Morning Glory Wave Clouds in Oklahoma: A Case Study

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ABSTRACT

Early in the morning on 9 June 1982 a system of traveling wave cloud lines passed over Oklahoma, and in particular, over the relatively dense mesonetwork of surface stations, including the instrumented 444 m KTVY television tower, operated by the U.S. National Severe Storms Laboratory. An analysis of the network and other data presented herein shows that, in structure, the associated disturbance was an internal undular bore propagating on a low-level stable layer, similar to "morning glory"-type disturbances, which are common at certain times of the year in parts of northern Australia. Moreover, the speed of propagation of the component bore-waves is in broad agreement with theoretical calculations.

There is evidence that the disturbance emanated late the previous evening from an area of rapidly growing intense thunderstorms located more than 400 km north-northwest of the surface mesonetwork. Possible mechanisms for generation are discussed, but the data pertaining to genesis are insufficient to be conclusive.

1. Introduction

"Morning glory" is the name given to a type of wind squall, or succession of such squalls, often accompanied by spectacular roll clouds, that occur in the Gulf of Carpentaria region of northern Australia. These squalls occur early in the morning and apparently at any time of the year, but are most common in the late dry season from early September until about mid-November. The leading squall is accompanied by an abrupt rise in surface pressure, often exceeding 1 mb in 10 min. This pressure rise is maintained and may continue slowly, but oscillations about the new value occur frequently following the initial jump and, in some cases, may persist for an hour or more. The disturbance front and roll clouds may exceed 100 km in length and generally move from the northeast sector at speeds of about 10 m s^{-1}. Following an observational expedition to the region in 1979 to study the phenomenon, these disturbances have been identified as internal undular bores propagating on the low-level maritime or nocturnal inversion, and their origin is believed to be the result of an interaction between the deeply penetrating east coast sea breeze over Cape York Peninsula and the developing nocturnal inversion (Clarke *et al.*, 1981; see also Clarke, 1983a,b).

A particularly surprising discovery made during the 1979 and subsequent observational expeditions is that, at least in the southern part of the Gulf region, morning glories may originate also from a sector approximately South. Moreover, while these appear to be less common than northeasterly ones, it is possible for both types to occur on the same morning.

Southerly morning glories are structurally similar to those from the northeast (Smith *et al.*, 1982), but less is known in detail about their origin. At least in some cases, it appears to be linked with the northeastward advance of a frontal trough across central Australia. It is suggested by Smith *et al.* (1982) that the advance of a front into a developing nocturnal inversion might generate waves, or borelike hydraulic disturbances, that propagate along this stable layer ahead of the front, in a manner envisaged by Tepper (1950). The efficacy of this mechanism is demonstrated by Smith *et al.* (1982) in a simple laboratory experiment. Assuming that such a mechanism *is* responsible for southerly disturbances, it would be reasonable to expect that morning-glory-type disturbances would sometimes occur as precursors to cold fronts in other parts of the world. If this is true, it would have important implications with regard to weather forecasting, since it is known that, when the air over the Gulf of Carpentaria is conditionally unstable, morning glory disturbances are effective triggering mechanisms for deep convection. Thus, in suitable conditions, the generation of borelike disturbances ahead of cold fronts could lead to the initiation of prefrontal showers or thunderstorms. A brief review of documented cases of possibly similar phenomena elsewhere is given by Smith *et al.* (1982). Two particularly well-documented events are described in detail by Shreffler and Binkowski (1981).

In this paper we present an analysis of data and some related theoretical calculations for a traveling borelike disturbance that occurred in Oklahoma in 1982.

2. The Oklahoma disturbance

Early in the morning on 9 June 1982, a series of rapidly moving cloud lines passed over Oklahoma and the associated disturbance was recorded by instruments in a network of surface observing stations operated by the National Severe Storms Laboratory (NSSL) and on the KTVY television tower about 15 km north of Oklahoma City. The clouds were recognized by Dr. R. P. Davies-Jones of NSSL as being similar to those associated with the morning glory in northern Australia. A photograph of two of the later cloud lines taken by Davies-Jones from NSSL at 0810 [all times are Central Standard Time (CST)] is reproduced in Fig. 1.

The cloud lines are clearly visible on nine successive high-resolution satellite photographs taken at 30 min intervals and commencing at 0601. These photographs reveal up to eight cloud lines, each about 250 km long, separated by a distance of 7–10 km and moving normal to their length in a more or less southerly direction with an estimated speed of ~15 m s^{-1}. Figure 2 shows the cloud system at 0801, shortly after it passed over NSSL.

a. The synoptic situation

The MSL isobaric analysis for 0800 (Fig. 3) shows an anticyclone centered over the eastern Rockies with an extended ridge to the south into northern Texas, a mesolow in northeast Oklahoma and a cold front extending from the low southwestward across central Oklahoma and into Texas. At this time, the cold front was about 40 km northwest of Oklahoma City and moving at an estimated speed of approximately 6.0 m s^{-1}.

b. Surface pressure data

Surface pressure records are available at the 12 stations shown in Fig. 4, including the KTVY tower. Typical of morning glories, the onset of the disturbance was heralded by a pressure jump followed by a series of small amplitude oscillations. This variation is exemplified by the microbarograph trace shown in Fig. 5 from a station about 2.5 km south of NSSL. The pressure rose by approximately 4 mb over a period of about 4.5 min and remained near this value for at least several hours. Oscillations about the mean with amplitude O(0.5 mb) persisted at most stations up to an hour after the initial rise. This pressure variation is characteristic of an undular bore.

A time of passage of the pressure jump, corresponding with the middle pressure of the steepest part of the trace, can be defined for each station. Assuming that the disturbance front is locally straight on the scale of the station network (this is confirmed by the satellite photographs) and that the disturbance moves with uniform speed, a least-square fit to the times of passage at individual stations gives a mean propagation velocity of 16.5 m s^{-1} from 348°. Thus, the disturbance moved faster than the component of motion in its direction of the cold front. The direction of motion of the surface cold front is estimated to make an angle of 30° to that of the disturbance.

c. Low-level wind and temperature structure

The disturbance front passed over the 444 m high KTVY tower shortly after 0600. As in the case of morning glories, the surface wind change occurs a few minutes prior to the peak pressure gradient; at the tower these onset times were 0602 and 0604, respectively. The tower is instrumented to measure horizontal wind speed and direction, vertical velocity and dry- and wet-bulb temperature, at the surface and at heights of 26, 45, 89, 177, 266 and 444 m. Pressure is measured at the surface and at the 444 m level. Data are recorded at 10 s intervals. From these data we have constructed time–height isotach cross

FIG. 1. A photograph of the last two cloud lines in the sequence taken at about 0810 CST by Dr. R. P. Davies-Jones. View is toward the southeast.

