Tropical Cyclone Eye Dynamics[1]

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ABSTRACT

A new perspective of the dynamics of a tropical cyclone eye is given in which eye subsidence and the adiabatic warming accompanying it are accounted for directly from the equations of motion. Subsidence is driven by an adverse, axial gradient of perturbation pressure which is associated principally with the decay and/or radial spread of the tangential wind field with height at those levels of the cyclone where the tangential winds are approximately in gradient wind balance. However, this pressure gradient is almost exactly opposed by the buoyancy force field due to adiabatic warming. This corroborates with observational data.

The relationship between the present view of eye dynamics and those of Malkus and Kuo and a recent study by Willoughby is discussed in detail.

1. Introduction

An essential characteristic of a mature tropical cyclone is its warm eye. Without this, central surface pressures as low as are observed and the associated high wind speeds could not occur; indeed, calculations show that such low surface pressures cannot be achieved simply by moist adiabatic ascent of low-level air in the clouds surrounding the eye (Riehl, 1954, p. 316). It is generally accepted that the relatively high temperatures observed in the eye, especially in the middle and upper troposphere, are due to dry adiabatic warming through subsidence, although the small vertical velocities involved, of course, cannot be measured directly. Nevertheless, a completely satisfactory theory which accounts for this subsidence has not been given (Anthes, 1974, Section C3). A contribution to this problem forms the subject of this paper.

Malkus (1958) and Kuo (1959) have suggested that subsidence in the eye is a result of supergradient winds[2] in the neighborhood of the eye wall, inward of the radius of maximum tangential wind speed. They argue that these lead to an outward radial drift of eye air into the eye wall cloud and that, by continuity, this air is replaced by subsidence within the eye. The existence of supergradient winds in a region extending several kilometers inside the radius of maximum wind has, indeed, been deduced from a composite analysis of observational data from many storms (Gray and Shea 1973; see Figs. 8–11), and Gray and Shea conclude that their data support the Malkus-Kuo hypothesis. However, data from individual storms show that gradient wind balance is closely satisfied over much of the storm at aircraft penetration levels[3] (Hawkins and Rubsam, 1968, p. 625; R. C. Sheets, personal communication) and it is possible that the relatively large gradient wind imbalance deduced by Gray and Shea is an artifice of their compositing procedure (Willoughby, 1979, p. 3177). In this paper we show that one can account for subsidence directly from the governing equations of motion, even when the horizontal winds are in close (but not exact) gradient wind balance.

2. Vortex-induced subsidence

A common (but not universal) feature of vortex flows is the occurrence of axial stagnation in the meridional motion, with reversed flow along all or part of the axis and a region surrounding it. This type of behavior is observed, for example, in dust devils (Sinclair, 1973) and in both laboratory simulations (Fitzgarrald, 1973) and numerical simulations (Smith and Leslie, 1975) of these vortices. The reasons are succinctly described by Morton (1966) in the context of "long-thin" vortices and the ideas involved have proved useful in understanding many aspects of vortex flows. It is shown below that they remain valid for "broad-shallow" vortices, including tropical cyclones, even though these are in hydrostatic balance to a high degree of accuracy.

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[2] Supergradient (subgradient) flow is defined when the sum of the mean centrifugal and Coriolis forces are larger (smaller) than the radial pressure gradient.

[3] These are mostly confined to pressure levels above about 400 mb.

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We consider an axisymmetric mean vortex for which the radial and vertical momentum equations are

$$\frac{Du}{Dt} - \frac{v^2}{r} - fv = -\frac{1}{\rho}\frac{\partial p_T}{\partial r} - F_1, \quad (1)$$

$$\frac{Dw}{Dt} = -\frac{1}{\rho}\frac{\partial p_T}{\partial z} - g - F_3, \quad (2)$$

respectively, where u, v and w are the *azimuthal-mean* velocity components in a cylindrical coordinate system r, ϕ, z, with its axis vertical and z measuring upward from the surface; t is the time, f the Coriolis parameter—assumed constant, p_T the mean total pressure, ρ the air density, g the acceleration due to gravity;

$$F_1 = \frac{1}{r}\frac{\partial}{\partial r}(\overline{ru'^2}) + \frac{\partial}{\partial z}(\overline{u'w'}) - \frac{\overline{v'^2}}{r}, \quad (3)$$

$$F_3 = \frac{1}{r}\frac{\partial}{\partial r}(\overline{ru'w'}) + \frac{\partial}{\partial z}(\overline{w'^2}), \quad (4)$$

are the radial and vertical components of the eddy stress, respectively, where the overbar denotes an azimuthal mean and a prime denotes a departure therefrom; and $D/Dt \equiv \partial/\partial t + u\partial/\partial r + w\partial/\partial z$.

