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Vortex motion in relation to the absolute vorticity gradient of the vortex environment

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SUMMARY

Recent attempts to establish a relationship between tropical cyclone motion and the absolute vorticity gradient of the cyclone environment are reviewed. Evidence is presented, both from analytic and numerical calculations, to show that for barotropic vortices there is no unique relationship between vortex motion, **c**, and the local absolute vorticity gradient of the (imposed) environment, $\nabla\bar{\zeta}_a$. Our results suggest that it is also unlikely that any similar relationship holds for tropical cyclones.

It is shown that conventional methods of averaging over an annular region about a tropical cyclone centre to determine the absolute vorticity gradient of the cyclone environment may be very sensitive to the size of the region chosen. For this reason as well as from theoretical considerations our results would indicate that there are certain intrinsic limitations for tropical cyclone track forecasting arising from statistically-derived relationships between **c** and $\nabla\bar{\zeta}_a$.

1. INTRODUCTION

The relevance of the magnitude and direction of the environmental absolute vorticity gradient on vortex motion has been recognized since the pioneering work of Kasahara and Platzman (1963): henceforth KP. They defined the 'vortex' as the azimuthally-averaged circulation about the local streamfunction minimum and the 'environment', or 'steering flow' as the residual flow field when the azimuthal average is removed. With these definitions, it was shown that the instantaneous motion of the vortex relative to the 'steering flow' across its centre can be expressed in terms of the relative and absolute vorticity gradient of the steering flow (see their Eq. (4.1)). Further, it was shown that a cyclonic vortex accelerates in the direction of the absolute (potential†) vorticity gradient of the steering flow. KP proposed also a numerical scheme by which the evolution of the 'steering flow' could be calculated in the presence of the moving vortex.

More recent numerical calculations by DeMaria (1985) using a non-divergent barotropic model confirmed the broad predictions of KP for a zonal shear flow, namely that the absolute vorticity gradient of the steering current results in a component of motion 90 degrees to the left of the gradient, and a component in the direction of the gradient. However, it should be noted that in contrast to KP's definition, DeMaria's 'steering current' refers to the initially imposed zonal flow. Used in this context the term 'steering current' is somewhat of a misnomer, being normally taken to be the current whose speed coincides with the vortex (or disturbance).

The question naturally arises: does the absolute vorticity gradient of a vortex environment uniquely characterize the instantaneous vortex drift speed relative to the environmental flow at the vortex centre? If the answer were yes, this might provide a useful basis for forecasting tropical cyclone monument, supposing of course that the data were adequate to allow the theory to be applied. The question has been studied in a theoretical framework in recent numerical model simulations (Evans *et al.* 1991; Ulrich and Smith 1991: henceforth US91; and Smith 1991: henceforth S91) and attempts have been made using observational data to determine whether such a relationship exists for tropical cyclones (Carr and Elsberry 1990; Franklin 1990). While the observational studies are equivocal, partly because of the limitations of the data, the numerical studies appear to be in conflict. Evans *et al.* conclude that "vortex propagation is highly correlated with the absolute vorticity gradient vector in the initial imposed environment", whereas the numerical calculations of US91, (supported by those of Shapiro and Ooyama 1990, see p. 183 therein) and the analytic theory of S91 show that no unique relationship exists between these quantities.

In the present note we seek to clarify the issues involved and present arguments which support our contention that, in barotropic models, vortex drift relative to the imposed environmental flow at the vortex centre is not uniquely related to the environmental absolute vorticity gradient. The inference would be that, even if the environmental flow of a tropical cyclone and the absolute

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† KP's model included the effects of divergence.

vorticity gradient thereof could be sufficiently accurately determined from the data, and even if one could demonstrate that barotropic dynamics were applicable, one could not be sure in a particular case of being able to determine the motion of the cyclone more accurately than by ignoring the vortex drift altogether. In any case, our calculations described in section 5 point to intrinsic difficulties in obtaining an accurate theoretically relevant measure of the absolute vorticity gradient of the cyclone environment from observational data.

2. Role of the absolute vorticity gradient

The precise role of the environmental absolute vorticity gradient on vortex motion in a horizontal shear flow on a beta-plane is clarified by the approximate analytic theory of barotropic vortex motion of S91; see also Smith and Ulrich (1990) for the special case of zero basic flow. The theory applies to a thought experiment in which an initially symmetric vortex is initialized in a prescribed environmental shear flow. From the initial instant, the vortex generates an asymmetric vorticity distribution which is predominantly azimuthal wavenumber-one in structure. The asymmetry is caused by the azimuthal advection of absolute vorticity in the presence of an environmental gradient of this quantity. We shall call this the zero-order vorticity asymmetry. Associated with this asymmetry is an asymmetric component of flow consisting of a pair of counter-rotating gyres. The flow between these gyres crosses the symmetric vortex centre, which may be defined as the absolute vorticity maximum, or, alternatively, to a very close approximation in the case of a tropical cyclone-scale vortex, the relative vorticity maximum. In the barotropic model, the absolute vorticity of a fluid parcel is advected by the total flow. It follows that the subsequent motion of the vortex centre can be attributed to the vector sum of the imposed environmental flow and the asymmetric gyre flow at that point.‡ This has been confirmed by numerical experiments and is an accurate result, even in the case when the relative vorticity maximum is used to define the vortex centre (Smith *et al.* 1990: henceforth SUD; US91). It transpires that the flow between the gyres is relatively uniform across the whole vortex core. Note that the foregoing scenario presupposes a partitioning of the total flow into the 'environment', which is taken to be the initially imposed environment; the 'vortex', which is taken to be the initial vortex, suitably relocated at the position of the maximum relative vorticity of the total flow; and the residual flow, which represents the vortex-induced asymmetries. We shall refer to the residual flow as the 'vortex asymmetry'.