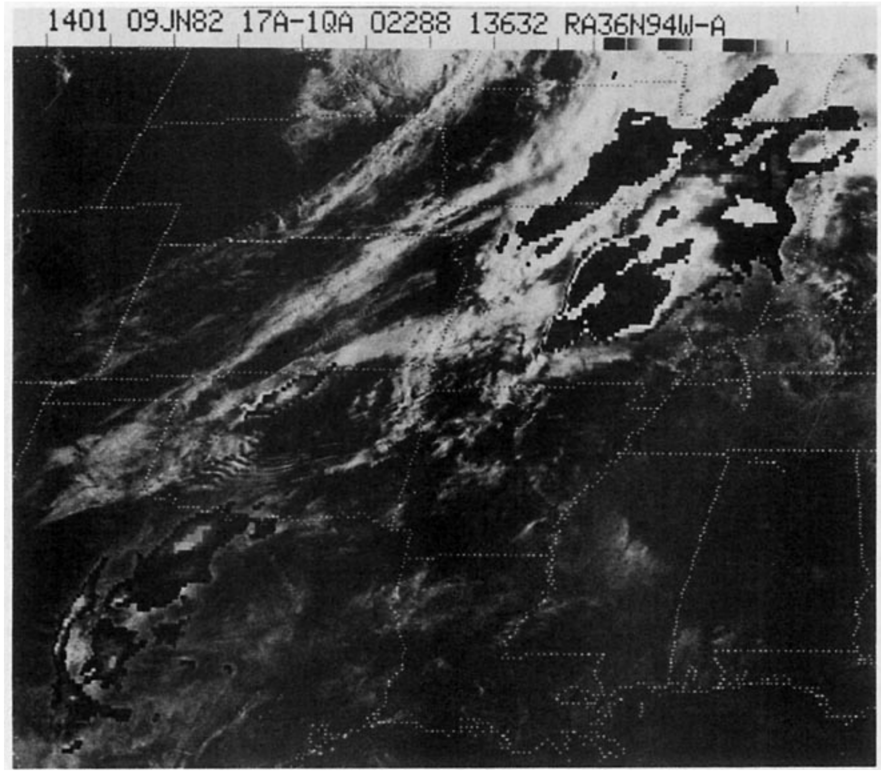


FIG. 2. High-resolution satellite photograph of the U. S. Midwest at 0801 CST (1401 GMT) on 9 June 1982.

sections of the horizontal wind components u, v in the direction of disturbance propagation (toward 168°) and at right angles to it (toward 78°), respectively, and of the vertical velocity w. The u and v isotachs are shown in Fig. 6 and the w isotachs in Fig. 7. To remove noise due to small-scale turbulent fluctuations, these data have been averaged over 70 s intervals. Additional smoothing in the vertical is performed on the w field.

From the u isotachs (Fig. 6a) we see that the flow below 300 m is almost everywhere in the *opposite direction* to that of the disturbance motion, except for some 6 min duration around the time of onset. However, maximum speeds in the disturbance direction did not exceed 3 m s^{-1} and are therefore much less than the disturbance speed itself. Toward the rear of the disturbance, from about 0645 onwards, the wind component above 300 m reversed in direction, but was again less than 3 m s^{-1}. Thus, there were no regions of advected flow at levels sampled by the tower, an observation that we return to later. Nevertheless, there was a prominent change in the isotach pattern following the disturbance onset with the wind fluctuating in strength with a period of approximately 10 min, in good agreement with the period of pressure oscillations after the leading pressure rise. These regular wind fluctuations persisted for about an hour, after which time a randomness set in, presumably as the convective phase of the planetary boundary layer began.

The transverse velocity component v (Fig. 6b) was generally much stronger than the u component and exhibited marked shear above 200 m. The onset of the disturbance brought coherent oscillations in v above about 180 m, again with a period close to 10 min. If the disturbance was exactly two-dimensional, there would be no fluctuating pressure gradient acting in the v direction and, hence, in the presence of positive mean vertical shear, the oscillations in v would be negatively correlated with those in w. That this was broadly the case for the disturbance may be judged by comparing Fig. 6b with Fig. 7a or b. The latter figure shows the calculated vertical velocity isotachs, obtained on the assumptions of two-dimensionality and stationariness by integrating the continuity equation and the time-averaged u isotachs. Stationariness implies the equivalence of time and space cross sections, subject to the appropriate rescaling of the abscissa in terms of the disturbance propagation speed. Comparing the observed and calculated vertical velocity fields in Fig. 7, we note good correspondence following the disturbance onset with both fields showing alternating regions of ascent and descent with a period of approximately 10 min, consistent with the u and v fields. Just ahead of the disturbance, where the pressure first began to rise

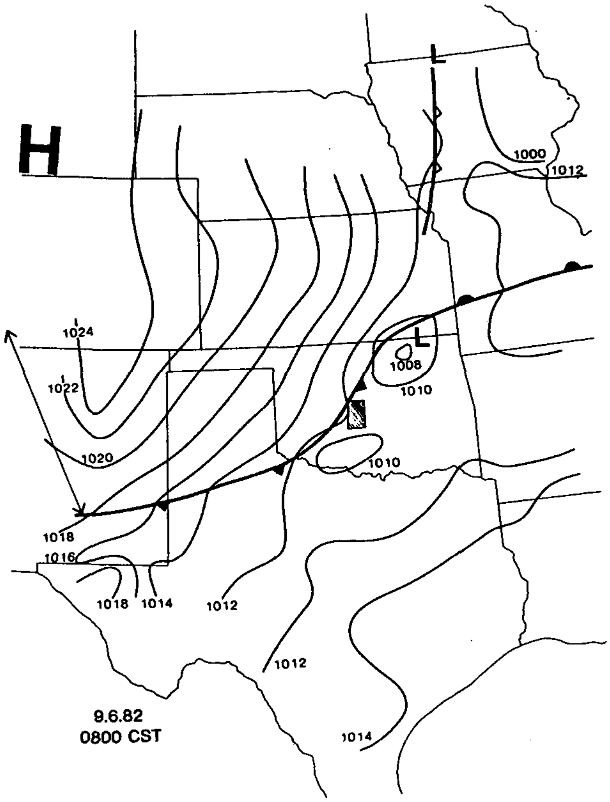


FIG. 3. MSL isobaric analysis for 0800 CST on 9 June 1982. The shaded rectangle indicates the location of stations in the NSSL network as detailed in Fig. 4.

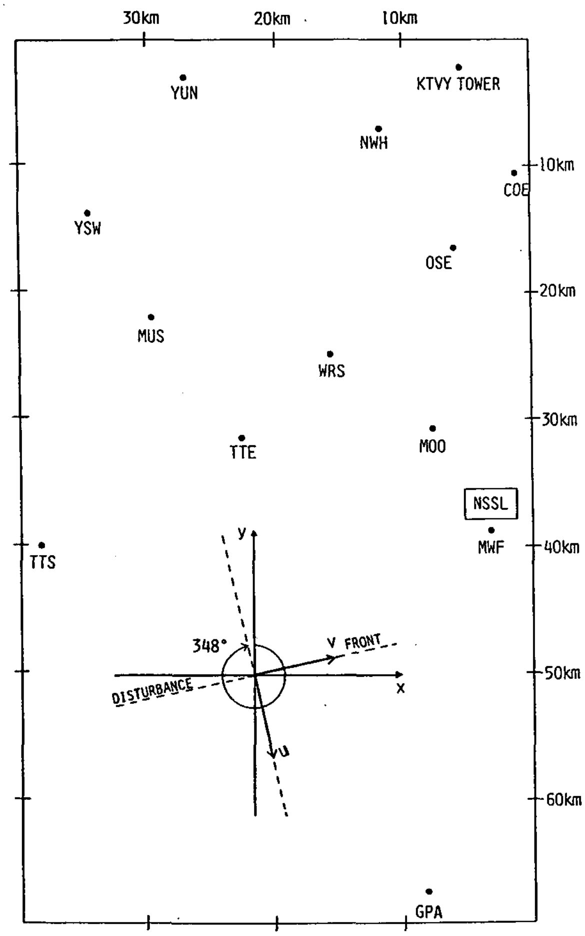


FIG. 4. Locations of the 12-station NSSL surface observational network including the KTVY television tower. The sketch shows the direction of the wind components u and v relative to the propagation direction of the disturbance.