It is convenient to remove from these equations a hydrostatic pressure distribution $p_0(z)$, which might be the ambient hydrostatic pressure field far from the storm, say, $p_a(z)$, or the horizontal areal average pressure $\langle p \rangle$, but in practice, these do not differ significantly and $p_a(z)$ is often a more convenient choice. Then with $p = p_T - p_0(z)$ and $\rho_0(z) = -g^{-1}dp_0/dz$, the equations become

$$\frac{Du}{Dt} - \frac{v^2}{r} - fv = -\frac{1}{\rho}\frac{\partial p}{\partial r} - F_1, \quad (5)$$

$$\frac{Dw}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \sigma - F_3, \quad (6)$$

where $\sigma = g(\rho_0 - \rho)/\rho$ is the *buoyancy force* per unit mass. In the fluid dynamics literature[4], p is usually referred to as the *dynamic pressure*[5] as it is this part of the pressure field which, in the absence of density variations, gives rise to motion. Meteorologists generally prefer the pseudonym *perturbation pressure* and we shall adhere to this terminology below. It is important to note that the definitions of perturbation pressure and buoyancy force are not unique and the partition of the vertical force field, $-\rho^{-1}\partial p_T/\partial z - g$, between the vertical gradient of perturbation pressure and the buoyancy force depends on the choice of reference pressure distribution $p_0(z)$. Eq. (6) shows that *in a state of no vertical motion, balance exists between the perturbation pressure gradient and the buoyancy force*, whereas *accelerated vertical motion is driven by an imbalance between the vertical gradient of perturbation pressure, the buoyancy force and the vertical stress.*

In concentrated vortex flows the radial acceleration in Eq. (5) is dominated over much of the flow by the centripetal contribution $-v^2/r$, and this results in a tight coupling between the azimuthal and axial components of motion through the pressure field. In such problems, it can be misleading to separate cause and effect artificially, but it is enlightening to diagnose the dynamical constraints of the equations of motion as follows.

We consider motion along the vortex axis ($r = 0$), where we can estimate the perturbation pressure by integrating $\rho \times$ Eq. (5) with respect to radius, from the axis to some large radius R at which the perturbation pressure can be assumed negligible. Differentiation with respect to height and division by the density then gives

$$-\frac{1}{\rho}\frac{\partial p}{\partial z}\bigg)_{r=0} = \frac{1}{\rho}\frac{\partial}{\partial z}\int_0^R \rho\left(\frac{v^2}{r} + fv\right)dr - \frac{1}{\rho}\frac{\partial}{\partial z}\int_0^R \rho\frac{Du}{Dt}dr - \frac{1}{\rho}\frac{\partial}{\partial z}\int_0^R \rho F_1 dr. \quad (7)$$

In particular, if the swirling motion is in gradient wind balance,

$$-\frac{1}{\rho}\frac{\partial p}{\partial z}\bigg)_{r=0} = \frac{1}{\rho}\frac{\partial}{\partial z}\int_0^R \rho\left(\frac{v^2}{r} + fv\right)dr. \quad (8)$$

The inner core of a tropical cyclone is essentially a high Rossby number flow in which the centrifugal acceleration is much larger than the Coriolis acceleration; hence, the right-hand side of Eq. (8) is dominated by the first term, and it follows that adverse axial (perturbation) pressure gradients ($-\rho^{-1}\partial p/\partial z < 0$) occur at levels where the tangential velocity field decays and/or spreads with height, i.e.,

$$\frac{\partial}{\partial z}\int_0^R (\rho v^2/r)dr < 0.$$

In a tropical cyclone, the decay of tangential velocity with height is especially prominent in the upper troposphere (see, e.g., Shea and Gray, 1973, Fig. 10) and we might surmise from Eq. (8) that $\rho^{-1}\partial p/\partial z$ will be large at these levels, say, above 500 mb. Haurwitz (1935) has shown that the vertical force balance in a tropical cyclone is very close to hydrostatic so that this large perturbation pressure gradient must be very nearly balanced by the buoyancy force in Eq. (6), requiring large temperature differences above 500 mb. Such large temperature differences are, indeed, observed at these levels. Of course, hydrostatic balance is not exact, except possibly at isolated points, but since the time scale for tropical cyclone intensification (at least half a day) is very

[4] See, e.g., Lighthill (1963, p. 10).