As the zero-order-vorticity asymmetry develops, it suffers distortion by the environmental shear as well as self-induced distortion by the gyres. Furthermore, both the gyre flow and the environmental shear lead to a distortion of the basic vortex. With the foregoing partitioning scheme, these distortion effects are manifested as additional contributions to the vortex asymmetry.

The analysis of S91 (see especially p. 688 and Fig. 1 therein) shows that the zero-order-vorticity asymmetry has the form of a pair of vortex dipoles with different strengths, their axes being at right angles to each other. The radial distribution of vorticity in each dipole is a function of the local angular rotation of the vortex and is linearly proportional to the magnitude of the environmental absolute vorticity gradient at the vortex centre. It is supposed in the theory that the environmental absolute vorticity gradient is in some sense slowly-varying over the scale of the vortex, not just over the vortex core scale which is characterized by the radius of maximum tangential wind speed, but on the scale of the entire vortex circulation, which might be measured by the radius of gale-force winds in the case of a tropical cyclone. Then it follows that the magnitude of the gradient at the vortex centre can be regarded as a measure for the gradient over the vortex as a whole. At zero order, the orientation of the combined dipole flow across the symmetric vortex centre is uniquely related to the direction of the absolute vorticity gradient vector. Indeed at this level of approximation there is a unique relationship between the vortex motion and the local environmental absolute vorticity gradient. However, in general, the relationship does not hold at higher levels of approximation.

A decisive demonstration of this fact is provided by three numerical calculations carried out by US91. These experiments compared the motion of a specific vortex in two zonal-shear flows with quadratic variation of the flow (i.e. linear shear) in the meridional direction with the case of zero basic flow. The shear flows, one on a beta-plane and one on an f-plane, were chosen to have the same absolute vorticity gradient as in the case of zero basic flow. It turns out that the vortex tracks differ markedly between these cases, the meridional displacement diminishing as the shear contribution to the environmental absolute vorticity gradient is increased. The precise reason for this behaviour was identified by S91. It was found that linear shear distorts the basic vortex vorticity

‡ Evans *et al.* (1991) refer to the asymmetric gyre flow at the vortex centre as the vortex propagation; see also section 6.

leading to a wavenumber-one contribution to the vortex asymmetry. The flow associated with this asymmetry has an equatorward component across the vortex centre, the strength of which increases with the magnitude of the shear. After 48 hours of model integration, the vortices in the extreme cases of zero-beta or zero-basic flow differ in meridional position by as much as 220 km. While this may be considered to be an acceptable error for a 48-hour forecast, it should be pointed out that it is the best-case situation, because we have chosen a parabolic velocity profile that is centred initially on the vortex centre. Since there is no uniform shear component, the zonal component of flow remains relatively small over the time period so that the difference between the tracks in the cases of zero-beta and zero-basic flow is primarily due to the different rates of meridional drift. In general we may also expect there to be a uniform shear component so that, as the vortex drifts meridionally at different rates, it will be subject also to an increasing advective component of motion in the zonal direction.

3. Slowly varying absolute vorticity gradient

In the experiments described above, the environmental absolute vorticity gradient was held constant; whereas a number of authors, including ourselves, have carried out calculations in which the gradient varies in the meridional direction, (e.g. DeMaria 1985; Shapiro and Ooyama 1990; Evans *et al.* 1991; US91). In the last of these papers we also studied flows in which the absolute vorticity gradient varies both zonally and meridionally. In all these situations there are reasons to expect some time delay in the response of the vortex asymmetry to the changing absolute vorticity gradient experienced by the moving vortex. The argument is based on the following considerations.

The calculations of SUD for the case without horizontal shear showed that, after the first 24 hours of integration, the maximum amplitude of the vorticity asymmetry is located more than 350 km from the vortex centre. At this radius, the tangential wind speed of the vortex is only about one quarter of the maximum wind speed. As time proceeds, the strength of the asymmetry and the radius at which the maximum occurs continue to increase until about 60 hours when the radius of the maximum stabilizes (see SUD, Fig. 5).

This increase in the strength and scale of the gyres in the model is easy to understand if we ignore the motion of the vortex. As we shall see, this effect is a limitation only at the later times. As shown by SUD, the change in relative vorticity of a fluid parcel circulating around the vortex is equal to its displacement in the direction of the absolute vorticity gradient times the magnitude of the gradient. For a fluid parcel at radius r the maximum possible displacement is $2r$, which limits the size of the maximum asymmetry at this radius. However, the time for this displacement to be achieved is $\pi/\Omega(r)$, where $\Omega(r)$ is the angular velocity of a fluid parcel at radius r. Since Ω is largest at small radii, fluid parcels there attain their maximum displacement relatively quickly, and as expected, at early times, the maximum displacement of any parcel occurs near the radius of maximum tangential wind (Fig. 1(a)). However, given sufficient time, fluid parcels at larger radii, although rotating more slowly, have the potential to achieve much larger displacements than those at small radii; as time continues, this is exactly what happens (Fig. 1(b)).