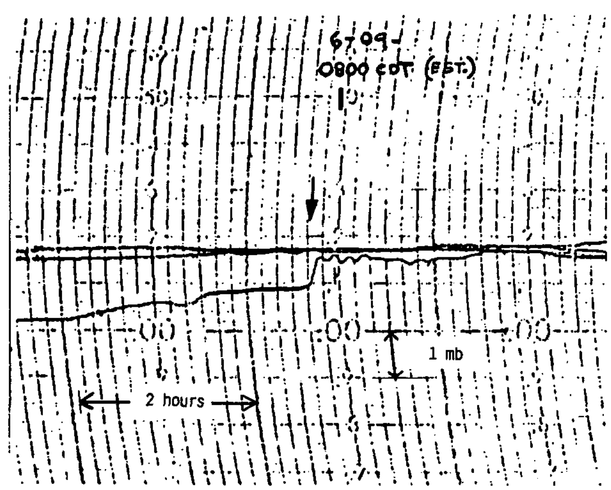


FIG. 5. Microbarograph trace from a station about 2.5 km south of NSSL on 9 June 1982. The arrow marks the calculated time of passage of the disturbance at 0642. Time and pressure increments are as labeled. The absolute pressure is unknown since the microbarograph had not been calibrated.

from its relatively stationary predisturbance value, both w fields showed upward motion, again as is observed in morning glories. Farther ahead, the two fields differ completely, as we might expect, since the two-dimensionality condition cannot be justified there.

Time series of wind speed and direction at a height of 6.3 m on the NSSL tower at Norman, Oklahoma, about 40 km south of the KTVY tower are summarized below and a section of these records is reproduced in Fig. 8. For several hours before the arrival of the disturbance until about 20 min before, the wind was southerly with a speed of $\sim$10 m s^{-1}. Thereafter, it decreased steadily in strength to almost calm just before the disturbance front passage, at which time it swung abruptly to 340° and increased sharply in strength to approximately 5 m s^{-1}. For the next 40 min it oscillated in direction between 340 and 180° and in strength, with a period of about 10 min, whereupon it reverted once again to southerly for a further 3 h. During the next hour it backed slowly and then at 1120, it changed abruptly to northerly, indicating the passage of the cold front itself. Taking into account the slightly larger predisturbance headwind at the KTVY tower location than at Norman, the wind perturbation at both places is essentially similar in structure. It is worth noting that

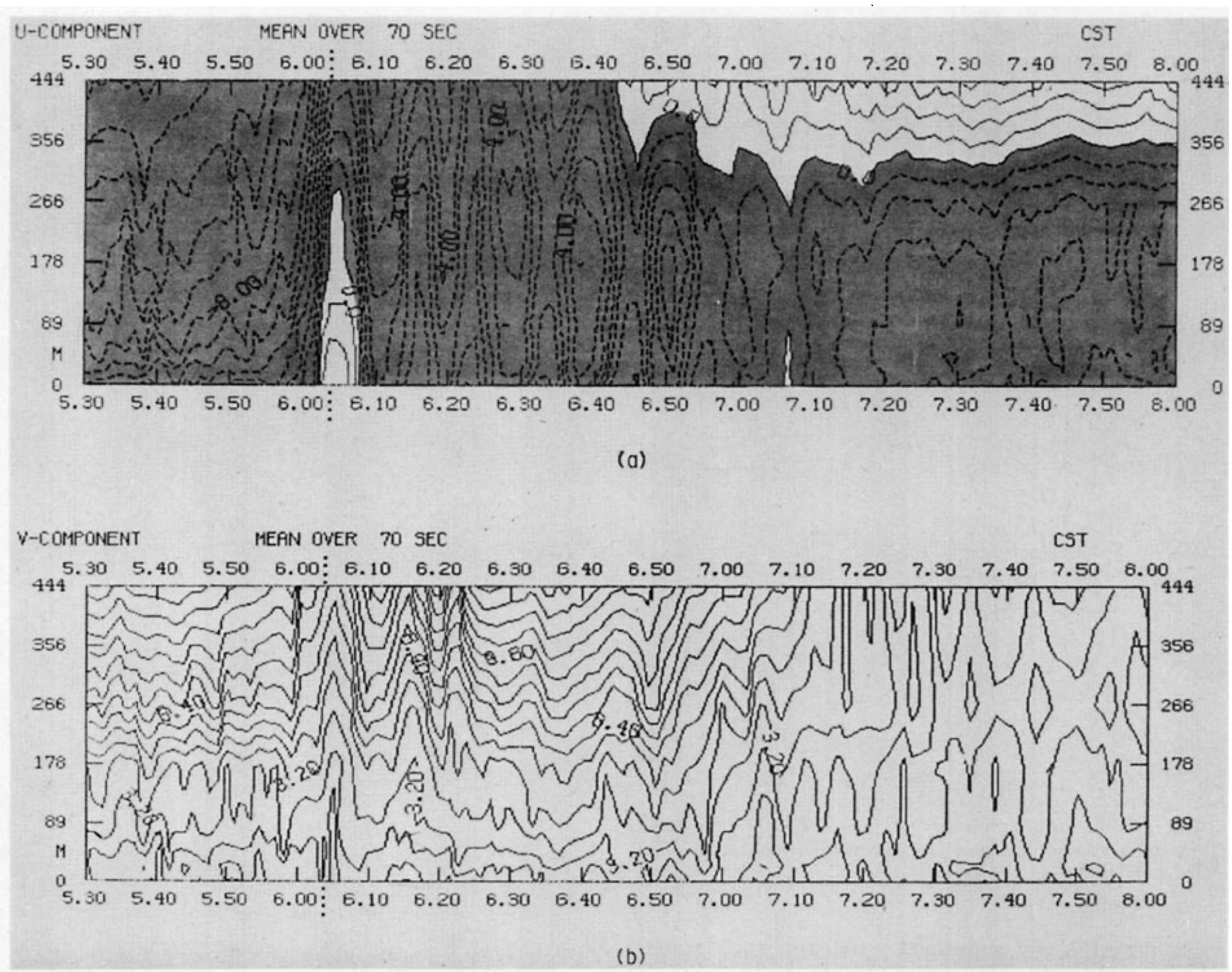


FIG. 6. Time–height cross sections of (a) the u and (b) the v isotachs derived from the KTVY tower data. The dotted lines indicate the calculated time of passage of the pressure jump as defined earlier. For the disturbance propagation speed of 16.5 m s^{-1}, 10 min of the abscissa corresponds to a horizontal distance scale of approximately 10 km. Negative isotachs are dashed and negative areas shaded.

the low-level wind component normal to and ahead of the disturbance is *toward* the disturbance, a feature characteristic of morning glories and of apparently similar surges; an outstanding example is the St. Louis pressure jump lines analyzed in detail by Shreffler and Binkowski (1981). As yet, the significance of this result is uncertain.