[5] Batchelor (1970, p. 176) chooses the pseudonym *modified pressure*.

long compared with the periods of non-hydrostatic inertia-gravity waves (typically of order $2\pi/N$, where N is the Brunt-Väisälä frequency),[6] vertical accelerations are orders of magnitude less than either the perturbation pressure gradient or the buoyancy force. It is therefore inaccurate to calculate the vertical velocity by integrating the vertical momentum equation. Accordingly, most numerical models of tropical cyclones realistically assume that hydrostatic balance is exactly satisfied, in which case the vertical velocity may be obtained by integrating the continuity equation (see, e.g., Rosenthal, 1970, p. 109). However, the vertical velocity also must be consistent with the thermodynamic equation which, for a Boussinesq fluid, can be expressed in the form

$$\frac{D\sigma}{Dt} + N^2 w = Q, \tag{9}$$

where Q represents diabatic sources (or sinks) of mean buoyancy together with the contribution due to eddy heat flux convergence.

As an example, if a tropical cyclone intensifies through successive states of approximate gradient wind balance, the buoyancy force on the axis is equal to the perturbation pressure gradient calculated from Eq. (8), and hence the vertical velocity from Eq. (9) is given by

$$w = \frac{1}{N_e^2}\left(Q - \frac{\partial\sigma}{\partial t}\right), \tag{10}$$

where $N_e^2 = N^2 + \partial\sigma/\partial z$ is the Brunt-Väisälä frequency in the eye.

Thus, *we may regard subsidence as associated with the developing axial pressure gradient and, provided the rate of development is small compared with the Brunt-Väisälä frequency, as it is in the eye of a tropical cyclone, the rate of subsidence is just that required to warm the air to the degree that its buoyancy remains in close hydrostatic balance with the pressure gradient.*

Clearly, in the absence of effects representing a sink of heat (i.e., $Q \equiv 0$), the presence of vertical motion in the mature (steady) state is not compatible with a steady, pressure gradient-buoyancy force balance.[7] This supports the deduction by Gray [reported in Gray and Shea (1973, p. 1569)] based on an observation during a flight into the center of a quasi-stationary storm (Hurricane Gladys, 20 September 1964, central pressure 960 mb) where the warm center air was apparently stagnant. Gray notes that "without storm motion there was no need to cycle air through the storm's center and that in comparison to eye ventilation, the radiation and purely turbulent diffusion heat losses were small."

The observations reported by Gray and Shea (1973) indicate that in general, "the eye is continually ventilated and reforming itself by new sinking and warming as it moves." This is again consistent with Eq. (10) which shows that even in the steady state, subsidence will occur when there is a sink of buoyancy in the eye. In this situation, the rate of subsidence is that required to produce just enough adiabatic warming to compensate for the cooling and hence maintain the buoyancy field in hydrostatic balance with the perturbation pressure gradient associated with the vortex.

3. Some deductions using observational data

It is instructive, using observational data, to estimate the axial perturbation pressure gradient on the basis of the centrifugal term alone in Eq. (7). We do this below using the distribution of mean tangential wind speed with height given by Shea and Gray (1973, Fig. 10) and the Tampa eye sounding reported by Riehl (1948). The latter is used to estimate the density at various levels in the eye as well as thicknesses between various pressure levels.

For analytic convenience we assume a Rankine-combined vortex with mean tangential velocity profile

$$v(r,z) = \begin{cases} v_m(z)r/r_m, & \text{if } 0 \le r < r_m \\ v_m(z)r_m/r, & \text{if } r \ge r_m, \end{cases}$$

where the maximum speed at height z, $v_m(z)$, is obtained from the data. If radial density variations are ignored, we find

$$\int_0^R \frac{\rho v^2}{r}\,dr = \rho v_m^2\left[1 + \frac{1}{2}\left(\frac{r_m}{R}\right)^2\right]$$
$$\approx \rho v_m^2, \quad \text{if } r_m \ll R.$$