Ultimately, of course, if $\Omega(r)$ decreases monotonically to zero, there is a finite radius beyond which the tangential wind speed is less than the translation speed of the vortex. As the maximum in the asymmetry approaches this radius the vortex motion can no longer be ignored (see Smith and Ulrich 1990, Fig. 12). However, the point to be made is that, even in the case of zero environmental shear, the vortex asymmetries do not achieve a steady configuration, but continue to evolve steadily on a timescale of days. Even if they achieve a quasi-steady state after this time, it is evident from the foregoing discussion that it would take the order of a day or two to achieve a new equilibrium with a changed environmental vorticity gradient.

The possibility of the gyres, and hence the vortex propagation (defined in the sense of Evans *et al.* 1991; see section 2 herein) achieving an equilibrium configuration for a given environmental absolute vorticity gradient is further reduced by horizontal shear. Even in the case of uniform shear, both the basic vortex and the vortex asymmetry suffer distortion by stretching in the direction of the shear. This leads inexorably to a larger and larger vorticity gradient on a finer and finer meridional scale, as is illustrated in Fig. 2. Another graphic example of this type of effect is the rapid homogenization of the vortex asymmetry in the vortex-core region as noted by SUD and Shapiro and Ooyama (1990). For non-uniform zonal shear, the local rate of distortion of the vortex asymmetry will change non-uniformly in the meridional direction as the vortex moves across the shear.

In summary, all the effects described above conspire to prevent the vortex asymmetry achieving some quasi-equilibrium configuration related to the local absolute vorticity gradient of the

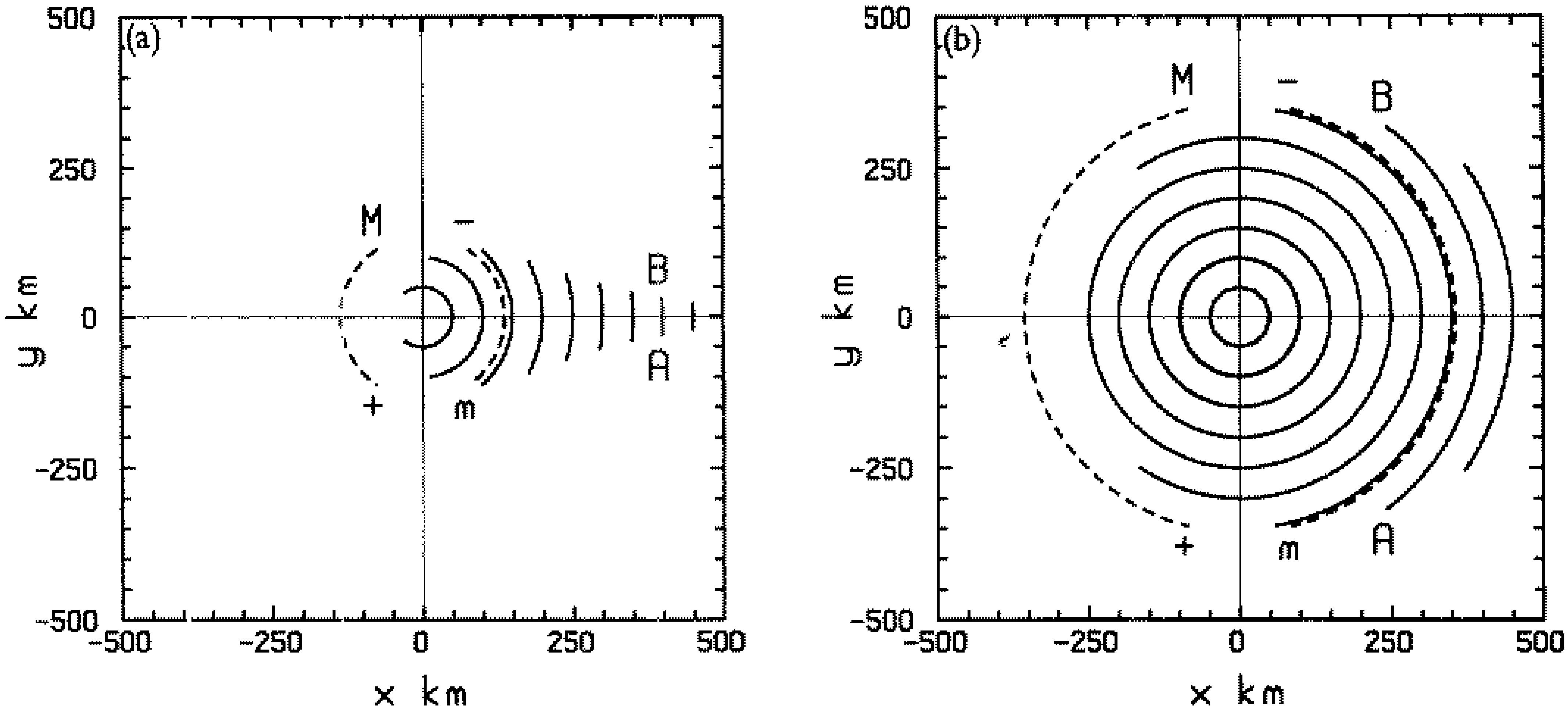

Figure 1. Approximate trajectories of fluid parcels which, for a given radius, give the maximum asymmetric vorticity contribution at that radius. The figures refer to the case of motion of an initially-symmetric vortex on a beta-plane with zero basic flow (a) at 2 hours, (b) at 24 hours. The particles are supposed to follow circular paths about the vortex centre (e.g. A → B) with angular velocity $\Omega(r)$, where Ω decreases monotonically with radius r. Solid lines denote trajectories at 50 km radial intervals. Dashed lines marked 'M' and 'm' represent the trajectories giving the overall asymmetric vorticity maximum and minimum, respectively. These maxima and minima occur at the positive and negative ends of the relevant lines.