The time–height cross section of temperature and potential temperature from the KTVY tower is shown in Fig. 9. In the layer extending from approximately 90 to 250 m, the air was only slightly stable (lapse rate ~ 0.8 K/100 m); this would appear to explain the relative absence of signal in the layer following the disturbance passage, since an air parcel would need to be displaced vertically through at least 100 m to produce a 0.2 K temperature perturbation at its new level. The temperature cross section suggests that vertical displacements were only a fraction of this. Higher than 250 m, lapse rates were less and the predisturbance air is consequently more stable. Because of this, and probably also because of larger parcel displacements above this level, the temperature fluctuations are more significant. Moreover, there is evidence of negative correlation between these and the vertical velocity fluctuations, consistent with internal gravity wave motion (compare Fig. 9 with Fig. 7). The surface temperature rose steadily from about the time of the disturbance passage, presumably due to insolation after sunrise. There is no indication of significant mixing of warmer air downward by turbulence accompanying the disturbance as is often the case for morning glories (Clarke *et al.*, 1981, p. 1731); indeed, the potential isotherms show that the shallow nocturnal radiation inversion (below 90 m) remains intact for some time after the disturbance passage.

d. *Upper air data*

Three rawinsonde soundings are available in the vicinity of the disturbances: one from Oklahoma City

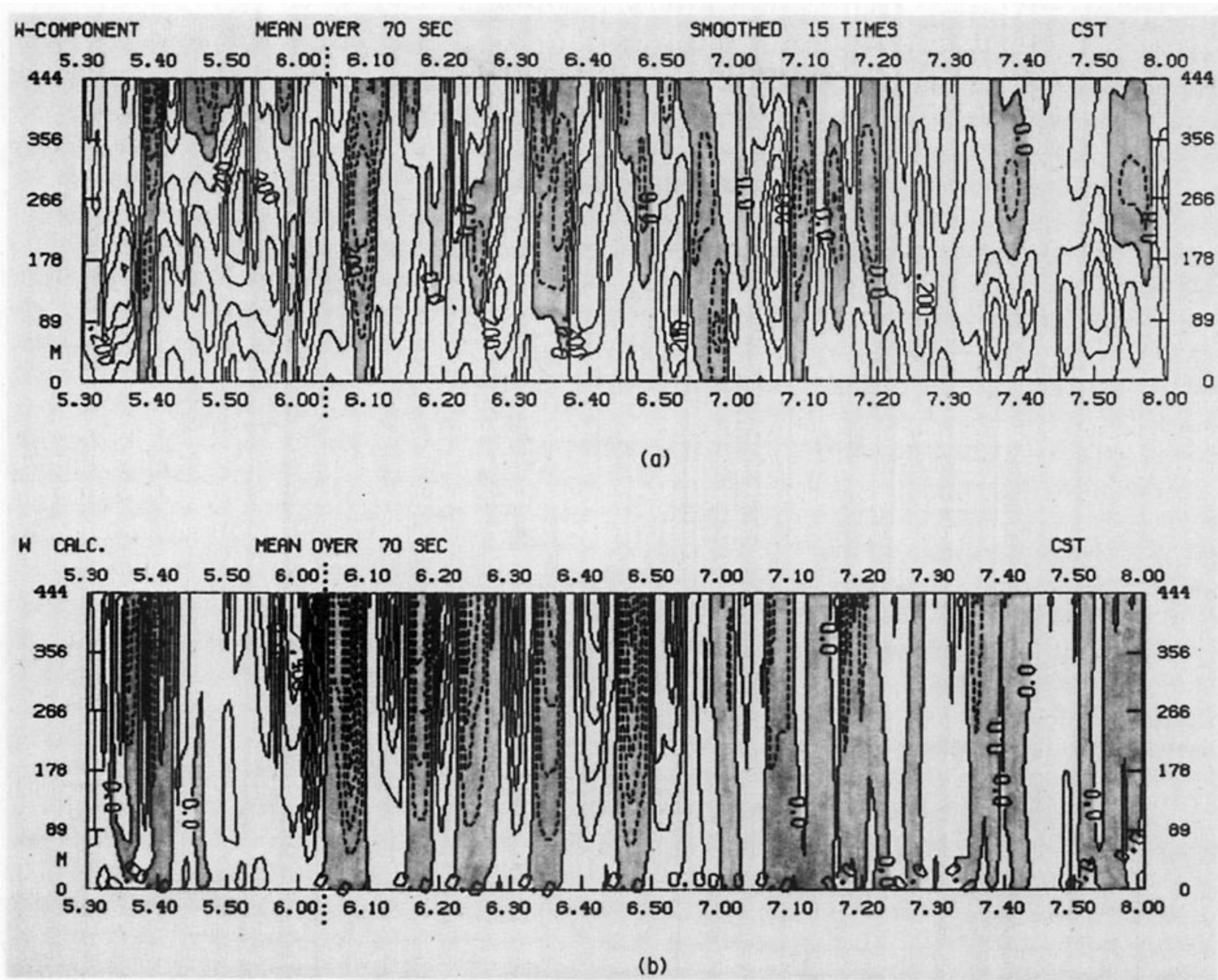


FIG. 7. As in Fig. 6 except for the w isotachs: (a) shows the measured w isotachs and (b) the ones calculated from the u-isotach field using the continuity equation and assuming two-dimensional flow.

at 0515 and two from Fort Sill, one at 0500 and the other at 0945. The first two of these were made, respectively, one hour and two-and-a-half hours before the passage of the disturbance front. The third was about two-and-a-quarter hours behind it, but was ahead of the surface cold front that followed the disturbance later in the morning (see Fig. 10). The time–height cross sections of the velocity component isotachs constructed from these soundings help to expose the change in wind structure following the disturbance passage. Consistent with the KTVY tower data at levels of overlap (i.e., below 444 m), the u cross section (Fig. 11a) shows a southerly[1] wind component at low levels (below about 1.5 km) ahead of the disturbance with a deep northerly component above. By 0945, the low-level southerlies have all but disappeared and a feature of special interest is the strong wind reversal that occurs in the layer between about 400 and 1600 m, with a wind maximum of 5.3 m s^{-1} in the direction of the disturbance at $\sim$1 km. It is tempting to associate this *internal surge* with the generation mechanism for the disturbance on the basis of its possible role analogous to the easterly sea breeze surge across Cape York Peninsula, which is the precursor of northeasterly morning glories in northern Australia. This possibility is considered further in the following section.

The winds normal to the disturbance propagation direction (Fig. 11b) are generally much stronger than the u component, with a maximum of 20.9 m s^{-1} at a height of about 1 km, and they are everywhere positive, i.e., essentially westerly.

The vertical distributions of virtual potential temperature θ_v derived from the three soundings are shown in Fig. 12, and the corresponding variation of Brunt–Väisälä period T_p computed for the Oklahoma

[1] Here, southerly and northerly are used loosely; strictly they refer to motion from 168 and 348 deg, respectively.