For our purpose, the approximation will suffice since with r_m typically 30 km, it gives 98% accuracy for R as small as 150 km. We then estimate the contribution to $-\rho^{-1}(\partial p/\partial z)_{r=0}$ using the formula $\bar{\rho}^{-1}\Delta(\rho v_m^2)/\Delta z$, where $\bar{\rho}^{-1}$ is the average density between two pressure levels, Δz the thickness between these levels and $\Delta(\rho v_m^2)$ the difference between values of the integral at these levels. The results are listed in Table 1, together with averages for the buoyancy force for the same pressure intervals $\bar{\sigma}$, computed from the Tampa eye sounding. As anticipated, the centrifugal term in Eq. (7) gives the largest contributions to the vortex-induced pressure gradient

[6] Assuming the eye to be in approximate solid body rotation and the Brunt-Väisälä frequency to be uniform, inertia-gravity waves have frequencies lying in the range $\zeta_a \rightarrow N$, where ζ_a is the absolute vorticity in the eye, typically 10^{-3} s^{-1}, and N is typically 10^{-2} s^{-1}. Therefore, the period of these waves ranges between about 10 min and 2 h. However, only the shorter period waves, with frequencies of order N, are appreciably non-hydrostatic.

[7] This is in marked contrast to a "two-cell" vortex in a homogeneous fluid where the adverse axial pressure gradient is unopposed by buoyancy and drives a circulation in the inner cell even in the steady state.

TABLE 1. Estimated contribution to the axial dynamic pressure gradient from the centrifugal term in Eq. (5) in various pressure intervals and mean buoyancy force in these intervals obtained from the Tampa eye; see text for details.

Pressure interval (mb)	$\frac{1}{\bar{\rho}}\frac{\Delta}{\Delta z}(\rho v_m^2)$ (m s^{-2})	$\bar{\sigma}$ (m s^{-2})
900–700	0.37	0.10
700–500	0.30	0.26
500–400	0.44	0.36
400–300	0.40	0.39
300–200	0.09	0.46

below 500 mb, and for the Tampa sounding at least, it accounts broadly for the dynamic pressure gradient required to balance the buoyancy force between 700 and 300 mb. At lower levels, the centrifugal term is much larger than the buoyancy force and for hydrostatic balance to obtain, it must be opposed by the last two terms in Eq. (7). Estimates for these require a much more detailed analysis of the data, but qualitative arguments show that they must act together to oppose the centrifugal term. Thus, at low levels, the radial flow is inward ($u < 0$), but experiences strong deceleration ($Du/Dt > 0$) as it approaches the radius of maximum wind; it is also subject to an outward frictional force ($F_1 > 0$). Above the inflow layer the radial velocity, and hence Du/Dt and F, are small, so that the integrals in the last two terms of Eq. (7) decrease with height to give a positive contribution to the axial dynamic pressure gradient force. In the outflow region above 300 mb, Du/Dt is positive, and since it is generally small in middle levels, we anticipate that

$$\partial\left[\int_0^R \rho(Du/Dt)dr\right]\Big/\partial z > 0,$$

so that the penultimate term in Eq. (7) has the same sign as the centrifugal term and gives a negative contribution to the axial dynamic pressure gradient. It is also possible that the Coriolis term is comparable with the centrifugal term, since the tangential velocities are much weaker in the outflow region than at low and middle levels.

TABLE 2. Subsidence rates ($-w$) for various eye cooling rates Q.

Q (K h^{-1})	$-w$ (m s^{-1})
0.1	0.007
0.25	0.02
1.0	0.07
4.0	0.28

Estimates for the rate of subsidence, using the formula $w = -Q/N_e^2$ [obtained from Eq. (10) with $\partial/\partial t \equiv 0$], are given in Table 2 for a range of Q values and for N_e^2 typical of the Tampa eye sounding between 300 and 400 mb, i.e., $2\pi N_e^{-1} = 9.5$ min. It is not easy to determine an appropriate value for Q for a given storm at a given time using hitherto published data, but values of cooling rates of $\sim$1 K h^{-1} lead to subsidence rates of order 0.1 m s^{-1}, consistent with estimates by Malkus (1958, p. 341). Such a value of Q is about one-quarter the rate of heating in the eye wall clouds due to latent heat release (Anthes, 1974, Table 3). If Q is assumed to incorporate the bulk effects of eye ventilation caused by storm motion, as well as turbulent mixing across the eye wall, evaporation of hydrometeors and radiation effects, cooling rates of this order of magnitude would appear to be not unreasonable.