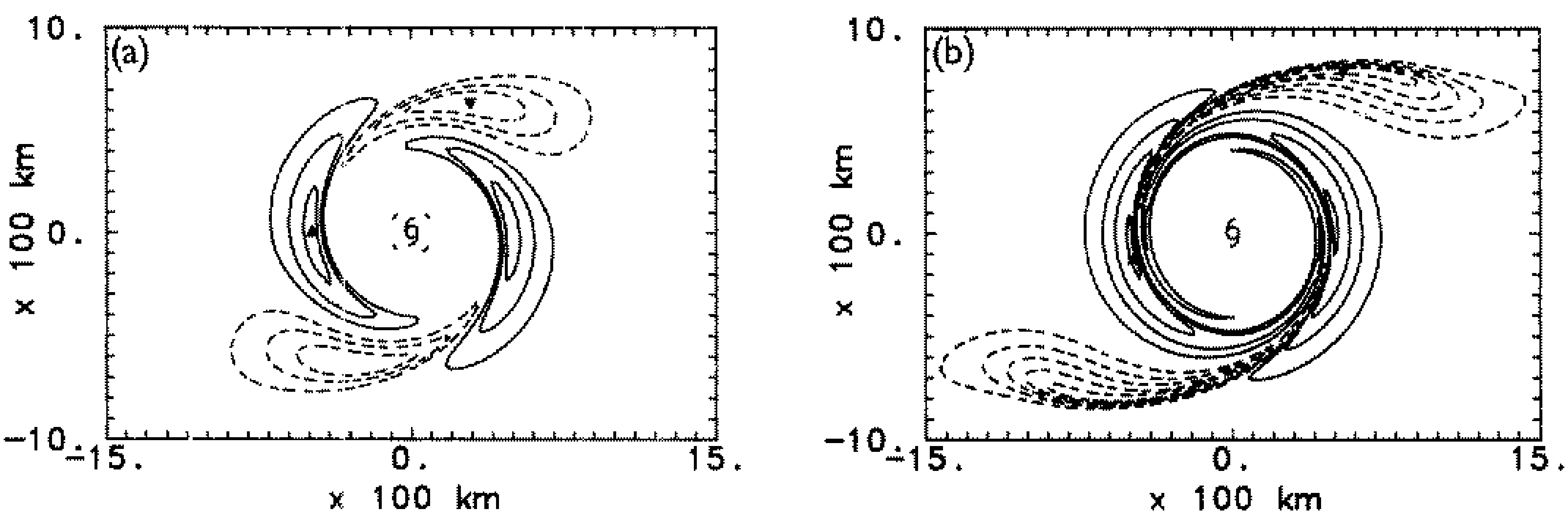

Figure 2. Vortex asymmetry in the case of an initially symmetric vortex in a uniform shear flow on an f-plane at times (a) 48 hours, (b) 96 hours. The shear magnitude corresponds with $5\,\mathrm{m\,s^{-1}}$ per 1000 km. The contour interval is $5 \times 10^{-6}\,\mathrm{s^{-1}}$; dashed contours indicate negative values.

environment. In general, this must prevent the streamflow across the vortex core from achieving quasi-equilibrium. We are led to conclude that it is not possible to relate vortex propagation to this gradient. In the following section we present an additional calculation in support of our contention.

4. Vortex motion in a large-scale planetary wave

We have carried out a series of numerical simulations of vortex motion in an environment provided by a stationary large-scale, finite-amplitude planetary wave, allowing the effects of zonal as well as meridional gradients of the absolute vorticity gradient to be explored (US91). We present a more general calculation here to study further the relationship between vortex propagation and the local absolute vorticity gradient of the imposed environment $(\nabla\zeta_a)_0$ at the vortex centre. Again we chose a basic state in the form of a finite-amplitude planetary wave, but added a uniform westerly horizontal shear of $3.2\,\mathrm{m\,s^{-1}}$ per 1000 km to enhance the easterly flow equatorward of the anticyclone. We took also a larger domain ($6000 \times 3000\,\mathrm{km}^2$) than US91 to reduce the influence of boundaries.

For the parameters chosen, the planetary wave propagates slowly westwards with a speed of $5.0\,\mathrm{m\,s^{-1}}$ and its structure changes slowly also on account of the basic shear. Details are given in the Appendix. Figure 3(a) shows part of the wave at 72 hours, by which time the ridge axis has moved 1200 km west from its initial position.

The vortex is the same as that used by US91. It is located initially in the easterly flow, south-west of the anticyclone centre, 500 km to the west of the initial ridge axis and 1000 km north of the southern boundary of the flow domain. For the first 48 hours it moves north-west, then recurves and moves westwards and finally north-east (Fig. 3(a)). Although the basic state may not be realistic of tropical cyclone environments in all details, it provides a suitable environment varying in time and space for our analysis of vortex motion.