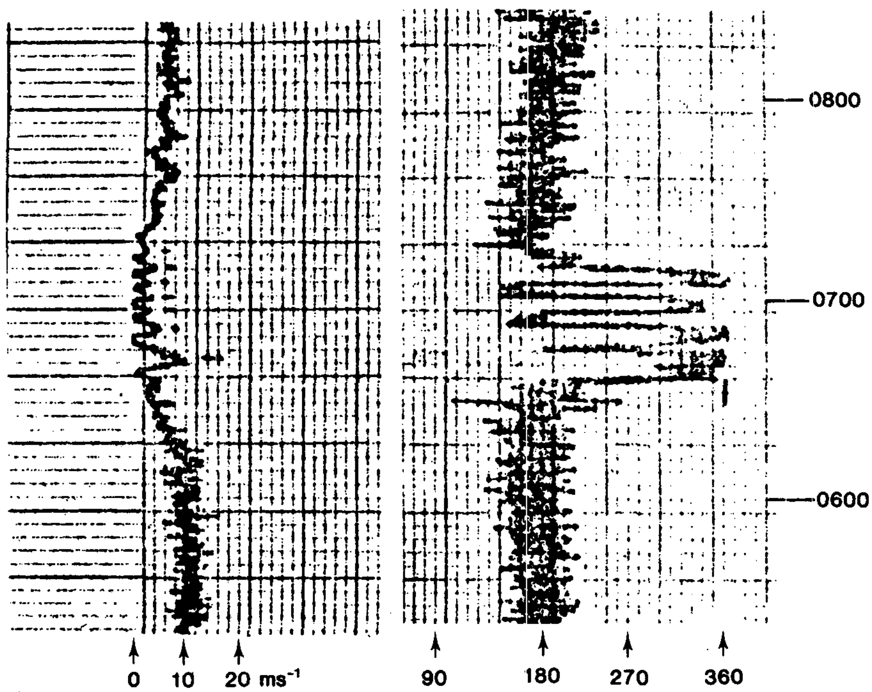
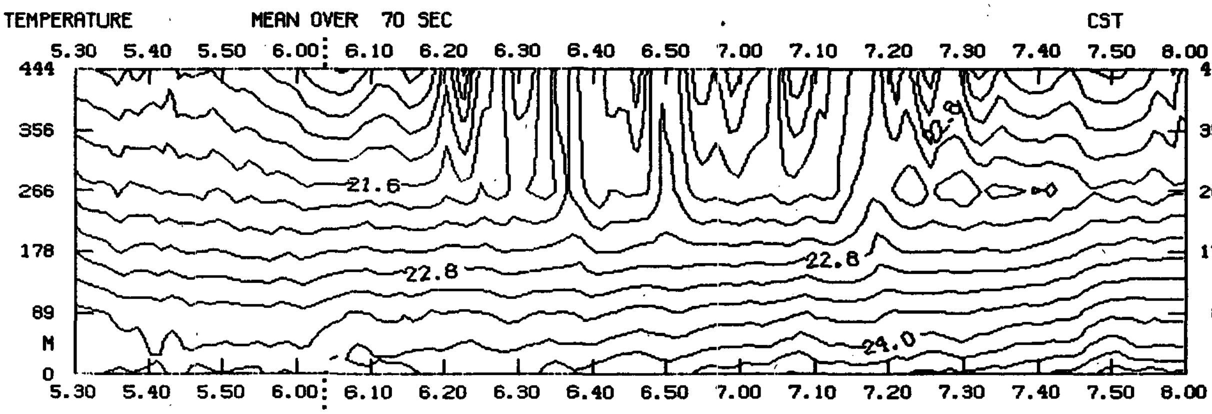
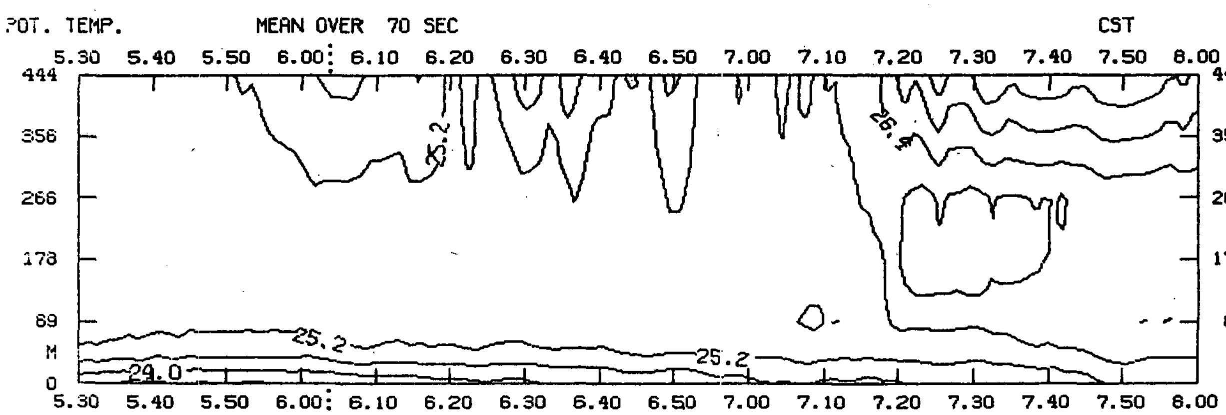


FIG. 8. Time series of wind speed and direction at a height of 6.3 m on the NSSL tower, about 40 km south of the KTVY tower.



(a)



(b)

FIG. 9. As in Fig. 6 except for (a) temperature and (b) potential temperature.

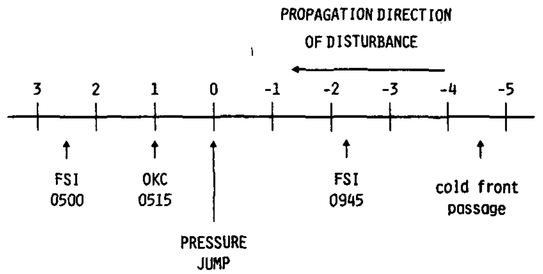


FIG. 10. Positions in time of the three rawinsonde soundings and the coldfront passage relative to the disturbance pressure jump.

City sounding, the closest predisturbance sounding to the disturbance front, is shown in Fig. 13 (here $T_p = 2\pi[\theta_v(z)/(gd\theta_v/dz)]^{1/2}$, where z measures height and g is the acceleration due to gravity). The two predisturbance soundings are very similar with strong stability ($T_p \sim 14.5$ min) between 1.4 and 5.5 km. These values are quite typical of the morning glory situation (see, e.g., Clarke *et al.*, 1981; Smith and Morton, 1984). Note that there is a very stable layer ($T_p \sim 4$ min) between about 600 and 800 mb.

The postdisturbance sounding is somewhat different below 4 km. In the first kilometer, the air has uniform θ_v and at most levels is considerably warmer than the predisturbance air, evidently a result of convective mixing. Between 700 m and 4 km, the air is cooler by as much as 4.5 K, the largest temperature decrease occurring at approximately 1.5 km. It follows that the internal "surge" is, indeed, cold and that at least at the time and location of the foregoing sounding, the cold air does *not* extend to the surface.

e. The disturbance origin

The satellite picture at 0801 shows that the disturbance cloud rolls were slightly curved as if they originated from a localized source in northern Kansas or southern Nebraska. Earlier pictures, of which Fig. 15 is typical, show two large thunderstorm complexes in the Midwest: one in southern Oklahoma, and a more extensive one centered over Missouri. On the