4. Discussion of the Malkus-Kuo theory and conclusions

Malkus (1958) and Kuo (1959) suggest that eye subsidence is a manifestation of radial acceleration due to the existence of supergradient winds in the neighborhood of the eye wall; these, they hypothesize, are caused by the inward turbulent flux of cyclonic angular momentum inside the radius of maximum winds, but Gray (1967) argues that vertical transport of angular momentum by the eye wall clouds may also play an important role. The idea is that the additional azimuthal-mean centrifugal force $\overline{v'^2}/r$, due to the tangential velocity fluctuation v', makes an important contribution to the azimuthal-mean momentum equation and precludes gradient wind balance. The arguments are best exposed by considering the following form of Eq. (5)[8];

$$\frac{Du}{Dt} = \underbrace{\left(\frac{v^2}{r} + \frac{\overline{v'^2}}{r} + fv\right)}_{\text{A}} - \underbrace{\left(\frac{v_{\rm gr}^2}{r} + fv_{\rm gr}\right)}_{\text{B}} - \underbrace{\left(\frac{1}{r}\frac{\partial}{\partial r}(r\overline{u'^2}) + \frac{\partial}{\partial z}(\overline{u'w'})\right)}_{\text{C}}, \tag{11}$$

where $v_{\rm gr}$ denotes the gradient wind defined by $\rho^{-1}\partial p/\partial r = v_{\rm gr}^2 r^{-1} + fv_{\rm gr}$. While there is some uncertainty of interpretation, the Malkus-Kuo theory appears tacitly to ignore the stress term C in Eq. (11) and to assume that term A exceeds term B by the amount $\overline{v'^2}/r$, so that the centrifugal force fluctuation drives an outward radial acceleration across the eye wall. By continuity, this would lead to subsidence in the eye. This picture may be broadly correct, but the underlying assumptions are *non sequitur*; there is no basis whatever for assuming

[8] Recall that we have omitted bars on mean velocities—see Section 2.

that $\overline{v'^2/r}$ leads to an imbalance of radial forces—one might equally suppose that the radial acceleration is very small and that v is accurately determined by Eq. (11) with the left-hand side equal to zero! Of course, any radial acceleration which does exist must be consistent with Eq. (11).

Even if the radial acceleration is positive, it does not necessarily follow that eye air is being accelerated outward; it is quite possible that inward flowing air is being decelerated inward. Thus the statement by Gray and Shea (1973, p. 1571) that "inside the radius of maximum wind, especially in the lowest layer, the winds are supergradient, supporting Malkus' (1958) hypothesis" is not correct; their observation neither supports nor detracts from this hypothesis. Nevertheless, a region of supergradient winds *is* expected just outside the eye, where the low-level inflow is decelerated as it ascends the ring of clouds surrounding the eye.

In earlier sections we show that eye subsidence is driven by an adverse, axial, perturbation pressure gradient which at middle levels of the cyclone is associated principally with the decay with height, and/or the radial spread, of the tangential wind field. The subsidence leads to adiabatic warming at a rate which is just sufficient to generate and maintain the buoyancy force in close hydrostatic balance with it. By continuity, the subsiding air must eventually flow outward and be accepted by the eye wall clouds, and the radial acceleration it experiences must be positive, implying supergradient winds, and must satisfy Eq. (17). This picture is consistent with observational data.

Because of the tight dynamic coupling between the meridional and tangential components of motion through the pressure field, it is equally acceptable to regard eye subsidence as the result of a positive radial acceleration due to an imbalance of forces on the right-hand side of Eq. (11). Subsidence follows from continuity and the vertical velocities involved must be such that they generate buoyancy forces in hydrostatic balance with the perturbation pressure gradients which, in turn, depend *inter alia* on the spatial distribution of tangential velocity. This alternative view contains the Malkus-Kuo theory but the latter fails to take account of the overall constraints. Moreover, it is not possible to infer, as do Malkus and Kuo, that "suction" by the eye wall clouds is *caused by* the turbulent mixing of angular momentum, or by the vertical transport of angular momentum as postulated by Gray, even though the centrifugal force fluctuation in Eq. (11) may be important. Such processes may simply result in a different balanced vortex state to that which would obtain in their absence. Note, however, that these remarks do not call into question the reasons advanced by Kuo (1959) for the existence of a calm eye in tropical cyclones.

