more rapid than $1/(\text{radius})$ as expected from Flierl *et al.*'s result.

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APPENDIX

Derivation of Eq. (1)

The Euler equation of motion for a homogeneous, incompressible, fluid rotating with angular velocity $\frac{1}{2}\mathbf{f}$ may be written as

$$\partial \mathbf{u} / \partial t + (\boldsymbol{\omega} + \mathbf{f}) \wedge \mathbf{u} = -\nabla(p/\rho + \frac{1}{2}\mathbf{u}^2) \quad (\text{A.1})$$

where \mathbf{u} is the velocity vector, $\boldsymbol{\omega}$ is the vorticity and p and ρ are the pressure and density. Let \mathbf{X} , T denote the position vector and time, respectively, in a coordinate system moving with velocity \mathbf{c} relative to the \mathbf{x} , t system. Then $\nabla_{\mathbf{x}} = \nabla_{\mathbf{X}}$ and, although $T = t$, $\partial/\partial T = \partial/\partial t + \mathbf{c} \cdot \nabla_{\mathbf{x}}$. Thus, in the moving coordinate system, Eq. (A.1) becomes

$$\partial \mathbf{U} / \partial t + (\boldsymbol{\omega} + \mathbf{f}) \wedge \mathbf{U} = -\nabla(p/\rho + \frac{1}{2}\mathbf{U}^2) - \mathbf{f} \wedge \mathbf{c} - d\mathbf{c}/dt \quad (\text{A.2})$$

where the vorticity vector is $\boldsymbol{\omega} = \nabla_{\mathbf{x}} \wedge \mathbf{u} = \nabla_{\mathbf{X}} \wedge \mathbf{U}$. The vorticity equation in the moving frame (the curl of A.2) is

$$\partial \boldsymbol{\omega} / \partial t + \mathbf{U} \cdot \nabla(\boldsymbol{\omega} + \mathbf{f}) = (\boldsymbol{\omega} + \mathbf{f}) \cdot \nabla \mathbf{U} - \mathbf{c} \cdot \nabla \mathbf{f}. \quad (\text{A.3})$$

If $\mathbf{f} = \mathbf{f}(\mathbf{x}) = \mathbf{f}(\mathbf{X} - \mathbf{c}T)$, then $\partial \mathbf{f} / \partial T = -\mathbf{c} \cdot \nabla \mathbf{f}$, whereupon (A.3) becomes

$$\partial(\boldsymbol{\omega} + \mathbf{f}) / \partial t + \mathbf{U} \cdot \nabla(\boldsymbol{\omega} + \mathbf{f}) = (\boldsymbol{\omega} + \mathbf{f}) \cdot \nabla \mathbf{U}. \quad (\text{A.4})$$

If the motion is two-dimensional in a plane perpendicular to \mathbf{f} , then $\boldsymbol{\omega}$ is perpendicular to $\nabla \mathbf{U}$ and (A.4) reduces to Eq. (1) when $\mathbf{f} = f\mathbf{k}$, and $\boldsymbol{\omega} = \zeta\mathbf{k}$.

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