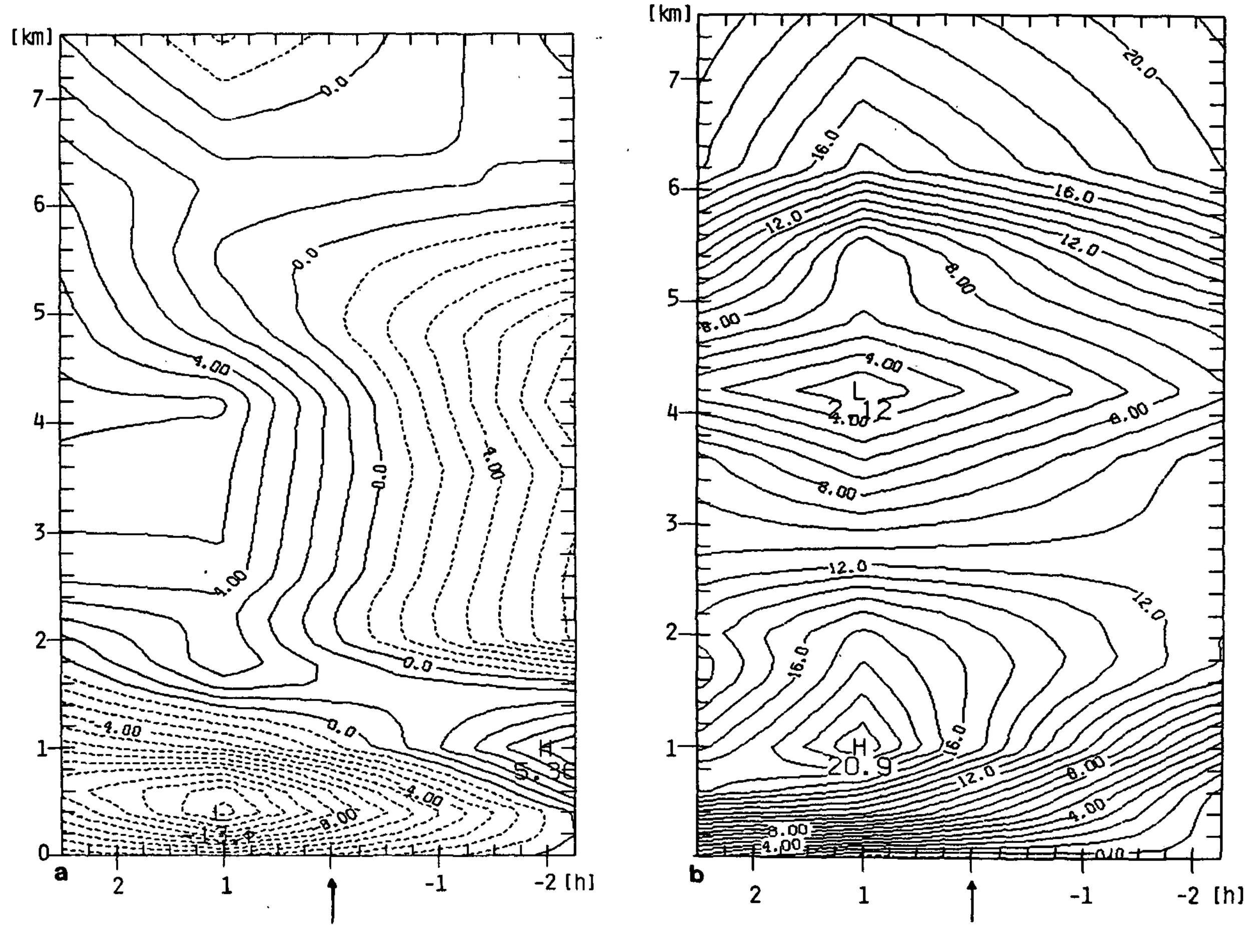


FIG. 11. Time–height cross section of (a) the u isotachs and (b) the v isotachs based on the three rawinsonde soundings (see Fig. 10). The arrow indicates the time of passage of the pressure jump.

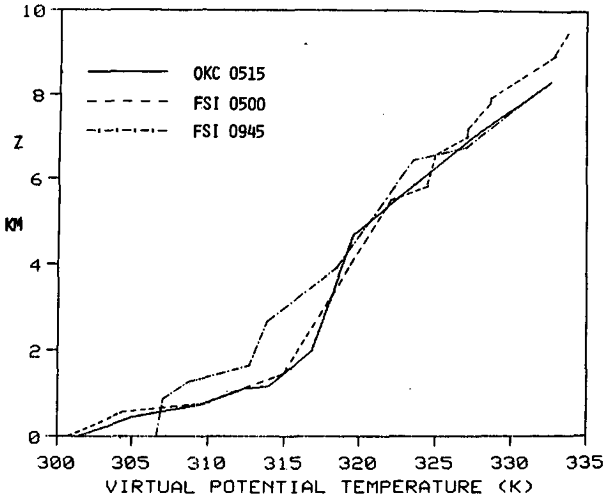


FIG. 12. Vertical profiles of virtual potential temperature θ_v derived from the three rawinsonde soundings.

western flank of the latter, rapidly growing cells were observed at about 2245 on the previous evening and by 2315 had formed an extensive anvil cloud over the eastern portion of the Kansas–Nebraska border, over 500 km north-northwest of Oklahoma City (Fig. 14). Radar observations from Grand Island, Nebraska, about 150 km northwest of the storms, showed that at this time the storms had reached extreme severity; the tops of several cells were in excess of 15 km, and echo intensities were indicative of the presence of hail. A hook echo was reported also. Extrapolation backwards to infer earlier positions of the disturbance front, assuming that its propagation speed remained uniform, shows that the disturbance front would have been in the location of these storms at about the time that the storms first appeared, evidence that these particular storms in some way initiated the disturbance (Fig. 15).

At least on the basis of satellite imagery, the wave clouds described here and their apparent thunderstorm origin show similarities with those described by Erickson and Whitney (1973), although in the latter case, the clouds appeared to be at middle tropospheric levels.

We turn now to the question of mechanisms. It is known that large thunderstorms are generally accompanied by a vigorous gust front, the leading edge of rain-cooled air at the surface, and that this cold outflow may move many tens of kilometers from the storm (Wakimoto, 1982; Weaver and Nelson, 1982). As suggested by Tepper (1950), if the gust front moves into a low-level stable layer at a speed less than the speed of propagation of long-wave disturbances on the layer, it will produce a wavelike disturbance moving ahead of it on the layer. Moreover, provided the stable layer acts as a duct, for example, if it underlies a sufficiently deep layer of weakly or neutrally stratified air as defined, say, by Maslowe and Redekopp (1980, Section 3), the disturbance may evolve ultimately into an internal undular bore. Christie *et al.* (1979) invoke this mechanism to account for the generation of nocturnal borelike disturbances that they have observed in surface pressure measurements in central Australia, and its efficacy has been demonstrated in laboratory experiments by Maxworthy (1980) and Smith *et al.* (1982). Indeed, on the basis of a large range of experiments, Maxworthy concludes that "if a given physical system is capable of supporting solitary wave motions then such motions will invariably arise from quite general excitations." In this connection, it should be remarked that Christie *et al.* and Maxworthy consider as solitary waves the constituent waves of an undular bore, on the basis of theoretical results concerning the evolution of finite amplitude long-wave disturbances from general initial states.

Recently Doviak and Ge (1984) have documented a single solitary wave disturbance, which appears to have been launched by a thunderstorm outflow in the manner described above and which moved at least 80 km away from the storm at a uniform speed of 13 m s^{-1}. In some cases, a thunderstorm outflow may be potentially warmer than the low-level stable layer and run above it. Indeed, an example of an intrusive cold outflow of this type, associated with a prefrontal squall line, is reported by Physick *et al.* (1984).