5. Relationship with Willoughby's theory

In a recent study of the meridional circulation in hurricanes, Willoughby (1979) finds that ". . . subsidence in the eye is forced by radial gradients of convective heating," a result not obviously in accord with the interpretation given in this paper. However, as is shown below, the two studies are complementary and not in conflict. Willoughby's analysis is more complete in the sense that it provides a recipe for solving the equations for a balanced vortex to obtain the instantaneous meridional circulation pattern for arbitrarily prescribed distributions of mean buoyancy and tangential velocity in hydrostatic and gradient (and hence thermal) wind balance with each other. Accordingly, his conclusion regarding subsidence is a global one which reflects the way in which the vortex is forced. In contrast, our interpretation gives consideration to local force balances in the eye, taking into account the same overall constraints included in Willoughby's theory. To show this, it is instructive to consider briefly Willoughby's approach. Essentially he considers an *evolving* axisymmetric mean vortex in thermal wind balance and described by the equations (in our notation),

$$\frac{v^2}{r} + fv = \frac{1}{\rho_0}\frac{\partial p}{\partial r}, \tag{12}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r} + w\frac{\partial v}{\partial z} + \frac{uv}{r} + fu = 0, \tag{13}$$

$$0 = -\frac{1}{\rho_0}\frac{\partial p}{\partial z} + \sigma, \tag{14}$$

$$\frac{\partial \sigma}{\partial t} + u\frac{\partial \sigma}{\partial r} + w\tilde{N}^2 = Q, \tag{15}$$

$$\frac{\partial}{\partial r}(\rho_0 ru) + \frac{\partial}{\partial z}(\rho_0 rw) = 0, \tag{16}$$

where $\tilde{N}^2 = N^2 + \partial\sigma/\partial z$. Using a technique introduced by Eliassen, he forms the time derivative of the azimuthal vorticity (i.e., the thermal wind) equation, i.e.,

$$\frac{1}{\rho_0}\frac{\partial}{\partial z}\left[\rho_0\left(\frac{2v}{r} + f\right)\frac{\partial v}{\partial t}\right] = \frac{\partial}{\partial r}\left(\frac{\partial \sigma}{\partial t}\right), \tag{17}$$

from which time derivatives can be eliminated using Eqs. (2) and (4). Then, the introduction of a streamfunction ψ, for the meridional components of the flow such that $u = -(r\rho_0)^{-1}\partial\psi/\partial z$, $w = (r\rho_0)^{-1}\partial\psi/\partial r$ leads to a diagnostic equation for ψ which can be solved for a specified heating source Q and for given spatial distributions of v and σ. It is necessary that v and σ are in thermal wind balance and that they satisfy conditions which ensure that the flow is everywhere centrifugally and statically stable, and hence that the equation for ψ is elliptic.

In this paper we have studied the local force fields associated with eye motions and the constraints on these fields, based essentially on Eqs. (12), (14) and (15), or generalizations thereof. Hence our interpretations are valid also in Willoughby's theory. On the other hand, Willoughby observes that the source term in his equation for ψ is $\partial Q/\partial r$ and concludes that "in the absence of Fickian diffusion of momentum and heat, both deep inflow in the outer vortex and subsidence within the eye are forced by radial gradients of convective heating." This represents a global view of meridional motion and one which certainly takes account of all the constraints. Nevertheless, it may be worth pointing out that Willoughby's theory applies strictly to evolving vortices, otherwise both sides of Eq. (17) are identically zero, and the diagnosed circulations would appear to depend on the rate of evolution; i.e., on the degree to which the initial fields chosen for v and σ are close to a steady state for the prescribed distribution of Q. Indeed, if a steady state exists, Eqs. (2) and (4) can be solved directly for u and w, i.e.,

$$u = \frac{Q\dfrac{\partial M}{\partial z}}{\dfrac{\partial \sigma}{\partial r}\dfrac{\partial M}{\partial z} - \tilde{N}^2\dfrac{\partial M}{\partial r}},$$

$$w = -\frac{Q\dfrac{\partial M}{\partial r}}{\dfrac{\partial \sigma}{\partial r}\dfrac{\partial M}{\partial z} - \tilde{N}^2\dfrac{\partial M}{\partial r}}, \tag{18}$$

where $M = rv + \frac{1}{2}r^2 f$ and v (or M) and σ are related through the thermal wind equation. Hence, according to Eq. (18), there can be no subsidence ($w = 0$) in the steady state when $Q = 0$, consistent with the results of Section 2.

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