We denote the environmental flow at the vortex centre by $\overline{\mathbf{U}}_0$ and the vortex velocity by $\mathbf{c}$, whereupon the vortex propagation is $\mathbf{c} - \overline{\mathbf{U}}_0$. Then, provided $|(\nabla\bar{\zeta}_a)_0| \neq 0$, we may express the propagation as

$$\mathbf{c} - \overline{\mathbf{U}}_0 = A(t)\,(\nabla\bar{\zeta}_a)_0 + B(t)\mathbf{k} \wedge (\nabla\bar{\zeta}_a)_0, \tag{1}$$

where $\mathbf{k}$ is the unit vector in the vertical and $A(t)$ and $B(t)$ are functions of time t. This representation differs from the one carried out by US91 in which the magnitude of $(\nabla\bar{\zeta}_a)_0$ was absorbed into the coefficients $A(t)$ and $B(t)$. The present version brings out more clearly the relationship (or lack thereof!) between $\mathbf{c} - \overline{\mathbf{U}}_0$ and the magnitude of $(\nabla\bar{\zeta}_a)_0)$. Note that the ratio B/A determines the direction of vortex propagation relative to the absolute vorticity gradient of the basic flow.

The variation of $A(t)$ and $B(t)$ with time is shown in Fig. 4. It is evident that all these functions undergo significant variation during the 196 hours of integration. Indeed $A(t)$ does not even have the same sign throughout this period. For the representation to be useful would require $A(t)$ and $B(t)$ to be independent, or at least approximately independent, of time. This is demonstrably not the case for any time period. Similar calculations were carried out for the cases studied by US91, with similar results. On the basis of these results we conclude that the representation of vortex propagation in relation to the absolute vorticity gradient has little to commend it, certainly for vortices embedded in an environmental flow that varies in time and space.

5. Observational studies

The fact that, even for barotropic vortices in simple models, there does not exist a unique relationship between vortex propagation and the imposed environmental vorticity gradient at the vortex centre would suggest that no such relationship exists for tropical cyclones. For these, the dynamics of motion is complicated by their baroclinic nature and by the nature of their environment. This view is consistent with the inconclusive results obtained by Carr and Elsberry (1990, see p. 545) in attempts to find such a relationship from composite data-sets on tropical cyclones. It is not inconsistent with the recent observational study of hurricane Josephine (1984) by Franklin (1990, see especially pp. 2740–2741). The environment of this storm was well sampled on three successive days by airborne Omega dropwindsondes, allowing an estimate of the environmental absolute vorticity gradient and the propagation vector to be made on each occasion. Franklin applied his analysis to the 300–850 mb layer mean flow and estimated the propagation vector $\mathbf{V}_p$ and the absolute vorticity gradient $\nabla(\zeta_e + f)$ by averaging these quantities over an annular region, either 3–5 or 5–7 degrees, about the storm. For the smaller annulus he found no clear relationship between the two vectors. For the larger one the magnitudes were proportional, although in two cases the directions differed by more than 30 degrees. Franklin regards these results as suggestive, although he was cautious about drawing strong conclusions from them because of the arbitrary nature of several of the assumptions on which the calculations were based.

We would argue that, since there is no unique relationship between $\mathbf{V}_p$ and $\nabla(\zeta_e + f)$ for barotropic vortices, one should not be surprised not to find one in the case of tropical cyclones.

In his paper, Franklin gives an excellent discussion of the problem of extracting a hurricane's environmental absolute vorticity gradient from observational data. Here, we explore this problem further using the numerical model calculation discussed in section 4.

A particular advantage of the model 'thought experiments' is the ability to partition 'the environment' as defined by KP into the 'initially imposed environment' and 'the vortex-induced asymmetry'. It was noted by US91 that this separation is not possible within an observational framework. However, the model results show that the vortex-induced asymmetry has fine-scale structure in the inner part of the vortex, and this can be expected to be the case for tropical