Because of the satellite data, it is, in our view, most probable that the Oklahoma morning glory described herein was generated by a cold air surge from the storms in southern Nebraska impinging on the preexisting low-level stable layer. The latter may have owed its existence to cold air outflows from storms over Oklahoma some 12 hours earlier. However, it would appear wrong to identify the internal cold air surge evident in the 0945 Fort Sill rawinsonde

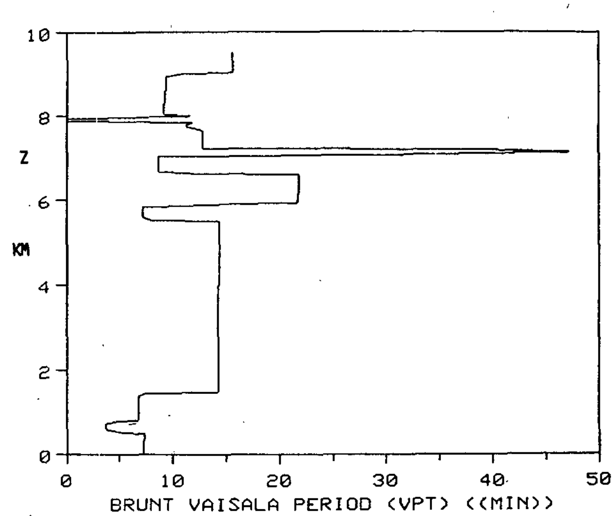


FIG. 13. Vertical profile of Brunt–Väisälä period T_p for the 0515 Oklahoma City sounding on 9 June 1982.

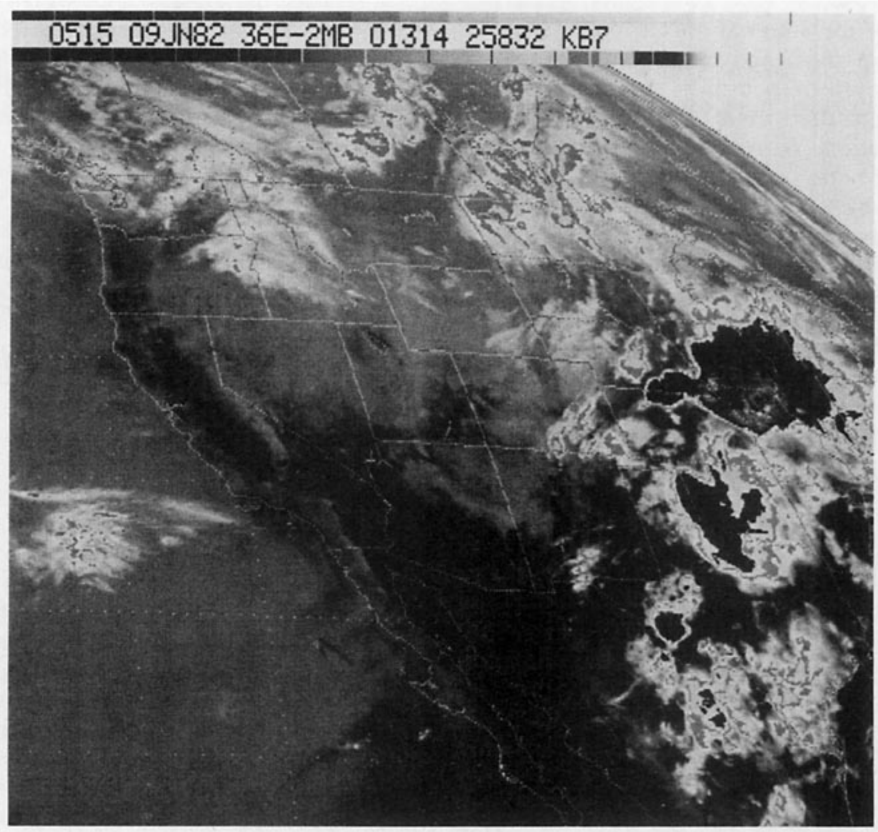


Fig. 14. IR satellite photo at 2315 CST (0515 GMT) on 8 June 1982 showing thunderstorms centered over Oklahoma and Missouri.

sounding with an elevated cold air outflow from the foregoing storms, since the winds at the surge levels have a substantial westerly component (Fig. 11b).

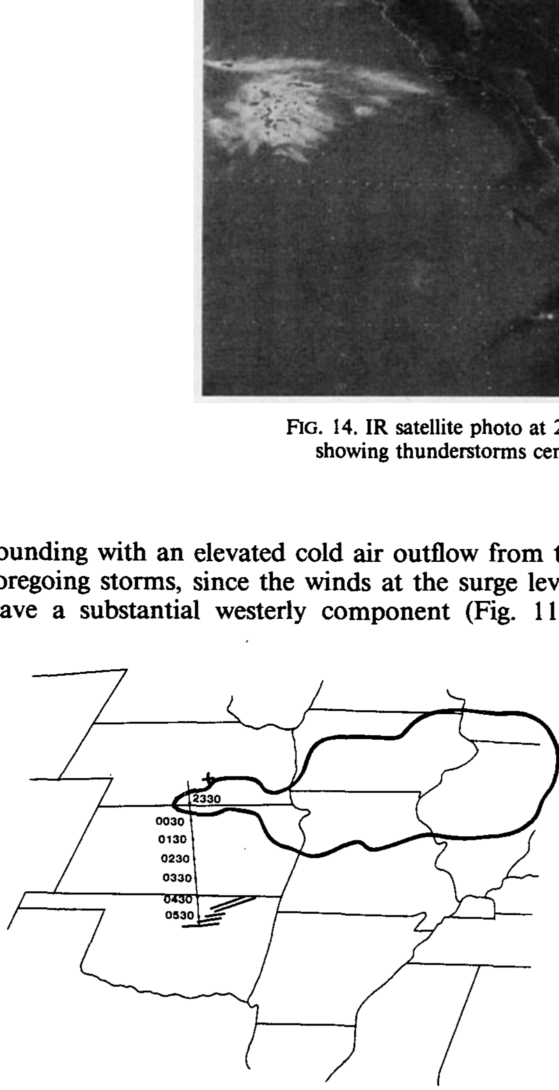


Fig. 15. Earlier inferred positions of the disturbance front obtained by backward extrapolation. The positions of the cloud lines are taken from the HR satellite picture at 0601; the position of the thunderstorm complex (thick solid contour) with the rapidly growing cell on its western flank is from the IR satellite picture at 2315 of the previous day (see Fig. 14). The cross indicates the location of the weather radar at Grand Island, Nebraska, referred to in the text.

Indeed, the wind at this location and time at a height of 1 km is approximately northwesterly with a speed of 7 m s^{-1} and Fort Sill is several hundred kilometers south-southwest of the location of the storms. It is, nevertheless, still possible that the internal cold surge evident in the 0945 Fort Sill sounding, whose existence may be a feature of air motion on a larger scale than the storms, could have been a key factor in generating the disturbance. A third possibility is that the waves originated as a result of frontal collapse as envisaged by Ley and Peltier (1978), but we have no way of assessing this from the available data.

3. Theoretical considerations

Benjamin (1966) shows that a steady internal undular bore can be viewed as a transition from a supercritical to subcritical flow state in analogy with the undular bore on a water surface. The role of the internal waves behind the bore is then to remove the energy liberated at such a transition. Except for exceedingly weak bores, these "bore waves" must be described in terms of finite amplitude theory. In the same paper, Benjamin presents a suitable theory of finite amplitude internal gravity waves of permanent form in a stably stratified fluid of finite depth. In a subsequent paper (Benjamin, 1967), he gives the analogous theory for internal waves on a shallow layer of stratified fluid above which there is an infinitely deep layer of neutrally stratified fluid, a flow configuration that more closely mirrors the atmospheric situation. With a straightforward extension to include the effects of ambient shear, the latter theory is found by Clarke *et al.* (1981) to provide satisfactory agreement in important respects with observations of the bore waves associated with two morning glory disturbances. A more detailed appraisal of the theory in relation to such disturbances is given by Noonan and Smith (1984). A comparison of the theory with the Oklahoma disturbance turns out to be of interest also and the results are summarized below.

In brief, the theoretical model relates to weakly nonlinear wave motions in a stably stratified shear flow of depth h with density and velocity structures $\rho(z)$ and $U(z)$, respectively, underlying an infinitely deep layer of homogeneous fluid with density $\rho(h)$ and moving with uniform speed $U(h)$. Here z measures vertically from the ground and by "weakly nonlinear" we imply that the wave amplitude a is small compared with h. It is assumed that each streamline is horizontal far upstream from the disturbance and that at a horizontal position x, measured in a reference frame moving with the disturbance at uniform speed c, the streamline has a vertical displacement $\psi(x, z)$ from its upstream height z. Disturbances are assumed to be independent of the cross-stream direction, normal to x and z. Then, in the lower layer, the vertical displacement is given by

$$\psi(x, z) = f(x)\phi(z), \tag{3.1}$$

where $\phi(z)$, normalized so that $\phi(h) = 1$, is determined by the solution of the linear eigenvalue problem for long waves, viz.,

$$[\rho(U - c_0)^2\phi_z]_z + \rho N^2\phi = 0, \tag{3.2}$$

subject to the boundary conditions

$$\phi(0) = 0 \quad \text{and} \quad \phi_z(h) = 0. \tag{3.3}$$

Here $N = [g(d\theta_v/dz)/\theta_v]^{1/2}$ denotes the Brunt–Väisälä frequency in the stable layer and c_0 is the linear phase speed of waves, the eigenvalues of which provide first approximations to possible values of c. Here we are interested only in the lowest eigenvalue, corresponding with the maximum value of c_0, and for which $\phi(z)$ increases monotonically from zero for $0 \leqslant z \leqslant h$. The function $f(x)$ satisfies a complicated equation whose coefficients depend on the linear eigenvalue and eigenfunction of the mode under consideration.

Benjamin (1976) shows that periodic (nonsinusoidal) solutions with wavelength λ exist for $f(x)$ having the form

$$f(x) = \frac{\frac{1}{4}a\sinh^2 p}{\cosh^2\left(\frac{1}{2}p\right) - \cos^2(\pi x/\lambda)}, \tag{3.4}$$

where a is the amplitude $f_{max} - f_{min}$ of the wave and p is a constant that depends on both a and λ and on the details of the linear eigenvalue problem. He shows also that a limiting case of (3.4) is the solitary wave solution

$$f(x) = \frac{a\lambda^2}{x^2 + \lambda^2}, \tag{3.5}$$

which represents a symmetrical "hump" profile with amplitude a and half-width λ. For either solution type, the propagation speed c is larger than that predicted by linear wave theory (i.e., c_0) by an amount which is an increasing function of a/h, and in the case of (3.5), λ is inversely proportional to a. Above $z = h$, the disturbance amplitude decays algebraically with height. For further details, refer to Clarke *et al.* (1981)[2] and/or Benjamin (1967).

From the predisturbance rawinsounding at Oklahoma City at 0515, the depth of the stable layer is about 1.4 km and the Brunt-Väisälä period averages approximately 7 min. With the observed wind component $U(z)$ and virtual potential temperature distribution $\theta_v(z)$, the calculated linear long-wave speed corresponding with the lowest eigenvalue c_0 is 10.8 m s^{-1}. The eigenvalue is found by integrating (3.2) forward in z by a Runge–Kutta method subject to the initial conditions $\phi(0) = 0$, $\phi'(0) = 1$ and successively adjusting c_0 until $\phi(z)$ attains its first maximum at $z = h$.

An estimate for a may be obtained from the rawinsonde data also. Neglecting mixing, the minimum vertical displacement necessary to produce cloud is deduced to be about 400 m from the undisturbed level of 650 m; above and below this level somewhat larger displacements are required, e.g., 480 m from 370 m and 1030 m from 740 m. It seems safe to infer that a/h is, therefore, at least 400/1400 = 0.29 and is possibly as high as 600/1400 = 0.43. For Benjamin's periodic solutions with wavelength 10 km, the corresponding wave speeds are 14.8 and 15.7 m s^{-1}, respectively, the higher value being a little less than, but close to, the observed speed of 16.5 m s^{-1}. For comparison, the calculated solitary wave speeds for the solution (3.2) are slightly lower than for the periodic solution: 13.9 m s^{-1} for $a/h = 0.29$ and 15.1 m s^{-1} for $a/h = 0.43$, and the corresponding half widths λ are 1.5 and 1.0 km, respectively. Hence, these solutions further underestimate the wave speed. In all of these comparisons, it should be borne in mind that the theory is worked out on the basis that $a/h \ll 1$, an assumption that is stretched in the foregoing calculations.

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[2] Attention is drawn to a number of misprints in this paper: Eqs. (A3) and (A4) should include factors $[1 - U(h)/c_0]^2$ in their denominators; $F(k)$ in (A5) should read $F(f)$; in Eq. (A9), $4\pi h/BL$ should be $4\pi/BLh$; the equation immediately above this should read $a \sinh p = 4\pi/BL$ and the expression immediately below should state "as $L/h \rightarrow \infty$."

The observed depth of the stable layer is larger than that of the typical Australian morning glory environment by a factor of about 1.5, implying on the basis of the theory that the wave amplitude increases more slowly with height and attains its maximum at a higher level. This may account partially for the relatively smaller surface disturbance associated with the Oklahoma bore system compared with a typical morning glory.

As in the case of the morning glory, the Brunt–Väisälä frequency is not zero above the stable layer, but probably half that in the stable layer itself. In such a situation, one might expect the waves to rapidly lose energy by upward radiation (see, e.g., Lindzen and Tung, 1976). However, Maslowe and Redekopp (1980) show that in contrast to infinitesimal waves, waves of sufficiently large amplitude may be ducted. The criterion they obtain for ducting is that $a/h > O(N_\infty/N)$, where N_∞ is the characteristic Brunt–Väisälä frequency above the stable layer. In the present case N_∞/N is about one-half, so that this criterion is approached when $a/h = 0.43$, the upper estimate for this quantity.

4. Conclusions

On the basis of the data available to us, we have shown that the Oklahoma wave disturbance of 9 June 1982 was, like morning glory disturbances of northern Australia, an internal undular bore propagating on a low-level stable layer and ducted by a deeper upper layer of near neutral stability. We have shown also that the observed phase speed of the component bore waves is in reasonable agreement with the speed calculated on the basis of Benjamin's (1967) weakly nonlinear wave theory.

There is good evidence from half-hourly satellite imagery that the disturbance was associated with a cluster of rapidly growing intense thunderstorm cells several hundred kilometers to the north. A plausible hypothesis is that the bore was generated as the cold air outflow from one or more of the storm cells moved into the low-level stable layer. That such generation is possible when the outflow moves more slowly than the speed of propagation of long-wave disturbances on the stable layer has been demonstrated in laboratory experiments, and essentially the same mechanism is believed to operate in the case of morning glory disturbances in northern Australia. Alternative possibilities for generation are discussed briefly, but the data are inadequate to permit firm conclusions to be drawn concerning these.

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