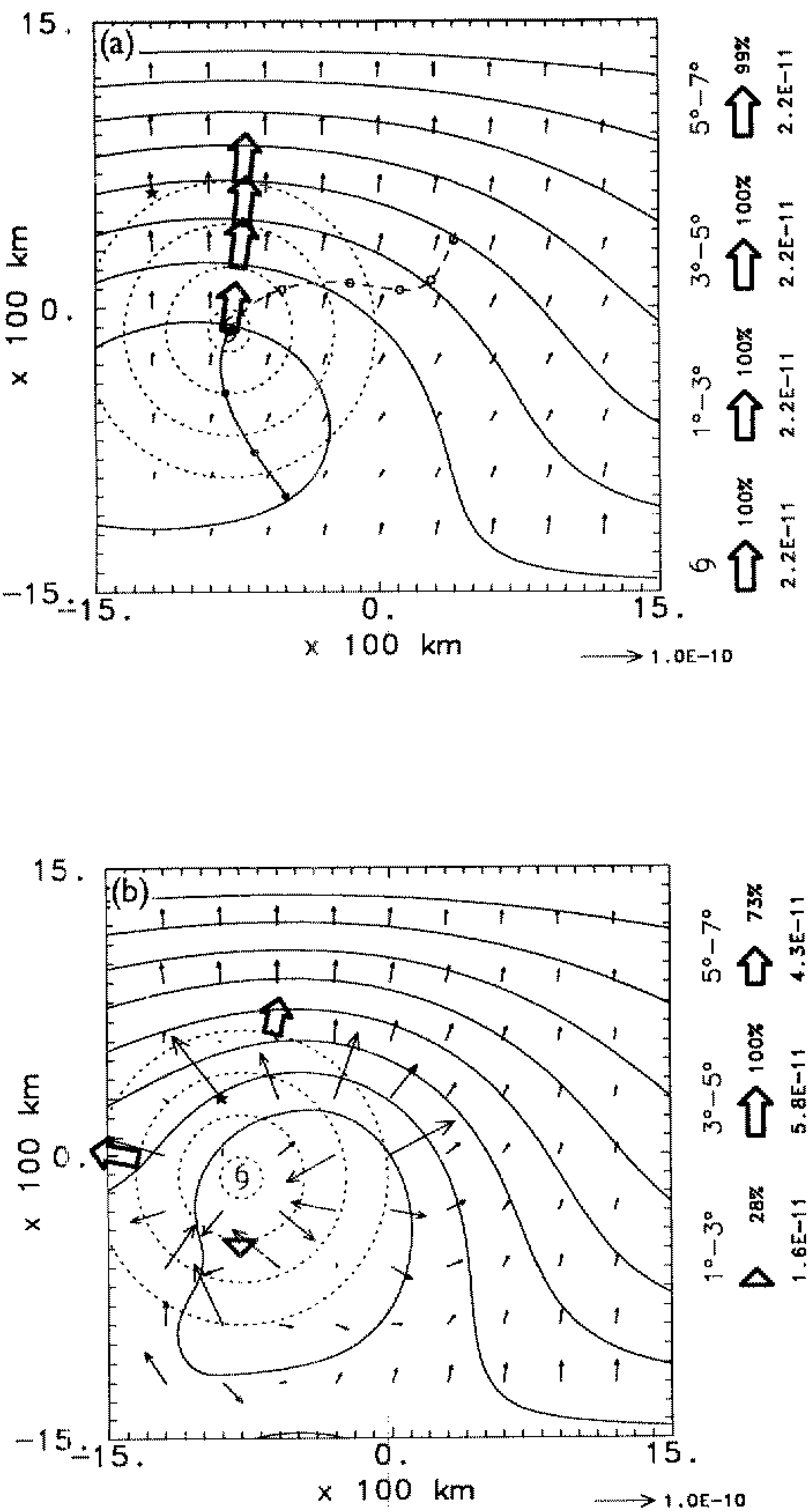

Figure 3. Absolute vorticity gradient (arrows) and streamlines for the calculation in section 4: (a) imposed environment and vortex track. The vortex position at 72 hours is marked by a cyclone symbol and at 24-hour intervals thereto by solid dots. The subsequent track is shown by the dashed line and open circles. The thick arrows show the relative magnitude and direction of the imposed environment averaged over annular regions 1–3, 3–5 and 5–7 degrees latitude compared with the analytic value at the vortex centre. The three annular regions are denoted by dashed circles. For clarity, the thick arrows are scaled differently from the smaller ones, those at the vortex centre denoting the same vector; (b) imposed environment plus vortex asymmetry at 72 hours. The absolute vorticity gradient vectors in this case are averaged over 100 km squares centred on the arrow tail. The vector scale is the same in each diagram and the vector with largest magnitude is indicated by a star. Note that the vortex induced asymmetry has a significant but irregular influence on the absolute vorticity gradient vector over a large area surrounding the vortex. This leads to considerable differences between the annular averages, all of which differ markedly from those calculated for the environment alone, and, in particular, from that at the vortex centre in (a) which is the value that is relevant to the theory.

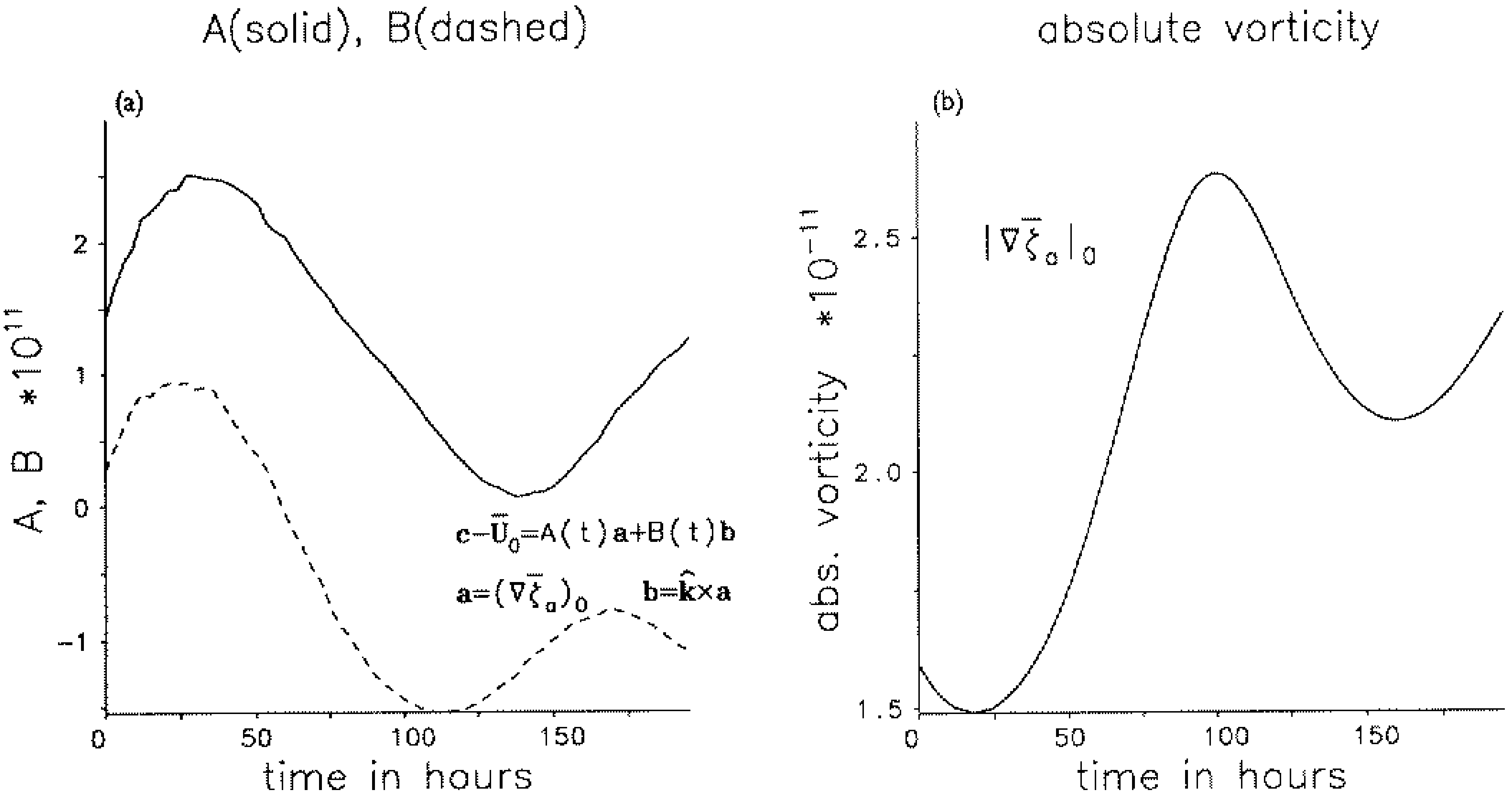

Figure 4. Time series of: (a) the coefficients $A(t)$ and $B(t)$ in Eq. (1) for the calculation described in section 4. Units are $10^{11}\,m^2$; (b) $|\nabla\bar{\zeta}_a|_0$ in units $10^{-11}\,m^{-1}\,s^{-1}$.

cyclones also. It was shown in section 3, that this fine structure is a consequence of the large-scale absolute vorticity gradient. Moreover, while it does not have an appreciable effect on vortex motion (see e.g. SUD, p. 351), its presence in nature does have implications for determining an appropriate definition for the environmental absolute vorticity gradient from observations to compare with theory (Franklin 1990, p. 2741). It may be interesting also to note that the evolving fine structure in the vortex asymmetry might have been a practical limitation in applying KP's original theory in which the absolute vorticity gradient of the total asymmetry was required at the vortex centre.

Even outside the vortex core region, the vortex asymmetries may make a significant contribution to the absolute vorticity gradient over a wide area surrounding the vortex. This is illustrated in Fig. 3(b) which shows the absolute vorticity gradient vectors of the imposed environment and of the total environment (the imposed environment plus the vortex asymmetry) for the calculation described in section 4 at 72 h. In the latter case the vectors were averaged over 100 km square domains centred on each vector tail. Despite this, there are significant and irregular differences between the two fields. Tests using finer resolution calculations show that the small-scale variability of the asymmetric vorticity field is a genuine effect of deformation and shear in the model and is not a consequence of numerical 'noise'. The differences between the streamlines for the imposed environment and the total environment, shown also in Fig. 3(b), are relatively small and are not suggestive of the large variability that exists in the vorticity gradient arising from the vorticity asymmetry.

The foregoing calculations can be used to assess the representativeness and accuracy of the azimuthal averages of the absolute vorticity gradient in various annular regions about a vortex: the procedure adopted by Franklin (1990) and others. As an illustration we show also in Fig. 3 vectors corresponding to the average over annuli of radii 1–3, 3–5 and 5–7 degrees of latitude, respectively. When these averages are applied to the imposed environment only (i.e. $\bar{\zeta}+f$), the calculated vectors are in the same direction as the absolute vorticity gradient at the vortex centre, but their magnitude decreases with increasing size of the annulus. However, when applied to the imposed environment plus the vortex asymmetry (i.e. $Z+f$), the direction and magnitude of the annular average differ considerably from one annular size to another and none is a good approximation to the value corresponding with that of the imposed environment at the vortex centre—the value of relevance in barotropic theory (S91). While the variation of the absolute vorticity gradient of the imposed environment in the foregoing calculation may not be typical of all tropical cyclones, the calculation highlights the uncertainties that may arise in attempting to establish relationships between the absolute vorticity gradient and vortex motion when the former quantity is estimated from an annular average of data about the storm. The calculation shows how unrepresentative this procedure can be, even when high-resolution data are available.

Results similar to those in Fig. 3 are obtained at other vortex positions along the track. Moreover, we checked the accuracy of our analysis for Fig. 3 by doubling and halving the resolution of the annular average analysis and the vortex integration to which it was applied. Doubling the resolution gave no appreciable difference, while halving it produced a small to moderate change in the magnitude of the average absolute vorticity gradient in each annulus, but little change in its direction. The change in magnitude was largest for the inner annulus (−44%), intermediate for the middle annulus (−10%) and smallest for the outer annulus (−7%). However, none is a useful approximation to the value for the imposed environment at the vortex centre.

6. Discussion

(*a*) *Comparison with Evans et al.'s results*

Evans *et al.* (1991) carried out an analysis similar to that described in section 4, but seemingly drew conclusions different from ours. In their abstract they state that ". . . vortex propagation is highly correlated with the absolute vorticity gradient in the imposed environment" and begin their conclusion with the statement that "The propagation vector . . . is a strong function of the absolute vorticity gradient of the environment". However, it appears that 'high correlation' and 'a strong function of' relate only to averages over a 24–48 hour period.

In practice it would be impossible to know whether a cyclone was in the appropriate time interval for the mean relationship to be applicable. In particular it is clear that a mean relationship would be of little use for deducing the vortex track in the examples discussed in sections 2 and 4. Indeed in the examples discussed in section 2, significant track differences were found although the absolute vorticity gradient was unchanged. We showed that these differences were associated with different proportionate contributions of the relative vorticity gradient to the absolute vorticity gradient of the environment. This factor may account for the finding of Evans *et al.* who state (p. 313) that "detailed study of the individual experiments suggests that significant deviations from the mean relationship occur". In view of this variability and of our own findings described above, we would argue that regression methods that relate vortex motion in barotropic models to ambient absolute vorticity gradients contain intrinsic inaccuracies because of their neglect of the various dynamical factors mentioned in section 3. Although such methods will always yield a result, suggestive that a relationship exists, this mean relationship cannot be more accurate in any individual case than the error which can arise, for example, by ignoring the relative contribution of the ambient shear to the absolute vorticity gradient of the environment. We have shown that such errors could be significant, and may limit the utility of such methods for forecasting tropical cyclone motion. The limitations are compounded by the difficulties of even diagnosing a relationship between vortex propagation and the environmental absolute vorticity gradient, even from relatively dense data in the environment of a tropical cyclone (Franklin 1990), a factor also considered by Evans *et al.* and in the previous section.

(*b*) *Terminology*

While we ourselves have succumbed to the use of the term 'vortex propagation' in the present paper so as to minimize confusion with other authors, we believe this term to be a misnomer. The numerical calculations of SUD and US91 show that, in the case of tropical cyclone-scale vortices, and with an appropriate definition for the partition between 'vortex' and 'environment', vortex motion can be simply and accurately associated with advection by the environmental flow across the vortex core. In practice, at least to the extent that vortex motion can be regarded as a barotropic process, inaccuracy in the determination from data of the environmental flow across the vortex core may lead to the inference of 'propagation'. To the unwary reader, this may be interpreted to mean that some special dynamical process is implied. Perhaps a neutral term such as 'relative motion' would avoid such misinterpretation.

7. Conclusion

We have reviewed the evidence, both theoretical and observational, for a relationship between vortex propagation (relative to some environmental flow) and the absolute vorticity gradient of the environmental flow. In doing so, we have sought to clarify the role of the absolute vorticity in determining the large-scale vortex asymmetries responsible for motion. We show that for barotropic vortices, no unique relationship exists in general, and suggest that the same is likely to be true also for tropical cyclones. Indeed, our calculations bring to light intrinsic difficulties in obtaining an accurate theoretically-relevant measure of the absolute vorticity gradient of the cyclone environment from observational data.

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APPENDIX

The basic-state planetary wave

The streamfunction for the basic-state planetary wave used in section 4 is

$$\psi(x,y) = \psi_0 \cos ly \sin(kx + p) - U_0 y\{1 + y/(2L_y)\}, \tag{A1}$$

where x, y are rectangular coordinates centred at the mid point of the computational domain, pointing east and north, respectively; $k = 2\pi/L_x$; $l = \pi L_y$; $p = 3\pi/8$; $L_x = 6000\,\mathrm{km}$; $L_y = 3000\,\mathrm{km}$; $\psi_0 = -U_m L_y/\pi$; $U_m = 5\,\mathrm{m\,s^{-1}}$; $U_0 = \beta/(k^2 + l^2)$. For a latitude of 22.5°N, $\beta = 2.11 \times 10^{-11}\,\mathrm{m\,s^{-1}}$, whereupon $U_0 = 9.6\,\mathrm{m\,s^{-1}}$.

REFERENCES

Carr, L. E. and Elsberry, R. L. 1990 Observational evidence for predictions of tropical cyclone propagation relative to environmental steering. *J. Atmos. Sci.*, **47**, 542–546

DeMaria, M 1985 Tropical cyclone motion in a non-divergent barotropic model. *Mon. Weather Rev.*, **113**, 1199–1210

Evans, J. L., Holland, G. J. and Elsberry, R. L. 1991 Interactions between a barotropic vortex and an idealized subtropical ridge. Part I: Vortex motion. *J. Atmos. Sci.*, **48**, 301–314

Franklin, J. L. 1990 Dropwindsonde observations of the environmental flow of hurricane Josephine (1984): Relationships to vortex motion. *Mon. Weather Rev.*, **118**, 2732–2744

Kasahara, A. and Platzmann, G. W. 1963 Interaction of a hurricane with a steering field and its effect upon the hurricane trajectory. *Tellus*, **15**, 321–335

Shapiro, L. J. and Ooyama, K. V. 1990 Barotropic vortex evolution on a beta-plane. *J. Atmos. Sci.*, **47**, 170–187

Smith, R. K. 1991 An analytic theory of tropical cyclone motion in a barotropic shear flow. *Q.J.R. Meteorol. Soc.*, **117**, 685–714

Smith, R. K. and Ulrich W. 1990 An analytical theory of tropical cyclone motion using a barotropic model. *J. Atmos. Sci.*, **47**, 1973–1986

Smith, R. K., Ulrich, W. and Dietachmayer, G. 1990 A numerical study of tropical cyclone motion using a barotropic model. Part I: The role of vortex asymmetries. *Q.J.R. Meteorol. Soc.*, **116**, 337–362

Ulrich, W. and Smith, R. K. 1991 A numerical study of tropical cyclone motion using a barotropic model. Part II: Spatially-varying large-scale flows. *Q.J.R. Meteorol. Soc.*, **117**, 107–124